A

COMPLETE TREATISE

ON

PRACTICAL MATHEMATICS:

INCLUDING

THE NATURE AND USE

OF

MATHEMATICAL INSTRUMENTS:

LOGARITHMIC TABLES.	LAND-SURVEYING.
TRIGONOMETRY.	GUNNERY.
MENSURATION OF HEIGHTS AND DISTANCES.	GAUGING.
——— OF SURFACFS & SOLIDS.	ARTIFICER'S MEASURING.
	MISCELLANEOUS EXERCISES.

WITH AN

APPENDIX ON ALGEBRA.

THE WHOLE CONDUCTED ON THE MOST APPROVED PLAN, WITH PROPER RULES, AND A VARIETY OF SUITABLE EXAMPLES TO EACH RULE.

Principally designed

FOR THE USE OF SCHOOLS AND ACADEMIES.

BY JOHN MACGREGOR,
TEACHER OF MATHEMATICS, EDINBURGH.

EDINBURGH:
PRINTED FOR BELL AND BRADFUTE,
AND
G. G. J. & J. ROBINSON, LONDON.

MDCCXCII.

Entered in Stationer's Hall.

TO

JOHN MACGREGOR OF MACGREGOR,

NOW CALLED AND KNOWN BY THE NAME OF

JOHN MURRAY,

COLONEL IN THE SERVICE, AND MILITARY AUDITOR-GENERAL

TO THE ARMY OF THE

HON. THE EAST INDIA COMPANY, IN BENGAL,

IN

TESTIMONY OF SINCERE ESTEEM,

THE

FOLLOWING SHEETS

ARE, WITH THE GREATEST RESPECT,

HUMBLY INSCRIBED,

BY HIS MOST DEVOTED,

AND VERY OBEDIENT SERVANT,

THE AUTHOR.

PREFACE.

NOTWITHSTANDING the many publications which have appeared on Mathematical ſubjects, and the great improvements that have been made in every department of the ſcience, a general treatiſe, on a cheap and accurate plan, ſeems as yet to be a deſideratum. Volumes have been wrote, not only upon every branch, but even upon particular parts of every branch. Theſe are more particularly adapted to the learned, and cannot all be purchaſed but at an extraordinary expence. This inconvenience has no leſs been felt by thoſe who have undertaken the charge of teaching, than by their pupils.

IT muſt be very diſagreeable to a teacher, before he can lead a claſs through a courſe of Practical Geometry, to make the ſtudents purchaſe a number of volumes on detached parts of the courſe: It is no

lefs perplexing to a fcholar to read them in order to retain what has been taught: Not to mention that the books fo purchafed may have a very different mode of expreffing the fame thing, which muft ftill encreafe the perplexity.

The author has frequently found, from his own experience, as well as from the report of others, that treatifes of this nature afford but imperfect materials for the exercife of youth. Hence teachers are under the neceffity of inventing what they efteem a proper fet of exercifes; and to this circumftance muft we chiefly afcribe the abfurd cuftom adopted by fome, viz, that of *teaching in their own way.*

With a view to obviate this inconvenience; to furnifh thofe who incline either to teach or learn practical Geometry, with a fyftem at once full and complete, for every purpofe in ordinary life; and to afford them this advantage at an eafy rate—the following treatife is refpectfully fubmitted to the attention of the Public. And, as the author has had fome years experience of its utility and convenience in the courfe of his practice, he thinks he can, with fome degree of confidence, recommend it as the only trea-

tife

tiſe that has yet appeared proper for being taught at academies and ſchools, the ingenious Dr Hutton's excepted. But though this work is excellent in its kind, its high price prevents its being generally uſeful;—an objection which, it is hoped, the following treatiſe will entirely remove.

The author is conſcious that there can *now* be but few claims to originality; yet he flatters himſelf that, even in this reſpect, he will not be found entirely deficient.

The arrangement is ſuch as ſeems beſt calculated for inſtruction—beginning with the ſimple rudiments, and, by gradual and eaſy ſteps, proceeding to that which is more complex, in ſuch order that what is prior paves the way for what is to follow. The greateſt care has been taken to ſelect the moſt important articles, and to introduce every neceſſary information, in ſo far as regards Menſuration.

To ſome of the problems two or more rules are annexed, and an example wrought at large to each, in ſuch a manner as to be intelligible to the moſt ordinary capacity; and, for exerciſe, copious ſets of unwrought

unwrought examples, with their anſwers, are inſerted; and, in order to render this work ſtill more acceptable, a great variety of miſcellaneous queſtions, with their anſwers, are propoſed, as a general exerciſe and concluſion to the work.

Upon the whole, It is hoped that, by means of the preſent treatiſe, the buſineſs of teaching and learning the practical part of Mathematics will be attended with leſs trouble and expence than formerly.

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CONTENTS.

Logarithmic

To

To

GEOMETRICAL DEFINITIONS.

1. A POINT is that which has no parts, neither length, breadth, nor thickneſs.

2. A line is length, without breadth or thickneſs.

3. A ſurface, or ſuperficies, is that which has length and breadth, without thickneſs.

4. A ſolid is that which has length, breadth, and thickneſs.

5. Points are the extremities of a line.

6. Lines are the boundaries of a ſuperficies.

7. Superficies are the boundaries of a ſolid.

8. A ſtraight line lies evenly between its extreme points. *See plate* 1. *fig.* 1.

9. Parallel lines are ſuch as are in the ſame plane, and keep the ſame diſtance, though produced ever ſo far.

10. An angle is the inclination of two lines of different directions, and meeting in a point. *See plate* 1. *fig.* 2.

N. B. When two lines, AB, and BC, meet in any point, B, the angle, may be expreſſed by three letters, putting B, the letter which is at the angular point, between the other two, thus: ABC, or CBA.

11. When one ſtraight line falls upon another ſtraight line, ſo as to make the adjacent angles equal to one another, each of them is a right angle and the ſtraight line which falls upon the other is perpendicular to it. *See plate* 1. *fig.* 3.

12. An angle which is leſs than a right angle, is called an acute angle. *See plate* 1. *fig.* 4.

13. An angle which is greater than a right angle, is called an obtuſe angle. *Plate* 1. *fig.* 5.

14. A ſigure is that which is incloſed by one or more boundaries.

15. A triangle is bounded by three ſtraight lines.

16. Quadrilateral ſigures are bounded by four ſtraight lines.

17. Polygons are bounded by more than four ſtraight lines.

18. An equilateral triangle is that which has all its ſides equal. *Plate* 1. *fig.* 6.

19. An iſoſceles triangle is that which has two of its ſides equal. *Plate* 1. *fig.* 7.

20. A ſcalene triangle is that whoſe ſides are all unequal. *Plate* 1. *fig.* 8.

21. A right-angled triangle is that which has one right angle. *Plate* 1. *fig.* 9.

22. The longeſt ſide of a right-angled triangle is called the hypothenuſe.

23. An acute angled triangle is that whoſe angles are all acute. *Plate* 1. *fig.* 10.

24. An obtuſe angled triangle is that which has one obtuſe angle. *See plate* 1. *fig.* 11.

25. A ſquare is a figure whoſe ſides are equal, and all its angles right angles. *See plate* 1. *fig.* 12.

26. An oblong is that whoſe parallel ſides only are equal, and all its angles right angles. *Plate* 1. *fig.* 13.

27. A rhombus is that which has all its ſides equal, but its angles not right angles. *Plate* 1. *fig.* 12.

28. A rhomboid is that whoſe oppoſite ſides only are equal, but its angles not right angles. *Plate* 1. *fig.* 13.

29. A trapezium is a four-ſided figure, which has none of its ſides parallel. *Plate* 1. *fig.* 14.

30. A trapezoid is a quadrilateral figure, with two of its ſides parallel. *Plate* 1. *fig.* 15.

31. A diagonal is a ſtraight line, which joins any two oppoſite angles of a quadrilateral figure. *Plate* 1. *fig.* 16.

32. A circle is a figure bounded by one curve line, which is called the circumference. *Plate* 1. *fig*. 17.

33. The centre of a circle is a point A, within the figure, equidiſtant from every point in the circumference.

34. The radius of a circle is the diſtance between the centre and circumference.

35. The diameter of a circle is a ſtraight line drawn through the centre, and terminated both ways by the circumference.

A is the centre.
AB the radius.
CD the diameter.

Note, The diameter is equal to twice the radius.

36. An arch is any part of the circumference.

37. The chord of an arch is a ſtraight line, drawn between the extremities of an arch.

38. The ſegment of a circle is that ſpace contained between the chord and arch of the ſame circle.

39. A regular polygon is that whoſe ſides are all equal.

40. An irregular polygon is a figure whoſe ſides are not all equal.

41. Polygons receive names according to the number of their ſides and angles.

Thus, A trigon has 3 ſides.
A tetragon 4
A pentagon 5
A hexagon 6
A heptagon 7
An octagon 8
An eneagon 9
A decagon 10, &c.

42. A mixed angle is that which is formed by one curved line meeting another ſtraight line.

43. A curve-lined angle is that which is formed by the meeting of two curved lines.

GEOMETRICAL PROBLEMS.

1. *To make an Equilateral Triangle upon a given line* AB.

FROM the centre A, at the diſtance AB, deſcribe an arch; and from the centre B, with the ſame radius, deſcribe another arch, cutting the former in C; join CA and CB. *Plate* 1. *fig*. 18.

PROBLEM II.

To biſect any given line AB *into two equal parts.*

Upon B for a centre, with a radius more than the half of AB, deſcribe an arch; and on A for a centre, with the ſame radius, deſcribe another arch, cutting the former in the points C,D: Join CD, and CD will biſect AB in the point E. *Plate* 2. *fig*. 19.

PROBLEM III.

To erect a perpendicular from a given point A, *in a given line* AB.

UPON any point, C for a centre, with the radius CA, deſcribe a circle, cutting the given line alſo in D; draw the diameter DCE, and join EA; then ſhall EA be the perpendicular. *Plate* 2. *fig*. 20.

PRO-

PROBLEM IV.

To erect a perpendicular from a given point A, *in a given line* AB, *another way.*

FROM the given point A, with any radius AC, describe an arch, cutting the given line in C; from C, with the same radius, cut the former arch in D and E; and upon these points as centres, describe arches cutting in R; join RA, and it will be the perpendicular required. *Plate* 2. *fig.* 21.

PROBLEM V.

From a given point C, *to drop a perpendicular upon a given line* AB.

On C, the given point, as centre, with any convenient distance, sweep an arch, cutting the given line in the points D,E; and from these points, with any radius more than half their distance, describe arches cutting each other either above or below the line; join the point of intersection and C, and it will be the perpendicular. *Plate* 2. *fig.* 22.

PROBLEM VI.

To bisect a given angle ABC.

From B the angular point as centre, describe an arch cutting the containing sides in D,F; on D,F for centres, describe arches of equal radii, cutting each other in E; join BE, which will bisect the angle ABC. *Plate* 2. *fig.* 23.

PROBLEM VII.

To trisect a right angle ABC.

FROM the angular point B, with any radius describe the arch AC; from C as centre, with the same radius, cut the arch AC in D; and from the centre A, with the same radius cut the arch AC in E; then join DB, EB, and they will trisect the angle. *Plate* 2. *fig*. 24.

PROBLEM VIII.

To draw a line parallel to a given line AB.

FROM any two points, D and E, describe arches of equal radii; draw CF to touch these arches, and CF will be parallel to AB.

PROBLEM IX.

To divide a line AB *into any number of equal parts.*

LET it be required to divide AB into seven equal parts, from A draw AD at any angle; and from B draw BC parallel to AD. On each of these parallel lines lay off as many equal parts as AB is to be divided into: Join the opposite points of division by straight lines, passing through AB, and they will divide AB as required. *Plate* 2. *fig*. 26.

PROBLEM X.

To find a fourth proportional to three given lines.

MAKE any angle ABC: Set off the first term from B to D, the second from D to A, the third from B to E; join DE, and through

through A draw AC parallel to DE; then EC will be the fourth proportional required. *Plate* 2. *fig.* 27.

PROBLEM XI.

To find a mean proportional between two given lines, AB, BC.

MAKE AC equal AB+BC; bisect the line AC in the point D, with the centre D, and radius DA, or DC, describe the semicircle AEC; erect the perpendicular BE, and it will be the mean proportional required. *Plate* 2. *fig.* 28.

PROBLEM XII.

To make a triangle with three given lines, AB, BC, CA.

TAKE any line AB for the base line; on the centre A, with the radius AC, describe an arch; on the centre B, with the radius BC, describe another arch, cutting the former in C; join CA and CB, and ABC is the triangle required. *Plate* 2. *fig.* 29.

PROBLEM XIII.

To measure any given angle from a line of chords.

FROM the angular point A, with the chord of 60° for a radius, describe an arch cutting the containing sides, produced, if necessary, in the points D,E; take the distance DE in your compasses, and apply it to the line of chords. Thus the quantity of any angle is obtained. *See fig.* 31. *plate.* 2.

Note, When the angle to be measured is obtuse, it must be taken off at twice. Thus, let the angle be 120°; first take 90° and 30°, or 60° and 60°, either of which will do.

PRO-

PROBLEM XIV.

To make an angle of any propoſed number of degrees, with a given line AB.

WITH the centre A, and radius 60° deſcribe an arch, cutting AB in C; then take the propoſed number of degrees in your compaſſes, and with this for a radius and centre C, deſcribe another arch, cutting the former in D; join AD, and the thing is done.

PROBLEM XV.

Upon a given line AB, *to deſcribe a ſquare.*

UPON the point AB erect a perpendicular AD, equal to AB; from the centre B, with the radius AB, deſcribe an arch; and on D as centre, with the ſame radius deſcribe another arch, cutting the former in the point C; join DC and BC, and it is done.

PROBLEM XVI.

To deſcribe a parallelogram of a given length and breadth.

Make BC perpendicular to AB; upon A, as centre and radius BC, deſcribe an arch; with the centre C, and radius AB, deſcribe another arch, cutting the former in D; then join DC and DA, and it is done.

PROBLEM XVII.

To deſcribe a circle in a given triangle, ABC.

BISECT any two of the angles with the lines AD and BD; from D drop a perpendicular DE, upon any one of the three ſides:

ſides; then upon D for a centre, and radius DE, deſcribe the circle, and it is done. *Plate* 2. *fig.* 34.

PROBLEM XVIII.

About any given triangle to deſcribe a circle.

BISECT any two ſides, BA, BC, by perpendiculars, DE, DF, with the centre D, and radius equal to the diſtance of any one of the angles, deſcribe a circle. *Plate* 3. *fig.* 35.

PROBLEM XIX.

To deſcribe a circle in or about a given ſquare.

DRAW two diagonals to the given ſquare; at the interſection D drop a perpendicular DE; on D as centre, with the radius DE, deſcribe a circle for the inſcribed circle; on D as centre, with half the diagonal for the radius, deſcribe another for the circumſcribed circle. *Plate* 3. *fig.* 36.

PROBLEM XX.

To deſcribe a ſquare in or about a given circle.

DRAW two diameters, AB, CD, at right angles to each other; join their extremities for the inſcribed ſquare ADBC, and, at the angular points of the inſcribed ſquare draw tangents, and they will form the circumſcribed ſquare, a b c d. *Plate* 3. *fig.* 37.

PROBLEM XXI.

To describe a circle through three given points, A,B,C, *which are not in the same straight line.*

JOIN the middle point to the other two; bisect their distances perpendicularly by straight lines meeting in D; then with the centre D, and distance of either of the three given points as radius, describe a circle, and it shall pass through A,B,C. *Plate* 3. *fig.* 38.

PROBLEM XXII.

A segment of a circle being given, to describe the circle of which it is the segment.

Draw AC the chord, and bisect it at right angles by BD; then join AB, and make the angle BAD equal to the angle ABD; draw AD, then with the point D as centre, and radius DA, DB or DC, describe the circle, and it is done. *Plate* 3. *fig.* 39. Or, take any three points in the segment, and bisect their distances, and the bisecting lines will intersect each other in the centre, as in prob. 21.

PROBLEM XXIII.

To describe a parallelogram that shall be equal to a given triangle, ABC.

Bisect BC in E; join AE, and draw CD equal and parallel to AE; then join AD, and AECD is the parallelogram required. *Plate* 3. *fig.* 40.

PRO-

PROBLEM XXIV.

To make a triangle equal to a given trapezium, ABCD.

Draw the diagonal BD, and through C draw CE parallel to BD, and meeting AD produced in E, join BE; and the triangle ABE is equal to the trapezium ABCD. *Plate* 3. *fig.* 41.

PROBLEM XXV.

To make a triangle equal to an irregular polygon, ABCDE.

Draw the diagonals, CA, CE, through B,D; draw DG and BF parallel to them, meeting the baſe AE, produced both ways in F and G; join CF and CG; ſo ſhall the triangle FCG be equal to the given figure ABCDE. *Plate* 3. *fig.* 42.

PROBLEM XXVI.

To divide the area of a given circle into any number of equal parts, by concentric circles, ſuppoſe into three equal parts.

Divide the ſemidiameter AC into three equal parts, in the points *a*,*b*; alſo biſect AC in the point x; and upon x as centre, with the radius Ax, or xC, deſcribe the ſemicircle AabC; and through the points of diviſion *a*,*b*, erect perpendiculars to meet the ſemicircle in *a*, and *b*; then, on C as centre, with the diſtances *b*,*a*, deſcribe circles, and it is done. *Plate* 3. *fig.* 43.

PROBLEM XXVII.

The fundamental projection of the diagonal ſcale.

Draw a line AE, of any convenient length; divide it into 12 equal parts; complete theſe into parallelograms of a conveni-

ent height, by drawing parallel lines; divide the altitude of these rectangles into ten equal parts, and, through each of these parts, draw parallel lines the whole length of the scale. Divide the first division AB into ten equal parts, also CD into as many, and connect these points of division by diagonal lines, and the scale is finished.

In taking measures from the diagonal scale---If the large divisions be reckoned units, the small divisions from A to B will be decimals. If the great divisions be 10, each of the small divisions is an unit; and if the great divisions be 100, then each of the small divisions is 10, and each division in the altitude is an unit.

If it were required to take off 456 from the scale; with one foot of the compasses on 4, extend the compasses till you have 4 of the great divisions and 5 of the lesser; then slide up your compasses with a parallel motion till you come to 6 on the parallel lines, and you have the extent required.

PROBLEM XXVIII.

The construction of the line of chords, sines, tangents, and secants.

About the centre C, with any convenient radius *, describe the semicircle ADB; erect the perpendicular CF, which will divide the semicircle into two quadrants, *viz.* AD, BD: divide the quadrant DB into nine equal parts, and upon the point B erect a perpendicular BT, then draw AD and BD.

On B as centre, transfer each of these divisions in the quadrant DB, to the straight line BD; then is BD a line of chords.

From the points 10, 20, 30, &c. in the quadrant BD, drop perpendiculars upon the diameter AB; transfer the perpendiculars

* The degrees are numbered from B to D.

culars to DC; ſo ſhall DC be a line of ſines, and CB a line of verſed ſines.

From the centre C, draw ſtraight lines through each diviſion in the quadrant BD, to meet the tangent BT; ſo ſhall BT be a line of tangents.

From the centre C, with the diſtances of each of the lines which meet the tangent, ſweep arches to cut CF; then ſhall CF be a line of ſecants.

If from the point A ſtraight lines be drawn to the ſeveral diviſions in the quadrant DB, they will divide the radius CD into a line of ſemitangents.

Again---Divide the quadrant AD into eight equal parts, and from A, transfer the diviſions to the line AD; then ſhall AD become a line of rhumbs, each diviſion anſwering to a point of the mariner's compaſs.

PROBLEM XXIX.

The angles, and one leg of a right-angled triangle being given, to conſtruct the figure, and find the other leg.

$$\text{Given}\left\{\begin{array}{l}\text{Angle A}=30^\circ\ 40'\\ \text{Angle C}=59^\circ\ 20'\\ \qquad \text{AB}=300\end{array}\right\}\text{Required BC.}$$

From the diagonal ſcale make AB 300; upon B erect a perpendicular of an indefinite length; and at the point A make an angle of 30° 40′; then draw the line AC, and it is done. If the angle at C be meaſured, it will be 59° 20′; and if the leg B C be applied to the ſame diagonal ſcale from which AB was taken, it will meaſure 177.9. *Plate* 3. *fig*. 43.

PRO-

PROBLEM XXX.

The hypothenuſe and all the angles being given, to find the legs.

Given $\left\{\begin{array}{r} AC=568 \\ \text{Angle } A=39^\circ\ 14' \\ \text{Angle } C=50^\circ\ 46' \end{array}\right\}$ Required AB, and BC.

Draw the line AB of an indefinite length, and draw AC equal 568, making with AB an angle of 39° 14′; and from C drop a perpendicular, cutting the base in B, and it is done: For if angle C be meaſured from the line of chords, it will meaſure 50° 46′; and if AB be meaſured from the ſame diagonal ſcale, it will meaſure 440, alſo BC 359.2. *Plate* 3. *fig.* 44.

PROBLEM XXXI.

The two legs of a right-angled triangle beeng given, to find the acute angles, and the hypothenuſe.

Given $\left\{\begin{array}{l} AB=150 \\ BC=160 \end{array}\right\}$ Req. angle A, angle C, and AC.

From any diagonal ſcale, draw AB=150, and from the ſame ſcale draw BC perpendicular to the former=160; join AC, and the triangle is conſtructed: for if angle A be meaſured from a line of chords, it will be 46° 51′; alſo angle C 43° 9′; and AC will be 219.3 equal parts. *Plate* 3. *fig.* 45.

PROBLEM XXXII.

The hypothenuſe and one of the legs being given, to find the acute angles and the other leg.

Given $\left\{\begin{array}{l} AC=150 \\ BC=\ \ 90 \end{array}\right\}$ Required ang. C, ang. A, and AB.

Draw

Draw the baſe AB, upon B erect the perpendicular BC equal 69; take 150 from the ſame ſcale, and with the centre C, and radius 150, deſcribe an arch to cut the baſe in A; join AC, and it is done: For angle A will meaſure 27° 23′, and angle C 62° 37′, and the baſe BG 133 equal parts. *Plate* 3. *fig*. 46.

PROBLEM XXXIII.

Given two angles of an oblique angled triangle, and the ſide oppoſite to one of them; to find the other ſides.

$$\text{Given}\left\{\begin{array}{l}\text{Angle C } 52^\circ\ 15'\\ \text{Angle A } 59^\circ\\ \text{A B } 276.5\end{array}\right\}\text{Required AC and BC.}$$

Find the ſupplement of the ſum of the two given angles, thus: $59^\circ\ 0' + 52^\circ\ 15 = 111^\circ\ 15'$.

And from 180° ſubtract 111° 15′, the remainder will be, 68° 45′; then draw AB equal 276.5: Draw AC, making angle A 59°, and from B draw BC, making angle B 68° 45′, and meeting AC in the point C, and it is done: then ſhall AC meaſure 325.9, and BC 299.7. *Plate* 3. *fig*. 47.

PROBLEM XXXIV.

Two ſides of an oblique angled triangle, and the angle oppoſite to one of them being given, to find the other angles and the third ſide.

$$\text{Given}\left\{\begin{array}{l}\text{AB}=26\\ \text{AC}=39.42\\ \text{Ang. B}=91^\circ\ 15'\end{array}\right\}\text{Required ang. A ang. C and BC.}$$

Draw the baſe AB equal 26, and at the point B make an angle of 91° 15′ by BC; then on A as centre, with the radius

dius 39.42, deſcribe an arch cutting BC in C, and join AC, and it is done.

So ſhall angle A meaſure 47° 30′, and angle C 41° 15′; alſo BC 29.07 equal parts. *Plate* 3. *fig*. 48.

PROBLEM XXXV.

Two ſides, and the contained angle of any triangle being given, to find the remaining angles, and the third ſide.

$$\text{Given}\left\{\begin{array}{c}AC=60\\ BC=50\\ \text{Ang. } C=45^\circ\end{array}\right\}\text{Required ang. A ang. B and AB.}$$

Draw AC equal 60, and BC equal 50 equal parts, meeting in C at an angle of 45°; then join AB, and it is done: For if you take AB in your compaſſes, it will meaſure 43.1 on the ſame ſcale of equal parts; alſo angle A will meaſure 55° 7′, and angle B 79° 53′, from the line of chords. *Plate* 3. *fig*. 49

LOGA-

LOGARITHMS.

LOGARITHMS are a ſet of artificial numbers, and may be conſidered as the indices of a ſeries of Geometrical proportionals, and are ſo related to the natural numbers, that the addition of Logarithms is equivalent to the multiplication of the correſponding numbers; alſo, the ſubtraction of logarithms is the ſame as the diviſion of the correſponding numbers; their difference being the logarithm of the quotient.

Here it may be obſerved, that common numbers are a ſeries whoſe differences are equal; ſuch as, 2, 4, 6, 8, 10, &c. where the common difference is 2, and are called a ſeries in arithmetical progreſſion.

Alſo a ſeries of numbers whoſe ratios are equal, are called a ſeries in Geometrical progreſſion; ſuch as, 2, 4, 8, 16, 32, 64, &c. the common ratio being 2.

The following table will, in ſome meaſure, illuſtrate theſe general obſervations.

Note, Column A is a ſeries in arithmetical progreſſion; the other columns are in Geometrical progreſſion, the common ratios being 2, 3, 4, 5, 10.

TABLE.

A	B	C	D	E	F
0	1	1	1	1	1
1	2	3	4	5	10
2	4	9	16	25	100
3	8	27	64	125	1000
4	16	81	256	625	10000
5	32	243	1024	3125	100000
6	64	729	4096	5625	1000000
7	128	2187	16384	78125	10000000

Now, let it be required to multiply 9 by 81, the product will be 729.

The terms in column A, correſponding to the factors, are 2 and 4; and which being added together, will give 6; over againſt 6 in column A, is 729, the product in column C.

Again—Let it be required to divide 78125 by 125, the quot will be 625. By the table it may be performed thus: Find the numbers

numbers in column A, anſwering to 78125, the dividend, and to 125 the diviſor (both in column E); ſubtract the leſſer from the greater, and over-againſt their difference in column A is 625 the quotient in column E.

By extending the foregoing table, many operations, both in multiplication and diviſion might be facilitated, provided the ſame numbers occur in the table; but as this ſeldom happens, the uſe of ſuch a table will be confined to a few inſtances. In order, therefore, to extend its utility, we ſhall ſhew a method by which this inconveniency is removed.

There was a method formerly in uſe in making logarithms: The firſt inventors choſe a ſet of numbers in arithmetical progreſſion, that ſhould anſwer to a ſet of geometrical ones; (this is entirely arbitrary;) and they choſe the decuple geometrical progreſſion as the moſt convenient, correſponding to the arithmetical ſeries 1, 2, 3, 4, 5, 6, 7, &c., as the ſimpleſt, whoſe common difference is, 1. as follows:

Arith. progreſſion, or log.	0,	1,	2,	3,	4.
Geo. prog. or numbers	1,	10,	100,	1000,	10000.

Hence it appears, that the logarithm of 1 is 0, of 10 is 1, of 100, is 2, &c.: but ſeveral numbers may be interpoſed between each of theſe; for, between 1 and 10 are 2, 3, 4, 5, 6, 7, 8, 9; to them alſo might indices be adapted, ſuited to each term between 1 and 10, conſidered in geometrical progreſſion. Likewiſe indices may be found to each term interpoſed between any two terms whatever, in geometrical progreſſion.

It is plain, that the indices to all the numbers under 10 is leſs than 1; that is, they are ſo many decimal parts; likewiſe, that the indices of numbers between 10 and 100 are 1 of an integer, and ſo many decimal parts, and ſo on of numbers greater than 100.

The integral part is commonly called the index, and the decimal part the logarithm.

 But

But ſince the above method is ſo intolerably laborious, the more learned mathematicians have thought of a more compendious one, by the menſuration of hyperbolic ſpaces, contained between the portions of an aſymptote, and right lines perpendicular to it and the curve of an hyperbola; but ſuch computations depend on principles that require the higher parts of Geometry, and cannot, therefore, according to our plan, be introduced here.

We ſhall ſubjoin the proceſs for obtaining the logarithm of 9, as derived from progreſſion.

Geo.

	Geo. pro.	Log.
A	1.00000	0.00000
C	3.16228	0.50000
B	10.00000	1.00000
B	10.00000	1.00000
D	5.62341	0.75000
C	3.16228	0.50000
B	10.00000	1.00000
E	7.49894	0.87500
D	5.62341	0.75000
B	10.00000	1.00000
F	8.65964	0.93750
E	7.49894	0.87500
B	10.00000	1.00000
G	9 30572	0.96875
F	8.65964	0.93750
F	8.65964	0.93750
H	8.97687	0.95312
G	9.30572	0.96875
H	8.97687	0.95312
I	9.13982	0.96094
G	9.30572	0 96875
H	8.97687	0.95312
K	9.05798	0.95703
I	9.13982	0.96094
H	8.97687	0.95312
L	9.01733	0.95508
K	9.05798	0.95703
H	8.97687	0.95312
M	8.99708	0.95410
L	9.01733	0.95708

	Geo. pro.	Log.
M	8.99708	0.95410
N	9.00720	0.95459
L	9.01733	0.95508
M	8.99708	0.95410
O	9.00214	0.95435
N	9.00720	0.95459
M	8.99708	0.95410
P	8.99961	0.95422
O	9.00214	0.95435
P	8.99961	0.95422
Q	9.00087	0.95428
O	9.00214	0.95435
P	8.99961	0.95422
R	9.00024	0.95425
Q	9.00087	0.95428
P	8.99961	0.95422
S	8.99992	0.95424
R	9.00024	0.95425
S	8.99992	0.95424
T	9.00008	0.95425
R	9 00024	0.95425
S	8.99992	0.95424
U	9.00000	0.95424
T	9.00008	0.95424

Here becauſe the 9 lies between 1=A, and 10.B, find a mean proportional C between them, and the logarithm of the ſame, is half the ſum of the two laſt logarithms. In like manner, is found, a mean proportional D between B and C; likewiſe the logarithm of D is half the ſum of the logarithms of B and C; ſo in the 18th ſtep of this proceſs, the logarithm of 9, is found to be 0,954242.

When the logarithms of prime numbers are thus calculated, the buſineſs becomes eaſier; for the logarithms of compoſite numbers may be obtained, by adding the logarithms of their component parts. Thus, the logarithm of 15 may be found, by adding the logarithm of 3 and 5 together; for 3+5=15, and ſo on of any other compoſite number.

The logarithms of roots are raiſed to any given power, by multiplying them by the exponent of the power, *& vice verſa.*

PROBLEM I.

To find the logarithm of any given number from the tables.

IT is uſual to divide logarithmic tables into 10 columns: In the left hand column, are the natural numbers between 100 and 1000, and at the top and bottom are marked, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9.

If the natural number is leſs than 100, its logarithm is found in the firſt page. If it exceed 100, and is leſs than 1000, the number is found in the left hand column, marked No. and its logarithm is found oppoſite to it in the adjacent column, under 0; but if the number exceed 1000, and is leſs than 10000, find the three higheſt figures in the column of numbers, and in the ſame line, titled by the unit at the top, is the logarithm required. The logarithm of 1786, may be found from the tables: thus, in the column No. look for 178, and in

in the ſame line, under 6, (the units place at he top) is 3.25188, the logarithm required.

Note, In every caſe, the index is leſs by 1, than the number of places; and, on the contrary, the number of places is greater by unity than the index.

The logarithms of mixed numbers, are found the ſame as if they were integers; but the integer alone determines the index.

Decimal fractions have negative indices, which are to be added when the logarithms are ſubtracted, and ſubtracted when the logarithms are added.

PROBLEM II.

To find the natural number correſponding to a given logarithm.

LET the given logarithm be 2.75976, it is required to find its correſponding number.

Look for the given logarithm, neglecting the index, and againſt it on the margin, you find 575, and 1 at top, which is 5751; but the index being 2, the integer muſt therefore conſiſt only of three places; and, by pointing off towards the right hand for decimais, the number will be 575.1.

It often happens, that the exact logarithm cannot be found in the tables, in which caſe we take the neareſt to it.

PROBLEM III.

To find the product of two given numbers by logarithms.

Rule, Add the logarithms of both factors together, and their ſum is the logarithm of the product.

Ex. Re-

Ex. Required the product of 15, multiplied by 70.

The logarithm of 15, is 1.17609.
The logarithm of 70, is 1.84510.

The log. of 1050, the product, 3.02119.

PROBLEM IV.

To find the quotient of two given numbers by logarithms.

Rule, From the logarithim of the dividend, subtract the logarithm of the divisor, and the remainder is the logarithm of the quotient.

Ex. Required the quotient of 425, divided by 15.

The log. of 425, is 2.62839.
The log. of 15, is 1.17609.

The log. of 28.33, the quotient 1.45230.

PROBLEM V.

To find the square, cube, or any higher power of a given number, by logarithms.

Rule, Multiply the logarithm of the root, by the exponent of the power, and the product is the logarithm of the power required.

Ex. Required the cube of 12.

The log. of 12, is 1.07918.
3.

The log. of $12^3 = 1728 = 3.23754$.

PROBLEM VI.

To extract the ſquare, cube, biquadrate, &c. root of a given number by logarithms.

Rule, Divide the logarithm of the given number, by the exponent of the power, and the quotient will give the logarithm of the root.

Ex. Required the cube root of 1728.

The logarithm of 1728, is 3.23754, which, if divided by 3, will quot 1.07918, the logarithm of 12 the root.

PROBLEM VII.

Three numbers being given, to find a fourth proportional to them.

Rule, From the ſum of the logarithms of the ſecond and third terms, ſubtract the logarithm of the firſt, and the remainder is the logarithm of the anſwer.

Ex. If 14 yards cloth, coſt 7 l., what will $70\frac{1}{2}$ yards coſt at that rate?

The log. of 14,	is	1.14613	firſt term.
of 7,	is	0.84510	ſecond term.
of 70.5,	is	1.84819	third term.
		2.69329.	ſum of the 2d and 3d terms.
Log. of 35.25,	is	1.54716.	remainder.
or 35 L. 5s.			

PROBLEM VIII.

To find a mean proportional between any two numbers by logarithms.

Rule, Add the logarithms of the two given numbers together, and half their ſum is the logarithm of the mean proportional.

Ex. Required a mean proportional between 8 and 32.

The log. of 8, is 0.90309.
The log. of 32, is 1.50515.

2) 2.40824.

The log. of 16, the mean prop. 1.20412.

for 8 : 16 :: 16 : 32.

PROBLEM IX.

To find the logarithm of the Sine, Tangent, Secant, belonging to any number of degrees and minutes required.

Rule, If the degrees required, be leſs than 45°, ſeek the degrees on the top, and the minutes in the left hand column titled M, in the ſame line under the propoſed name at the top, ſtands the ſine, tangent and ſecant required. If the degrees given, exceed 45°, ſeek the degrees at the bottom, and the minutes in the right hand column marked M, and the propoſed name at the bottom.

Note, If the degrees at the top and the minutes in the left hand column, be added to the degrees at the bottom and minutes in the right hand column, the ſum will be 90°. Hence they are complements of each other.

TRIGO-

TRIGONOMETRY.

PLANE Trigonometry is that part of Geometry, which teaches how to meaſure the ſides and angles of plane triangles. It is divided into right-angled and oblique-angled trigonometry.

The circumference of any circle, is divided into 360 equal parts, called degrees, and each degree into 60 equal parts, called minutes, and each minute into 60 equal parts, called ſeconds, and ſo on.

Note, Degrees are frequently marked °, and minutes ′. Thus, 30 degrees, 14 minutes, are marked 30°, 14′.

A ſemi-circle contains 180°, and a quarter of a circle or quadrant, 90°. Thus the arch ABD, is 180°, and BD is 90°.

DEFINITIONS.

1. THE complement of an arch, is what it wants of 90°, or of a quadrant. Thus, the complement of the arch ED, is EB. *See fig.* 50. *Plate* 3.

 2, The

2. The ſupplement of an arch, is what it wants of a ſemi-circle, Thus the ſupplement of the arch ED, is EBA.

Note, An arch and an angle meaſure each other.

3. A line drawn through one extremity of an arch perpendicular upon the diameter paſſing through the other extremity, is called the ſine of that arch. Thus, EH is the ſine of the arch ED, or of the angle ECD.

4. The ſegment of the diameter intercepted between the ſine and extremity of an arch, is called the verſed ſine of that arch. Thus, HD is the verſed ſine of the arch ED, or of the angle ECD.

5. A ſtraight line paſſing through D, one extremity of an arch, and meeting the diameter produced through E, the other extremity, is called the tangent of the arch. Thus, GD is the tangent of the arch ED, or of the angle ECD.

6. A ſtraight line drawn from the centre, through one extremity of an arch, meeting the tangent drawn through the other extremity, is called the ſecant of that arch. Thus, CG is the ſecant of the arch ED, or of the angle GCD.

Corollary 1. The ſine, tangent and ſecant of any arch, is the ſine, tangent, or ſecant of its ſupplement.

BK is the tangent, CK the ſecant, and EL the ſine of the arch BE, according to definitions 3, 5, and 6, but BE is the complement of the arch ED; therefore LE, BK and CK, are the ſine complement, tangent complement, and ſecant complement of the arch ED. But for brevity's ſake, they are called the co-ſine, co-tangent, and co-ſecant of the arch ED, or of the angle ECD.

Corol.

Corol. 2. Since the triangle CEH and GCD are ſimilar, CH : CD (=CE) : : CE : CG. Hence,

In words, The radius is a mean proportional between the co-ſine and ſecant of any arch.

Corol. 3. Becauſe the triangles BKC, GCD are ſimilar, GD : DC (=CB) : : CB : BK. Hence,

In words, The radius is a mean proportional between the tangent and co-tangent of any arch.

Note, The leaſt poſſible ſecant, the tangent of 45°, and the ſine of 90°, are each of them equal to the radius.

In every triangle, there are ſix things to be conſidered, *viz.* three ſides and three angles.

All the angles in a triangle, are together equal to two right angles, or 180°. If, therefore, two angles of a triangle are given, the third is alſo given, for it is found, by ſubtracting the ſum of the other two from 180°.

When one angle of a triangle is given, the ſum of the other two may be found, by ſubtracting the given angle from 180°.

When one angle of a triangle is a right-angle, the other two are acute, and are together equal to one right-angle, and conſequently are complements of each other.

PROPO-

PROPOSITION.

IN any right angled plane triangle, if the hypothenuſe be made radius, the legs become the ſines of the oppoſite angles : but if either of the legs be made radius, the other leg becomes the tangent of the oppoſite angle, and the hypothenuſe becomes the ſecant of the ſame angle. Fig. 51. plate 3.

LET ABC be a right angled triangle, if the hypothenuſe BC be made radius, the ſide AC will be the ſine of the oppoſite angle ABC ; and if either ſide, BA be made radius, the other leg AC will be the tangent of the oppoſite angle ABC, and the hypothenuſe BC, the ſecant of the ſame angle.

With the centre B, and radii BC, BA, deſcribe two arches CD, EA, meeting BC, BA in E and D. Since CAB is a right angle, BC being radius, AC is the ſine of the angle ABC, by definition 3, and BA being radius, AC is the tangent, and BC the ſecant of the angle ABC, by def. 5, 6.

Since circles are to one another as their radii, ſimilar arches of the ſame circles will be in the ſame proportion ; therefore, the ſines, tangents, and ſecants of ſimilar arches, that is, of equal angles, are as their radii ; conſequently, the tabular radius is to the tabular ſine, tangent or ſecant of either of the acute angles of a right angled triangle, as the radius of the given triangle, is to the ſine, tangent or ſecant, in the ſame triangle.

And, becauſe any one of the three ſides may be called the radius, any of the ſides required, may be obtained by three analogies or varieties.

N. B. All the varieties which can occur in the ſolution of right angled triangles, may be comprehended under two problems.

Firſt,

First, When all the angles and one side are given, to find the other two sides.

2*d*, When two sides and the right angle are given, to find two acute angles and the third side.

We come now more fully to shew how each of these problems are solved by logarithms.

PROBLEM I.

CASE I. *The angles and one of the legs given, to find the hypothenuse and the other leg. Plate* 3. *fig.* 43.

Ex. 1. In the triangle ABC, right-angled at B, suppose AB 300 equal parts, as feet, yards, miles, &c., and the angle at A 30° 40′, (and consequently the angle at C 59° 20′) Required the sides BC, AC.

Variety 1. making AB rad. BC becomes the tangent, and AC the secant of angle A. Whence arise the following proportions:

To find BC.		To find AC.	
radius 90° - - -	10.00000	As rad. 90° - -	10.00000
is to AB 300 -	2.47712	is to AB 300 -	2.47712
So tang. ang. A 30° 40′	9.77303	So sec. A 30° 40′ -	10.06543
To BC 177.9 - -	2.25015	To AC 348.8 - -	2.54255

Variety 2. making BC rad. BA becomes the tangent, and AC the secant of the angle at C. Hence the following proportions:

Variety

To find BC.		To find AC.	
As tang. ang. C 59° 20′	10.22697	As tan. C 59° 20′ -	10.22697
is to AB 300 -	2.47712	is to AB 300 -	2.47712
So rad. 90° - -	10.00000	So sec. C 59° 20′ -	10.29239
			12.76951
To BC 177.9 -	2.25015	To AC 348.8 - -	2.54254

Variety 3. Making AC rad. BC becomes the sine of angle A, and AB sine angle C. Hence the following proportions.

To find BC.		To find AC.	
As sine C 59° 20′ -	9.93457	As sine C 59° 20′ -	9.93457
is to AB, 300, -	2.47712	is to AB 300 -	2.47712
So sine A 30° 40′ -	9.70761	So is rad 90° - -	10.00000
	12.18473	To AC 348.8 - -	2.54255
To BC 177.9 - -	2.25016		

Ex. 2. In the right angled triangle ABC, right angled at B, suppose BC 4876 equal parts, angle A 53° 3′, and angle C 36° 57′. Required AB; AC. *Fig* 52. *plate* 4.

Construction. Draw AB, upon B erect the perpendicular BC 4876, and at C draw CA, making an angle of 36° 57′ with BC; then shall angle A be 53° 3′, AB 3668, and AC 6101 equal parts.

Variety 1. Making AB rad. BC becomes the tangent, and AC the secant of angle A. Hence arise the following proportions.

To find AB.		To find AC.	
As tan. ang. A 53° 3′	10.12367	As tan. A 53° 3′ -	10.12367
to BC 4876 -	3.68806	to BC 4876 -	3.68806
So rad. 90 - - -	10.00000	So sec. A 53° 3′ -	10.22104
			13.90910
To AB 3668 -	3.56439	To AC 6101 - -	3.78543

Variety

Variety 2. Making BC rad. AB becomes the tangent, and AC the ſecant of angle C. Hence ariſe the following proportions.

To find AB.		To find AC.	
As rad. 90° - -	10.00000	As rad. 90° - -	10.00000
Is to BC 4876, -	3.68806	Is to BC 4876, -	3.68806
So tan. C 36° 57′ -	9.87633	So ſec. C 36° 57′ -	10.09737
To AB 3668 - -	3.56439	To AC 6101 - -	3.78543

Variety 3. Making AC rad. AB becomes the ſine of the angle at C, and BC the ſine of the angle at A. Whence ariſe the following proportions.

To find AB.		To find AC.	
As ſine A 53° 3′ -	9.90263	As ſine A 53° 3′ -	9.90263
Is to BC 4876, -	3.68806	is to BC 4876 -	3.68806
So ſine C 36° 57′ -	9.77896	So is rad. 90° - -	10.00000
	13.46702	To AC 610.1 -	3.78543
To AB 366 8 - -	3.56439		

Case II.

The angles and the hypothenuſe being given to find the legs.

In the triangle ABC, right angled at B. ſuppoſe AC 568 equal parts, angle A 39° 14′, and angle C 50° 46′. Required AB, BC. *Fig.* 44. *plate* 3.

Variety 1. Making AB radius, BC becomes the tangent, and AC the ſecant of A. Whence the following proportions.

To find BC.		To find AB.	
As ſec. A 39° 14′ -	10.11094	As ſec. A 39° 14′ -	10.11094
Is to AC 568, - -	2.75435	Is to AC 568 - -	2.75435
So is tan. A 39° 14′	9.91198	So is rad. 90 - -	10.00000
	12.66633	To AB 440 - -	2.64341
To BC 359.2 - - -	2.55539		

Variety 2. Making BC rad. AB becomes the tangent, and AC the ſecant of the angle C. Whence the following proportions.

To find BC.		To find AB.	
As ſec. C 50° 46′ -	10.19895	As ſec. C 50° 46′ -	10.19895
is to AC 568 -	2.75435	is to AC 568 -	2.75435
So is rad. 90 -	10.00000	So is tan. C 50° 46′	10.08802
To PC 359.2 -	2.55540		12.84237
		To AB 440 -	2.64342

Variety 3. Making AC rad. BC becomes the ſine of angle A, and AB the ſine of the angle at C. Whence the following proportions.

To find BC.		To find AB.	
As rad 90° - -	10.00000	As rad. 90° - -	10.0000
is to AC 568 -	2.75435	is to AC 568 -	2.75435
So is ſine A 39° 14′	9.80105	So is ſine C 50° 46	9.88906
To BC 359.2 - -	2.55540	To AB 440 - -	2.64341

PROBLEM II.

Two ſides and the right angle given, to find the acute angles, and the third ſide. Fig. 46. plate 3.

In the triangle ABC, right angled at B, ſuppoſe the hypothenuſe AC 150, and the leg. CB 69. Required the angles A and C and the leg. BA.

Variety

Variety 1. Making AC rad. then BC becomes the ſine of the angle at A. Whence, the following proportion.

To find angle A.		To find angle C.
* As AC 150 -	2.17609	Since the two angles of a right angled triangle, are complements of each other, angle C may be found, (by ſubtracting angle A=27° 23′ from 90°,) to be 62° 37′
is to rad. 90°	10.00000	
So BC 69 - -	1.83885	
To ſine A 27° 23′	9.66276	

Variety 2. Making BC rad. then AC becomes the ſecant of the angle at C. Whence the following proportion.

Te find angle C.		To find angle A.
As BC 69 - - -	1.83885	If from 90° you ſubtract 62° 27′, the remainder 27° 23′ will give angle A.
is to radius 90°	10.00000	
So is AC 150 -	2.17609	
To ſec. C. 62° 37′	10 33724	

Now there are other three varieties to find AB.

Variety 1. making AB radius, to find AB.

To find AB.		To find AB.	
As ſec. A, 27° 23′	10.05161	As tang. ang. A 27° 23′	9.71431
is to AC 150 -	2.17609	is to BC 69 - -	1.83885
So is rad. 90 - -	10.00000	So is rad. 90 - -	10.00000
To AB 133.2 -	2.12448	To AB 133.2 -	2.12454

 To

* When an angle is required, the length of a line is made the firſt and third terms, alſo a ſide that is neither given nor required, cannot be admitted into the proportion, or made radius.

To find AB.		To find AB.	

Variety 2. making BC rad. to find AB.

As rad. 90 -	10.00000	As sec. C 62° 37′ -	10.33730
to BC 69 -	1.83885	to AC 150 - -	2.17609
So tan. C 62° 37′ -	10.28569	So tan. C 62° 37′ -	10.28569
To AB 133.2 - -	2.12454		12.46178
		To AB 133.2 - -	2.12448

Variety 3. Making AC rad. to find AB.

To find AB.		To find AB.	
As rad. 90 -	10.00000	As sine A 27° 23′ -	9.66270
to AC 150 -	2.17609	to BC 69 -	1.83885
So sine ang. C 62° 37′	9 94839	So Sine C 62° 37′	9 94839
To AB 133.2 - -	2.12448		11.78724
		To AB 133 2 - -	2.12454

OBLIQUE

OBLIQUE ANGLED TRIGONOMETRY.

THE solution of all plane triangles, may be deduced from the three following theorems.

THEOREM I.

In any plane triangle, the sides are in the same proportion, as the sines of the opposite angles. Fig. 53. plate 4.

Dem. From the angles A and B, draw BE and AD perpendicular, to the opposite sides, BC and AC produced if necessary. Because the triangles ADB, AEB, are right angled triangles, the side AD becomes the sine of the angle ABD, and BE the sine of the angle BAE; if AB the hypothenuse, common to both the triangles, be made the radius; but the two triangles ADC, BEC, have each a right angle at D and E, likewise the common angle ACB, therefore, they are similar, and consequently, BC is to CA, as BE is to AD; that is, the sides are in the same proportion as the sines of the opposite angles.

THEO-

Theorem II.

In any plane triangle, the ſum of any two ſides is to their difference : as the tangent of half the ſum of the angles at the baſe, is to the tangent of half their difference. Fig 54. plate 4.

Dem. Let ABC be a plane triangle, AB+BC : AB—BC :: tan. $\frac{\text{ang. A+Ang. C}}{2}$: tan. $\frac{\text{ang. C—ang. A}}{2}$, upon A as centre with AB the longeſt ſide for a radius, deſcribe a circle, meeting AC produced in E and F; produce BC to D, join DA, FB, EB, and draw FG parallel to BC, meeting EB in G.

The angle EAB is equal to the ſum of the angles at the baſe, and the angle EAB at the centre is double the angle EFB at the circumference, therefore, EFB is half the ſum of the angles at the baſe; but the angle ACB, is equal to the angles CAD, and ADC, or ABC together, therefore, FAD is the difference of the angles at the baſe, and FBD is half that difference, but FBD is equal to the alternate angle BFG; ſince the angle FBE in a ſemi-circle, is a right angle, FB being radius, BE, BG will be tangents of the angles EFB, BFG; but it is plain, that EC is the ſum of the ſides, BA and AC, alſo that CF is their difference; and ſince EG and BC are parallel, EC : CF as EB : BG, that is, the ſum of the ſides is to their difference : as the tangent of half the ſum of the angles at the baſe, is to the tangent of half their difference.

THEO-

Theorem III.

In a plane triangle, the base is to the sum of the sides, as the difference of the sides, is to the difference of the segments of the base, made by the perpendicular upon it from the vertex. Fig. 55. plate 4.

Let ABC be a plane triangle, if from B the vertex a perpendicular BD be dropped on the base, AC : AB+BC : : BC—AB : DC—AD. Upon B as centre with BC, the greater side for a radius, describe a circle meeting BA and CA, produced in F and E. It is manifest, that AF is the difference of the sides, and that EA is the difference of the segments of the base, for ED and DC are equal, and AG is the sum of AB and BC; but, because FG and EC cut each other within a circle in the point A, the rectangle contained by the segments of the one, is equal to the rectangle contained by the segments of the other, that is, FA×AG=EA×AC, and by Euclid vi. 16. AC : AG : : FA : EA. Wherefore, in any plane triangle, the base is to, &c.

Note, The sum and difference of two magnitudes being given, to find each of them.

Rule, To half the sum, add half the difference, the sum will be the greater, and from half the sum, subtract half the difference, the remainder will be the less.

In plane triangles may be given,

The three angles and one side.	to find the other parts.
Two sides and the angle opposite to one of them.	
Two sides and the angle contained between them.	
The three sides.	

PRO-

oblique Ang.d

PROBLEM I.

The angles and one side given, to find the two remaining sides. *Plate* 3. *fig*. 47.

Ex. 1. Given $\begin{cases} \text{Ang. C } 52^\circ 15 \\ \text{Ang. A } 59^\circ \\ \text{AB } 276.5 \end{cases}$ Req. AC and BC

$180 - 52^\circ 15 + 59^\circ = 68^\circ 45' = $ an. B.

To find AC.

As sine ang. C 52° 15′	9.89801
is to A B 276.5 -	2.44170
So is sine an. B 68° 45′	9.96942
	12.41112
To AC 325.9 - -	2.51311

To find BC.

As sine ang. C 52° 15′	9.89801
is to AB 276.5 -	2.44170
So is sine ang. A 59°	9.93307
	12.37477
To BC 299.8 - -	2.47676

EXAMPLE 2. *plate* 4. *fig*. 56.

Given $\begin{cases} \text{AB} = 2600 \\ \text{Ang. A } 47^\circ 30' \\ \text{Ang. C } 41^\circ 15' \end{cases}$ Req. AC BC.

$180 - 47^\circ 30 + 41^\circ 15' = 91^\circ 15'$ ang. B.

To find AC.

As sine ang. C 41° 15′	9.81911
is to AB 2600	3.41497
* So is sine ang. B 91° 15′	9 99990
	13.41487
To AC 3942	3.59576

To

* The sine, tangent, secant, &c. of any arch, is the sine, tangent, secant, &c. of its supplement. Hence the sine of 91° 15′ may be obtained thus, 180°—91° 15′=88° 45=the supplement of 91° 15′.

To find BC.

As sine ang. C 41° 15′	9.81911
is to AB 2600	3.41497
So is sine ang. A 47° 30′	9.86763
	13.28260
To BC 2907	3.46349

PROBLEM II.

Two sides and the angle opposite to one of them being given, to find the other angles and the third side, Fig. 48. plate 3.

Ex. Given $\left\{\begin{array}{lll} AB & & 26 \\ AC & - & 39\ \ 42 \\ ang.\ B & & 91° \ 15' \end{array}\right\}$ Req. ang. A, ang. C and BC.

180°—ang. C+ang. B.=ang. A 47° 30′.

To find angle C.

As AC 39.42 -	1.59572
is to sine ang. B 91° 15′	9.99990
So is AB 26 - -	1.41497
	11.41487
To sine C 41° 15′	9.81915

To find BC.

As sine ang. C 41° 15′	9.81911
is to AB 26 - -	1.41497
So is sine ang. A 47° 30′	9.86763
	11.28260
To BC 29.07 -	1.46349

PROBLEM III.

Two sides and the angle contained being given, to find the remaining angles, and the third side. Fig. 49. plate 3.

Ex. Given $\left\{\begin{array}{lll} AC & - & 60 \\ BC & - & 50 \\ ang.\ C & & 45° \end{array}\right\}$ Req. ang. A, ang. B. and AB.

To

To find the angles.	
As AC+BC 110 - - - -	2.04139
to AC—BC 10 - - - - -	1.00000
So is tan. $\frac{\text{ang. B}+\text{ang. A}}{2}$ 67° 30′	10.38278
To tang. $\frac{\text{ang. B}-\text{ang. A}}{2}$ 12° 23′	9.34139

To find ½ sum ang. A & ang. B.

$$\begin{array}{r} 180 \\ 45 \\ \hline 2)135 \text{ sum.} \\ \hline 67^\circ\ 30' = \tfrac{1}{2} \text{ sum.} \end{array}$$

To ½ sum - - - - - - - -	67° 30′
Add ½ difference - - - - -	12° 23′
The greater - - - - -	79° 53′
From ½ sum - - - - - - -	67° 30′
Subtract ½ difference - - -	12° 23′
The less - - - - - - - - -	55° 7′

Now the greater angle is subtended by the greater side; therefore, angle B opposite 60, will be 79° 53′, and angle A 55° 7′.

To find AB.

As sine ang. A 55° 7′	9.91398
is to BC 50 - -	1.69897
So sine ang. C 45° -	9.84948
	11.54845
To AB 43.1	1.63447

EXAMPLE II. *Fig.* 56. *plate* 4.

Given $\left\{\begin{array}{ll} \text{AB} & 180 \\ \text{BC} & 200 \\ \text{ang. B} & 69^\circ \end{array}\right\}$ Required angles A, C, and AC.

To

To find the sum of ang. A & C.	To find the angles A and C.	
180°	As CB+AB 380 - -	2.57978
69	is to CB—AB 20 -	1.30103
2)111 sum	So is tan. ang. $\frac{A+C}{2}$ 55° 30′	10.16287
		11,46390
55° 30′ half sum of ang. A and C.	Is to tan. ang. $\frac{A-C}{2}$ 4° 23′	8.88412

To find AC		To find AC.	
To ½ the sum -	55° 30′	As sine A 59° 53′	9.93702
Add ½ the dif. -	4° 23	is to BC 200	2.30103
		So is sine B 69° -	9.97015
The greater -	59° 53′		12.27118
From ½ the sum -	55° 30′	To AC 215.9 - -	2.33416
Take ½ the diff. -	4° 23′		
The less, -	51° 7′		

PROBLEM IV.

The three sides of any triangle being given, to find the angles.
Fig. 57. plate 4.

Ex. 1. Given $\left\{\begin{array}{ll}AB & 100\\ BC & 80\\ AC & 60\end{array}\right\}$ Required all the angles.

AB:

$$AB : AC+BC :: BC-AC : BD-AD$$

$$100 : 140 :: 20 : 28$$

In all cases of this kind, the greater segment is adjacent to the greater side, and *vice versa*.

To $\frac{1}{2}$ the base	- -	50
Add $\frac{1}{2}$ difference	-	14
The greater seg.	-	64
From $\frac{1}{2}$ the base,	-	50
Take $\frac{1}{2}$ diff.	- -	14
The lesser seg.	-	36

To find angle A.		To find angle B.	
As AD 36 - -	1.55630	As BD 64 - -	1.80618
is to rad 90° -	10.00000	is to rad. 90 -	10.00000
So is AC 60 -	1.77815	So is BC 80 - -	1.90309
To sec. ang. A 53° 8′	10.22185	To sec. ang. B 36° 52′	10.09691

Angle C may be found thus: From 180, subtract the sum of angles A and B, the remainder will give angle C. Or add the complements of the angles A and B together, and the sum is angle C.

The preceding problem is frequently wrought according to the following Rule.

Add the three sides together, and, from half the sum, subtract the sides severally; then add the complements of the logarithms of the half sum, and of the difference between the half sum, and the side opposite to the angle sought, to the logarithms of the differences of the two other sides and half sum; and half their sum will be the tangent of half the angle required. Thus, let angle A be required.

80

Oblique ang.

80
60
100

ſum) 240

$\frac{1}{2}$ ſum 120

half ſum 120	Complement	7.92082
the dif. bet. 80 & 120 40	Complement	8.39794
the other differences { 60	Log. - -	1.77815
{ 20	Log. - -	1.30103
		2)19.39794
Tangent of 26° 34′		9.69897
2		
53° 8′ The angle CAB.		

The angles BC may be ſound by problem 1. of oblique angled trigonometry.

We come now to the application of trigonometry, to the menſuration of heights and diſtances.

MENSU-

MENSURATION OF HEIGHTS AND DISTANCES.

THE instruments commonly made uſe of in meaſuring heights and diſtances, are the Geometrical Quadrant, the Theodolite and the Geometrical ſquare.

The Geometrical quadrant is uſed for inveſtigating vertical angles; whether they be angles of * altitude, or angles of depreſſion.

The Theodolite ſerves for meaſuring angles on a horizontal plane, or on an inclined plane.

A vertical plane, is that which is at right angles with the horizon. A horizontal plane, is that which is parallel to the horizon.

The Geometrical quadrant, is the fourth part of a circle, and is divided into 90°, to which two ſights are adapted, and a plumb line ſuſpended from the centre; it is commonly made of braſs or wood. *Fig.* 1. *plate* 4.

The

* *N. B.* When the object is higher than the meaſurer's eye, it is ſaid to ſubtend an angle of elevation, but when lower, an angle of depreſſion.

The Theodolite is a ſemi-circle divided into 180°, with an index which turns about on its centre, and retains any ſituation given it, on which are two ſights, called the moveable ſights; there are alſo two other ſights fixed on the diameter of the theodolite, which are called the fixed ſights. *Fig.* 2. *plate* 4.

Sights are ſmall pieces of wood or braſs, having ſmall holes or ſlits in them, to view the object through;—They are fixed perpendicular to the plane of the theodolite, but parallel to the plane of the quadrant.

The geometrical ſquare may be made of braſs, wood, or any ſolid body, having equal ſides and angles; from one of the angles, a thread is ſuſpended, with a ſmall weight at the end, ſo as to point always to the centre. The two ſides oppoſite to the centre of ſuſpenſion, are divided each of them into 100 equal parts; there is alſo an index, which, (when occaſion ſerves), may be fixed to the centre of ſuſpenſion, and is made ſo as to turn round, and retain any ſituation; on this index, are two ſights. *See fig.* 3. *plate* 4.

Heights and diſtances are of two kinds, viz. acceſſible and inacceſſible: acceſſible objects are houſes, growing trees, &c. inacceſſible ones are all mountains, celeſtial bodies, alſo houſes and trees, in certain ſituations.

PROBLEM I. *See Plate* 4. *fig.* 58.

To meaſure acceſſible heights.

EXAMPLE I.

Let AB be a horizontal plane and BC a tower, whoſe height is required: From B, the foot of the tower, meaſure any convenient diſtance, 80 feet upon the horizontal plane AB. Suppoſe the tower to ſubtend an angle of 39° 49′ from A. What is its height?

As cofine ang. elev. 39° 49′	-	9.88542
Is to rad. 90′ - -	-	1.90309
So is fine ang. elev. 39° 49′	-	9.80641
		11.70950
To the height of the tower 66.69	=	1.82408

EXAMPLE II.

A tower, furrounded by a ditch 40 feet broad: from the other fide of the ditch, the tower fubtends an angle of 53° 13′. Required the height of the tower, alfo the length of a ladder fufficient to fcale the tower. *See fig.* 58. *plate* 4.

To find the height of the tower.

As radius - -	90.10000
is to the breadth of the ditch 40 -	1.60206
So is tan. el. 53° 13′	10.12631
To the height of the tower 53.5	=1.72837

To find the length of the ladder.

As radius 90° - -	10.0000
is to the br. of ditch 40	1.60206
So is fec. elev. 53° 13′	10.22256
To ladder 66.78	= 1.82462

EXAMPLE III. *Plate* 4. *fig.* 59.

From the top of a fhip-maft 100 feet above the level of the water, I took an angle of depreffion of another fhip's hull, 74° 15′; required the diftance of the other fhip.

As radius	90°	-	10.00000
Is to the height of the maft	100	-	2.00000
So is tang. depreffion 74° 15′	- -		10.54971
To the dift. 354.6	-	-	2.54971

PRO-

PROBLEM II.

To measure inaccessible heights and distances.

EXAMPLE I. *Plate* 4. *fig.* 60.

At the foot of a hill, I took an angle of elevation of its top, and found it to be 50° 42′. I then measured back 120 yards on the horizontal plane, and observed the angle to be 40° 12′. Required the perpendicular height of the hill.

N. B. When any side AB of the triangle ADB is produced, the exterior angle DBC is equal to both the interior and opposite angles DAB, ADB; therefore the angle ADB will be 10° 30′.

To find BD.		To find DC the height.	
As sine ADB=10°30′	9.26063	As rad. 90 -	10.00000
is to AB 120 -	2.07918	is to BD 425 -	2.62839
So is sine an. A 40°12′	9.80987	So is sine DBC 50°42′	9.88865
	11.88905	To the height 328.9	2.51704
To BD 425 - -	2.62842		

EXAMPLE II. *Plate* 4. *fig.* 67.

I observed an object on the other side of a river, on a level with the place where I stood; behind me was a regular declivity, which I might reckon a straight line. I marked my station by the side of the river, and measured back 170 yards, when I observed I was higher than the object. I took the angle of depression of the mark by the river side 42° 18′, of the

 bottom

bottom of the object 72° 8′, and of its top 78° 20′. Required the height and distance of the object.

Here, because the angle ABC is 42° 18′ the angle BAC is 47° 42; consequently, its supplement, the angle BAD will be 132° 18. And since all the angles of a triangle are equal to two right angles, and that the angle DBA is 29° 50′, the remaining angle BDA will be 17° 52′ Again, the angle CDE is a right angle, of which the angle BDC is a part; therefore, the angle BDE is 72° 8′, and the angle at E 101° 40′; also the angle DBE will be 6° 12′.

To find the dist. of the object.		To find BD.	
As sine ADB 17° 52′	9.48686	As sine BDA 17° 52′	9.48686
is to AB 170 -	2.23045	is to AB 170 -	2.23045
So is sine ABD 29° 50′	9.69677	so is sine BAD = 132° 18′	9.86902
	11.92722		12.09947
To the dist. 275.7	2.44036	To BD 409.8 - -	2.61261

To find the height of the object.

As sine ang. E 101° 40′ -	9.99093
is to BD 409.8 -	2.61257
So sine DBE 6° 12′ - -	9.03342
	11.64599
To the height 45,19 - -	1.65506

EXAMPLE III. *Plate* 5. *fig.* 1.

Being on a horizontal plane, I took the angle of elevation of the summit of a hill, and of the top of a tower built upon it, and found them to be 48° 20′ and 61° 25′. I then measured back 150 yards, and found the angle subtended by the height of the tower above the plane to be 38° 19′. Required the height of the tower.

The

The exterior angle CBD, is equal to both the interior and opposite angles, CAB, ACB; but CAB is 38° 19′; therefore, ACB will be 23° 6′: and since all the angles of a triangle are equal to two right angles, angle ABC will be 118° 35′. Or it is the supplement of the angle CBD; also angle BCD is 28° 35′, and CEB will be 138° 20′.

To find BC.

As sine an. ACB	23° 6′	9.59366
is to AB -	150	2.17609
So is sine an. A	38° 19′	9.79240
		11.96849
To BC 237 - -		2.37483

To find the tower's height.

As sine CEB	138° 20′	9.82269
is to BC	237 -	2.37475
So is sine CBE	13° 5′	9.35481
		11.72956
To the height of the tower 80.7 -		1.90687

EXAMPLE IV. *Plate* 5. *fig*. 2.

From a window on a level with the bottom of a steeple, I took the angle of elvation of the top of the steeple 50°; from another window, 20 feet perpendicular above the former, I took another angle of the top of the steeple 45° 15′ Required the height and distance of the steeple.

Because the angle ACD is a right angle, of which the angle SCD=50° is a part, the angle SCA will be 40°, consequently, the alternate angle CSD will also be 40°. And since the angle SAB is 45° 15′, and the angle BAD a right angle: therefore, the whole angle SAC 135° 15′, and the angle ASC 4° 45′.

To find CS.

As sine ASC	4° 45′	8.91807
is to AC	20	1.30103
So is sine SAC	135° 15′	9.84758
		11.14861
To CS 170 -		2.23054

To find the height of the steeple.

As sec. ang. SCD	50°	10.19193
is to SC	170 -	2.23045
So is tan. SCD	50°	10.07619
		12.30664
To the height SD 130.2 feet. -		2.11471

To find the diſtance of the ſteeple.

As co-ſecant SCD 50°		10.11575
is to SC 170	-	2.23045
So is co-tan. SCD 50°		9.92381
		12.15426
To the diſt. of the ſteeple.	109.3	2.03851

EXAMPLE V. *Plate* 5. *fig.* 3.

From the top of a tree 70 feet high, I took the angle of depreſſion of two other trees, lying directly in a ſtraight line from me, and on the ſame horizontal plane with the tree on which I then ſtood, viz. that of the nearer 36°, and of the other, 55° 30′. Required their diſtance from the tree from which the obſervation was taken, and from one another.

To find the diſt. of the nearer.		To find the diſt of the other.	
As radius 90 -	10.00000	As rad. 90 -	10.00000
to height of tree 70	1.84510	is to height of tree 70	1.84510
So is tan. dep. 36°	9.86126	So is tan. 2. depr. 55° 30′	10.16287
To the diſt. 50.86	1.70636	To the diſt. 101.9	2.00797

The diſtance of the fartheſt	-	101.9 feet.
The diſtance of the nearer	-	50.86 feet.
Their diſtances from one another		51.04 feet.

EXAMPLE VI. *Plate* 5. *fig.* 4.

Wanting to know the diſtance between a houſe and a tree, the tree being on the other ſide of a river; I took my firſt ſtation at the houſe, and marked my ſecond at B; the angle ſubtended by the diſtance between my ſecond ſtation, and the tree is 60°. I then meaſured the diſtance between my firſt and ſecond ſtations, 380 yards, and found the angle ſubtended by the houſe and tree to be 43°. Required the diſtance between the houſe and the tree.

As

As sine angle D 77°	9.98872
is to AB 380	2.57978
To sine ang. B 43°	9.83378
	12.41356
To the distance 266	2.42484

EXAMPLE VII. *Plate* 5. *fig*. 5.

I wished to know the distance between a kirk and a mill, which were upon the other side of a river, I choose two stations, A and B, distant 400 links, and found the angles MAK 40°, KAB 64° 25′, and ABM 56° 15′, MBK 50° 8′. Required the distance between K the Kirk, and M the Mill.

40° 00 MAK
64° 25 KAB
104 25 ang. MAB.

50° 8 MBK
56 15 ABM
106 23 ang. ABK.

104° 25′ MAB
56 15 ABM
160 40
180 00
19 20 ang. AMB

106° 23′ ABK
64 25 KAB
170 48
180 00
9 12 ang. AKB.

In the triangle AKB to find AK.

As sine ang. AKB 9° 12′	9.20380
is to AB 400 - -	2.60206
So is sine ang. ABK 106° 23′	9.98200
	12.58406
To AK 2400 - -	3.38026

In the triangle AMK to find AM.

As sine ang. AMB 19° 20′ -	9.51991
is to AB 400 -	2.60206
So is sine ang. MBA 56° 15′	9.91985
	12.52191
To AM 1005 -	3.00200

In

In the triangle AKM, to find the angles AMK, MKA.

As AK+AM 3405	3.53212	to ½ sum 70° 00
is to AK—AM 1395	3.14457	add ½ dif. 48° 23′
So is tan. $\frac{\text{AMK+MKA}}{2}$ 70°	10.43893	
	13.58350	the greater 118 23
To tan. $\frac{\text{AMK—MKA}}{2}$ 48° 23′	10.05138	the less 21 37

To find the distance between M and K.

As sine angle MKA 21° 37′		9.56631
is to MA 1005	-	3.00217
So is sine angle MAK 40°		9.80807
		12.81024
To the dist. of the objects 1754		3.24393

Note, The foregoing example may be performed, by using MB and BK as the containing sides.

EXAMPLE VIII. *Plate* 5. *fig*. 6.

If the Peak of Teneriff be four miles above the level of the sea, and the angle of depression taken from the farthest visible point, be 87° 25′ 55″. Required the diameter of the earth, also the farthest visible point that can be seen from the Peak.

If the square of the visual ray, being a tangent to the earth, be divided by the height of the spectator's eye, above the level of the sea, the quotient will give the earth's diameter, and the height of the spectator's eye above the level more.

Demon. Because the straight line AC is equally divided at E, and produced to the point D, the rectangle AD, DC, together with the square of EC, is equal to the square of ED, but the square of ED is equal to the squares EB, BD, because DBE is a right angle; therefore, the rectangle AD, DC, together with

with the ſquare of EC=EB, is equal to the ſquares EB, BD; take away the common ſquare EB, and the remaining rectangle AD, DC, is equal to the ſquare of BD the viſual ray. And becauſe the rectangle AD, DC, is equal to the ſquare of BD, (Euclid. 17th. 6.) DC : DB : : DB : AD.: Therefore, $\frac{DB^2}{DC}=AD$ and AD—DC=CA the diameter.

To find FD.

As rad. 90°	-	10.00000
is to DC 4	-	0.60206
So ſec. 87° 25′ 55″		11.34866
To FD 89.27		1.95072

To find CF.

As rad. 90°	-	10.00000
is to DC 4	-	0.60206
So is tan. 87° 25′ 55″		11.34822
To CF 89.18		1.95028

Here it muſt be obſerved, that if from any point without a circle, two ſtraight lines be drawn to touch the circle, they are equal to one another, (Eucl 37. 3.); therefore, FC is equal to FB, but BF and FD make up BD the viſual ray; conſequently, it will be 89.18+89.27=178.45=BD, and $\frac{178.45^2}{4}=7961$ =AD, and 7961—4=7957, the earth's diameter nearly.

To find BE the ſemidiameter.

As rad. 90°	-	10.00000
is to BD 178.4	-	2.25139
So is tang. 87° 25′ 55″	-	11.34822
To BE the ſemidiameter,	3978	3.59961
	2	
The diameter of the earth	7956	

Several

Several methods have been invented to find the earth's diameter. Mr Picart of the Academy of ſciences at Paris, has propoſed an exact method, by which, not only the equatorial and polar diameters may be known, but alſo the figure of the earth determined.

According to Mr Picart, ' a degree of the meridian at the ' latitude of 49° 21′, was 57.06 French toiſes, each of which con- ' tains 6 feet of the ſame meaſure; from which it follows, that ' if the earth be an exact ſphere, the circumference of a great ' circle of it, will be 123.249,600 Paris feet, and the ſemi- ' diameter of the earth, 19.615,800 feet: but the French ma- ' thematicians, who, of late, examined Mr Picarts obſervations, ' aſſure us, that a degree in that latitude, is 57.183 toiſes. ' They meaſured a degree in Lapland, in the latitude of 66° 20′, ' and found it to be 57.438 toiſes. By comparing theſe degrees, ' as well as by the obſervations on pendulums, and the theory ' of gravity, it appears, that the earth is an oblate ſpheriod; ' and the axis or diameter that paſſes through the poles, will be ' to the diameter of the equator, as 177 is to 178, or the earth ' will be 22 miles higher at the equator, than at the poles. A ' degree has likewiſe been meaſured at the equator, and found ' to be conſiderably leſs than in the latitude of Paris, which ' confirms the oblate figure of the earth. Hence it appears, ' that if the earth were of an uniform denſity from the ſurface ' to the centre, then according to the theory of gravity, the me- ' ridian would be elliptical, and the equatorial would exceed ' the polar diameter, by about 44 miles.'

PROBLEM III. *Plate* 5. *fig.* 9.

To find the height of an object, by means of one ſtaff.

Suppoſe the pole AB of an unkown height, BC a horizontal plane, and ED a ſtaff of a known length. At any convenient

venient distance from the pole, fix your staff perpendicular in the ground, then move backwards or forwards, till you find the point C, whence you may view the top of your staff, E, in a line with A the top of the object, then say, as CD : DE : : CB : BA the height of the object. *Fig.* 67. *plate* 5.

EXAMPLE.

Let BC be 80 feet, CD 5, and DE 4, required AB.

$$\begin{array}{r} 5 : 4 : : 80 \\ 4 \\ \hline 5)320 \\ \hline 65 = AB. \end{array}$$

PROBLEM IV.

To measure the height of an object from the length of its shadow.

Place any staff of a known length in the same plane with the object; then say, as the length of the staff's shadow, is to the length of the staff; so is the length of the object's shadow: to its height.

EXAMPLE.

Wanting to know the height of a steeple, whose shadow I found to be 200 feet, I fixed my staff perpendicular to the horizontal plane, the length of the staff, is $4\frac{1}{2}$ feet, and of the shadow, 6 feet, required the height of the steeple.

$$
\begin{array}{r}
6 : 4\frac{1}{2} :: 200 \\
4\frac{1}{2} \\
\hline
800 \\
100 \\
\hline
6)900 \\
\hline
\end{array}
$$

Anſ. 150 feet high.

PROBLEM V.

To meaſure the height of an object, by a plane mirror, or by a bucket full of water. See fig. 69

Place the mirror or bucket between you and the object. So that the top of the object may appear in the middle of the horizontal ſurface, then ſay, As the diſtance between the object, ſhadow, and your feet, is to the height of the eye; ſo is the diſtance between the object's ſhadow, and the object; to the height of the object.

PROBLEM VI.

Diſtances may alſo be meaſured by loud ſounds, ſuch as, the firing of a cannon, the tolling of a bell, thunder, &c.

It has been found, by many exact experiments, that the uniform velocity of ſound, is 1142 feet, *per* ſecond of time. If, therefore, the ſeconds elapſed, be multiplied by 1142, the product will be the anſwer in feet.

EXAM-

EXAMPLE I.

After seeing a flash of lightning, it was 8 seconds before I heard the thunder, required the distance.

$$\begin{array}{r}
1142 \\
8 \\
\hline
5280)9136(1 \\
5280 \\
\hline
3)3856 \\
\hline
1285\frac{1}{3}
\end{array}$$

Ans. 1 mile $1285\frac{1}{3}$ yards.

EXAMPLE II.

After observing the firing of a cannon, 24 seconds elapsed, before I heard the report, required the distance. *Ans.* 5 miles 336 yards.

EXAMPLE III.

After observing a man striking a bell with a hammer, 5 seconds elapsed before I heard the sound. What was the distance? *Ans.* 1 mile 430 feet.

PROBLEM VII.

To find the velocity of the wind.

Observe the shadow of a cloud at any particular place, then count the number of seconds elapsed, before it reach any other particular place; then say, As the number of seconds elapsed

is to one hour. So is the diſtance of the two places, to the diſtance the wind, will paſs over in one hour.

Note, By a ſimilar experiment, the velocity of running waters may be computed.

PROBLEM VIII.

Heights or depths may be eſtimated from the velocities acquired by falling bodies, and the ſpaces fallen through in given times, or from the time of falling.

In ſucceſſive equal parts of time, ſuch as 1, 2, 3, 4, &c., the ſpaces paſſed over, are in the ſeries of the odd numbers, 1, 3, 5, 7, 9, 11, &c., and the acquired velocities, as 1, 2, 3, 4, &c. Hence, it is plain, that the velocities are as the times, and the ſpaces paſſed over, are as the ſquare of the times of falling. Thus, in a quarter of a ſecond, from the inſtant of beginning to fall, a body will fall 1 foot; in half a ſecond, it will have fallen 4 feet, in three quarters, 9 feet, and in one ſecond, 16 feet. In the next ſecond, it will fall through $16 \times 3 = 48$, which added to the velocity at the end of the former ſecond, will give 64, the whole ſpace fallen through in two ſeconds. In the third ſecond, the body will fall through $5 \times 16 = 80$, which being added to the laſt ſum, 64, will give 144, the ſpace paſſed over in 3 ſeconds, and ſo on continually.

For the continued addition of the odd numbers, gives the ſquares of all numbers from unity and upwards.

Thus, In 1 ſecond, a body will fall 16 feet, which is $1^2 \times 16$.

In 2 ſeconds, $1+3=4=2^2 \times 16=64$.

In 3 ſeconds, $1+3+5=9=3^2$ & $9 \times 16=144$ and ſo on.

EXAM

The velocity acquired at the end of any given time may be found thus. Suppoſe a body begins to move with a celerity conſtantly encreaſing in ſuch a manner as would carry it through 16 feet in one ſecond, at the end of this ſpace it will have acquired ſuch a degree of velocity as would carry it 32 feet in the next ſecond, though it ſhould then receive no new impulſe from the cauſe by which its motion had been accelerated. But as the ſame accelerating cauſe continues conſtantly to act, it will move 16 feet farther the next ſecond, conſequently it will have run 64 feet, and acquire ſuch velocity as would, in the ſame time, carry it over double the ſpace. And ſo on.

EXAMPLE I.

How far will a body fall in 6 ſeconds?

$6^2=36$

$36\times16=576$ feet.

EXAMPLE II.

In what time will a body deſcend through 11664 feet?

```
16)11.664(729(27 ſeconds.
   112     4
   ---    -----
     46  47)329
     32     329
    ---     ---
    144
    144
    ---
```

EXAMPLE III.

Required the laſt acquired velocity, when a body has fallen 8 ſeconds of time.

32 the additional velocity *per* ſecond.
8 the time.

256 the laſt acquired velocity is 256 feet *per* ſecond.

EXAMPLE

EXAMPLE IV.

If a body move at the rate of 1376 feet *per* ſecond, How far muſt it fall to acquire that velocity?

$$\begin{array}{l} 32)1376(43 \text{ ſeconds, time of falling.} \\ \underline{128} \\ 96 \qquad \text{and } 43^2 \times 16 = 29584 \text{ feet.} \\ \underline{96} \end{array}$$

In the following Table, the column titled T denotes the ſeconds of time from 1″ to 60″; S the ſpaces paſſed over in any ſecond of time. The third column gives the heights from which a body would fall at the end of any given time, from 1″ to 60″; and column 4th denotes the laſt acquired velocity at the end of any given time. Thus, at the end of 22 ſeconds, the body has fallen from the height of 7744 feet, and moves with a velocity of 704 feet *per* ſecond.

TABLE

TABLE OF FALLING BODIES.

T.	S.	*Height.*	Last acq. vel.	T.	S.	*Height.*	Last acq. vel.
1	1	*Feet* 16	32	31	61	15376	992
2	3	64	64	32	63	16384	1024
3	5	144	96	33	65	17424	1056
4	7	256	128	34	67	18496	1088
5	9	400	160	35	69	19600	1120
6	11	576	192	36	71	20736	1152
7	13	784	224	37	73	21904	1184
8	15	1024	256	38	75	23104	1216
9	17	1296	288	39	77	24336	1248
10	19	1600	320	40	79	25600	1280
11	21	1936	352	41	81	26806	1312
12	23	2304	384	42	83	28224	1344
13	25	2704	406	43	85	29584	1376
14	27	3136	448	44	87	30976	1408
15	29	3600	480	45	89	32400	1440
16	31	4096	512	46	91	33856	1472
17	33	4624	544	47	93	35344	1504
18	35	5184	576	48	95	36864	1536
19	37	5776	608	49	97	38416	1568
20	39	6400	640	50	99	40000	1600
21	41	7056	672	51	101	41616	1632
22	43	7744	704	52	103	43264	1664
23	45	8464	736	53	105	44944	1696
24	47	9216	768	54	107	46656	1728
25	49	10000	800	55	109	48400	1760
26	51	10816	832	56	111	50176	1792
27	53	11664	864	57	113	51984	1824
28	55	12544	896	58	115	53824	1856
29	57	13456	928	59	117	55696	1888
30	59	14400	960	60	119	57600	1920

PROBLEM IX.

To measure heights and distances by the geometrical square.

When the plane is horizontal, the instrument is to be supported and placed horizontally at any point A, and it is to be turned till the remote point F, whose distance is to be measured, is seen through the fixed sights; then turn the index, till, through the sights upon it, you see any accessible object B; then place the instrument at the point B, directing the fixed sights to the first station A, and the moveable ones to the point F; and if the index cut the reclined side of the square, as in the point E, then, from similar triangles, ES : SB : : as BA : AG; but if the index cut the right side of the square K, it will be BR : RK : : BA : AF. In either of these cases, the distance required may be found by the rule of three *.

Perpendicular heights, when accessible, may be obtained by the quadrant only. For example, If you wanted the height of a house, tree, &c. approach towards or retire from the object, till it subtends an angle of 45°; then shall the height of the object be equal to its horizontal distance. *Euclid*, I. 6.

A similar observation may be made of the other instruments used for heights and distances; but this, and many more, will daily occur in practice.

* The side DE is called the right side, E the reclined side.

TABLES.

LOGARITHMIC TABLES;

CONTAINING,

I. A TABLE OF THE LOGARITHMS OF NUMBERS FROM 1 TO 10000.

II. A TABLE OF LOGARITHMIC SINES, TANGENTS, SECANTS, AND VERSED SINES, TO EVERY *DEGREE* AND *MINUTE* OF THE *QUADRANT*.

III. A TABLE OF LOGARITHMIC SINES, TANGENTS, AND SECANTS, TO EVERY *POINT*, *HALF POINT*, AND *QUARTER POINT* OF THE *COMPASS*.

A TABLE *of the* LOGARITHMS *of* NUMBERS *from* 1 *to* 10000.

Nº.	Log.	Nº.	Log.	Nº.	Log	Nº.	Log.	Nº.	Log.
1	0.00000	21	1.32222	41	1.61278	61	1.78533	81	1.90848
2	0.30103	22	1.34242	42	1.62325	62	1.79239	82	1.91381
3	0.47712	23	1.36173	43	1.63347	63	1.79934	83	1.91908
4	0.60206	24	1.38021	44	1.64345	64	1.80618	84	1.92428
5	0.69897	25	1.39794	45	1.65321	65	1.81291	85	1.92942
6	0.77815	26	1.41497	46	1.66276	66	1.81954	86	1.93450
7	0.84510	27	1.43136	47	1.67220	67	1.82607	87	1.93952
8	0.90309	28	1.44786	48	1.68124	68	1.83251	88	1.94448
9	0.95424	29	1.46240	49	1.69020	69	1.83885	89	1.94939
10	1.00000	30	1.47712	50	1.69897	70	1.84510	90	1.95424
11	1.04139	31	1.49136	51	1.70757	71	1.85126	91	1.95904
12	1.07918	32	1.50515	52	1.71600	72	1.85733	92	1.96379
13	1.11394	33	1.51851	53	1.7[illegible]428	73	1.86332	93	1.96848
14	1.14613	34	1.53148	54	1.73239	74	1.86923	94	1.97313
15	1.17609	35	1.54407	35	1.74036	75	1.87506	95	1.97772
16	1.20412	36	1.55630	56	1.74819	76	1.88081	96	1.98227
17	1.23045	37	1.56820	57	1.75587	77	1.88649	97	1.98677
18	1.25527	38	1.57978	58	1.76343	78	1.89209	98	1.99123
19	1.27875	39	1.59106	59	1.77085	79	1.89763	99	1.99563
20	1.30103	40	1.60206	60	1.77815	80	1.90309	100	2.00000

N°.	0	1	2	3	4	5	6	7	8	9
100	00000	00043	00087	00130	00173	00217	00260	00303	00346	00389
101	00432	00475	00518	00561	00604	00647	00689	00732	00775	00817
102	00860	00903	00945	00988	01030	01072	01115	01157	01199	01242
103	01284	01326	01368	01410	01452	01494	01536	01578	01620	01662
104	01703	01745	01787	01828	01870	01912	01953	01995	02036	02078
105	02119	02160	02202	02243	02284	02325	02366	02407	02449	02490
106	02531	02572	02612	02653	02694	02735	02776	02816	02857	02898
107	02938	02979	03019	03060	03100	03141	03181	03222	03262	03302
108	03342	03383	03423	03463	03503	03543	03583	03623	03663	03703
109	03743	03782	03822	03862	03902	03941	03981	04021	04060	04100
110	04139	04179	04218	04258	04297	04336	04376	04415	04454	04493
111	04532	04571	04610	04650	04689	04727	04766	04805	04844	04883
112	04922	04961	04999	05038	05077	05115	05154	05192	05231	05269
113	05308	05346	05385	05423	05461	05500	05538	05576	05614	05652
114	05690	05729	05767	05805	05843	05881	05918	05956	05994	06032
115	06070	06108	06145	06183	06221	06258	06296	06333	06371	06408
116	06446	06483	06521	06558	06595	06633	06670	06707	06744	06781
117	06819	06856	06893	06930	06967	07004	07041	07078	07115	07151
118	07188	07225	07262	07298	07335	07372	07408	07445	07482	07518
119	07555	07591	07628	07664	07700	07737	07773	07809	07846	07882
120	07918	07954	07990	08027	08063	08099	08135	08171	08207	08243
121	08279	08314	08350	08386	08422	08458	08493	08529	08565	08600
122	08636	08672	08707	08743	08778	08814	08849	08884	08920	08955
123	08991	09026	09061	09096	09132	09167	09202	09237	09272	09307
124	09342	09377	09412	09447	09482	09517	09552	09587	09621	09656
125	09691	09726	09760	09795	09830	09864	09899	09934	09968	10003
126	10037	10072	10106	10140	10175	10209	10243	10278	10312	10346
127	10380	10415	10449	10483	10517	10551	10585	10619	10653	10687
128	10721	10755	10789	10823	10856	10890	10924	10958	10992	11025
129	11059	11093	11126	11160	11193	11227	11260	11294	11327	11361
130	11394	11428	11461	11494	11528	11561	11594	11628	11661	11694
131	11727	11760	11793	11826	11860	11893	11926	11959	11992	12024
132	12057	12090	12123	12156	12189	12222	12254	12287	12320	12352
133	12385	12418	12450	12483	12516	12548	12581	12613	12646	12678
134	12710	12743	12775	12808	12840	12872	12905	12937	12969	13001
135	13033	13066	13098	13130	13162	13194	13226	13258	13290	13322
136	13354	13386	13418	13450	13481	13513	13545	13577	13609	13640
137	13672	13704	13735	13767	13799	13830	13862	13893	13925	13956
138	13988	14019	14051	14082	14114	14145	14176	14208	14239	14270
139	14301	14333	14364	14395	14426	14457	14489	14520	14551	14582
140	14613	14644	14675	14706	14737	14768	14799	14829	14860	14891
141	14922	14953	14983	15014	15045	15076	15106	15137	15168	15198
142	15229	15259	15290	15320	15351	15381	15412	15442	15473	15503
143	15534	15564	15594	15625	15655	15685	15715	15746	15776	15806
144	15836	15866	15897	15927	15957	15987	16017	16047	16077	16107
145	16137	16167	16197	16227	16256	16286	16316	16346	16376	16406
146	16435	16465	16495	16524	16554	16584	16613	16643	16673	16702
147	16732	16761	16791	16820	16850	16879	16909	16938	16967	16997
148	17026	17056	17085	17114	17143	17173	17202	17231	17260	17289
149	17319	17348	17377	17406	17435	17464	17493	17522	17551	17580
150	17609	17638	17667	17696	17725	17754	17782	17811	17840	17869
151	17898	17926	17955	17984	18013	18041	18070	18099	18127	18156
152	18184	18213	18241	18270	18298	18327	18355	18384	18412	18441
153	18469	18498	18526	18554	18583	18611	18639	18667	18696	18724
154	18752	18780	18808	18837	18865	18893	18921	18949	18977	19005
155	19033	19061	19089	19117	19145	19173	19201	19229	19257	19285
156	19312	19340	19368	19396	19424	19451	19479	19507	19535	19562
157	19590	19618	9645	19673	19700	19728	19756	19783	19811	19838
158	19866	19893	19921	19948	19976	20003	20030	20058	20085	20112
159	20140	20167	20194	20222	20249	20276	20303	20330	20358	20385
N°.	0	1	2	3	4	5	6	7	8	9

Nº.	0	1	2	3	4	5	6	7	8	9
160	20412	20439	20466	20493	20520	20547	20575	20602	20629	20656
161	20683	20710	20736	20763	20790	20817	20844	20871	20898	20925
162	20951	20978	21005	21032	21059	21085	21112	21139	21165	21192
163	21219	21245	21272	21299	21325	21352	21378	21405	21431	21458
164	21484	21511	21537	21564	21590	21617	21643	21669	21696	21722
165	21748	21775	21801	21827	21854	21880	21906	21932	21958	21985
166	22011	22037	22063	22089	22115	22141	22167	22194	22220	22246
167	22272	22298	22324	22350	22376	22401	22427	22453	22479	22505
168	22531	22557	22583	22608	22634	22660	22686	22712	22737	22763
169	22789	22814	22840	22866	22891	22917	22943	22968	22994	23019
170	23045	23070	23096	23121	23147	23172	23198	23223	23249	23274
171	23300	23325	23350	23376	23401	23426	23452	23477	23502	23528
172	23553	23578	23603	23629	23654	23679	23704	23729	23754	23779
173	23805	23830	23855	23880	23905	23930	23955	23980	24005	24030
174	24055	24080	24105	24130	24155	24180	24204	24229	24254	24279
175	24304	24329	24353	24378	24403	24428	24452	24477	24502	24527
176	24551	24576	24601	24625	24650	24674	24699	24724	24748	24773
177	24797	24822	24846	24871	24895	24920	24944	24969	24993	25018
178	25042	25066	25091	25115	25139	25164	25188	25212	25237	25261
179	25285	25310	25334	25358	25382	25406	25431	25455	25479	25503
180	25527	25551	25575	25600	25624	25648	25672	25696	25720	25744
181	25768	25792	25816	25840	25864	25888	25912	25935	25959	25983
182	26007	26031	26055	26079	26102	26126	26150	26174	26198	26221
183	26245	26269	26293	26316	26340	26364	26387	26411	26435	26458
184	26482	26505	26529	26553	26576	26600	26623	26647	26670	26694
185	26717	26741	26764	26788	26811	26834	26858	26881	26905	26928
186	26951	26975	26998	27021	27045	27068	27091	27114	27138	27161
187	27184	27207	27231	27254	27277	27300	27323	27346	27370	27393
188	27416	27439	27462	27485	27508	27531	27551	27577	27600	27623
189	27646	27669	27692	27715	27738	27761	27784	27807	27830	27852
190	27875	27898	27921	27944	27967	27989	28012	28035	28058	28081
191	28103	28126	28149	28171	28194	28217	28240	28262	28285	28307
192	28330	28353	28375	28398	28421	28443	28466	28488	28511	28533
193	28556	28578	28601	28623	28646	28668	28691	28713	28735	28758
194	28780	28803	28825	28847	28870	28892	28914	28937	28959	28981
195	29003	29026	29048	29070	29092	29115	29137	29159	29181	29203
196	29226	29248	29270	29292	29314	29336	29358	29380	29403	29425
197	29447	29460	29491	29513	29535	29557	29579	29601	29623	29645
198	29667	29688	29710	29732	29754	29776	29798	29820	29842	29863
199	29885	29907	29929	29951	29973	29994	30016	30038	30060	30081
200	30103	30125	30140	30168	30190	30211	30233	30255	30276	30298
201	30320	30341	30363	30384	30406	30428	30449	30471	30492	30514
202	30535	30557	30578	30600	30621	30642	30664	30685	30707	30728
203	30750	30771	30792	30814	30835	30856	30878	30899	30920	30942
204	30963	30984	31006	31027	31048	31069	31091	31112	31133	31154
205	31175	31197	31218	31239	31260	31281	31302	31323	31345	31366
206	31387	31408	31429	31450	31471	31492	31513	31534	31555	31576
207	31597	31618	31639	31660	31681	31702	31723	31744	31765	31785
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685	83569	83575	83582	83588	83594	83601	83607	83613	83620	83626
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692	84011	84017	84023	84029	84036	84042	84048	84055	84061	84067
693	84073	84080	84086	84092	84098	84105	84111	84117	84123	8413[illegible]
694	84136	84142	84148	84155	84161	84167	84173	84180	84186	84192
695	84198	84205	84211	84217	84223	84230	84236	84242	84248	84255
696	84261	84267	84273	84280	84286	84292	84295	84305	84311	84307
697	84323	84330	84336	84342	34348	84354	84361	84367	84373	84379
698	84386	84392	84398	84404	84410	84417	84423	74429	84435	84441
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703	84696	84702	84708	84714	84720	84726	84733	84739	84745	8475[illegible]
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887	94792	94797	94802	94807	94812	94817	94822	94827	94832	94836
888	94841	94846	94851	94856	94861	94866	94871	94876	94880	94885
889	94890	94895	94900	94905	94910	94915	94919	94924	94929	94934
890	94939	94945	94949	94954	94959	94963	94968	94973	94978	94983
891	94988	94993	94998	95002	95007	95012	95017	65022	95027	95032
892	95036	95041	95046	95051	95056	95061	95066	95071	95075	95080
893	95085	95090	95095	95100	95105	95109	95114	95119	95124	95129
894	95134	95139	95143	95148	95153	95158	95163	95168	95173	95177
895	95182	95187	95192	95197	95202	95207	95211	95216	95221	95226
896	95231	95236	95240	95245	95250	95255	95260	95265	95270	95274
897	95279	95284	95289	95295	95299	95303	95308	95313	95318	95323
898	95328	95332	95337	95342	95347	95352	95357	95361	95366	95371
899	95376	95381	95386	95390	95395	95400	95405	95410	95415	95419
900	95424	95429	95434	95439	95444	95448	95453	95458	95463	95468
901	95472	95477	95482	95487	95492	95497	95501	95506	95511	95516
902	95521	95525	95530	95535	95540	95545	95550	95554	95559	95564
903	95569	95574	95578	95583	95588	95593	95598	95602	95607	95612
904	95617	95622	95626	95631	95636	95641	95646	95650	95655	95660
905	95665	95670	95674	95679	95684	95689	95694	95698	95703	95708
906	95713	95718	95722	95727	95732	95737	95742	95746	95751	95756
907	95761	95766	95770	95775	95780	95785	95789	95794	95799	95804
908	95809	95813	95818	95823	95828	95832	95837	95842	95847	95852
909	95856	95861	95866	95871	95875	95880	95885	95890	95896	95899
910	95904	95909	95914	95918	95923	95928	95933	95938	95942	95947
911	95952	95957	95961	95966	95971	95976	95980	95985	95990	95995
912	95999	96004	96009	96014	96019	96023	96028	96033	96038	96042
913	96047	96052	96057	96061	96066	96071	96076	96080	96085	96090
914	96095	96099	96104	96109	96114	96118	96123	96128	96133	96137
915	96142	96147	96152	96156	96161	96166	96171	96175	96180	96185
916	96190	96194	96199	96204	96209	96213	96218	96223	96227	96232
917	96237	96242	96246	96251	96256	96261	96265	96270	96275	96280
918	96284	96289	96294	96298	96303	96308	96313	96317	96322	96327
919	96332	96336	96341	96346	96350	96355	96360	96365	96369	96374
920	96379	96383	96388	96393	96398	96402	96407	96412	96417	96421
921	96426	96431	96435	96440	96445	96450	96454	96459	96464	96468
922	96473	96478	96483	96487	96492	96497	96501	96506	96511	96515
923	96520	96525	96530	96534	96539	96544	96548	96553	96558	96562
924	96567	96572	96577	96581	96586	96591	96595	96600	96605	96609
925	96614	96619	96624	96628	96633	96638	96642	96647	96652	96656
926	96661	96666	96670	96675	96680	96685	96689	96694	96699	96703
927	96708	96713	96717	96722	96727	96731	96736	96741	96745	96750
928	96755	96759	96764	96769	96774	96778	96783	96788	96792	96797
929	96802	96806	96811	96816	96820	96825	96830	96834	96839	96844
930	96848	96853	96858	96862	96867	96872	96876	96881	96836	96890
931	96895	96900	96904	96909	96914	96918	96923	96928	96932	96937
932	96942	96946	96951	96956	96960	96965	96970	96974	96979	96984
933	96988	96993	96997	97002	97007	97011	97016	97021	97025	97030
934	97035	97039	97044	97049	97053	97058	97063	97067	97072	97077
935	97081	97086	97090	97095	97100	97104	97109	97114	97118	97123
936	97128	97132	97137	97141	97146	97151	97155	97160	97165	97169
937	97174	97179	97183	97188	97192	97197	97202	97206	97211	97216
938	97220	97225	97230	97234	97239	97243	97248	97253	97257	97262
939	97267	97271	97276	97280	97285	97290	97294	97299	97304	97308
N°.	0	1	2	3	4	5	6	7	8	9

N°.	0	1	2	3	4	5	6	7	8	9
940	97313	87317	97322	97327	97331	97336	97340	97345	97350	97354
941	97359	97364	97368	97373	97377	97382	97387	97391	97396	97400
942	97405	97410	97414	97419	97424	97428	97433	97437	97442	97447
943	97451	97456	97460	97465	97470	97474	97479	97483	97488	97492
944	97497	97502	97506	97511	97516	97520	97525	97529	97534	97539
945	97543	97548	97552	97557	97562	97566	97571	97575	97580	97584
946	97589	97594	97598	97603	97607	97612	97617	97621	97626	97630
947	97635	97640	97644	97649	97653	97658	97663	97667	97672	97676
948	97681	97685	97690	97695	97699	97704	97708	97713	97717	97722
949	97727	97731	97736	97740	97745	97749	97754	97759	97763	97768
950	97772	97777	97781	97786	97791	97795	97800	97804	97809	97813
951	97818	97823	97827	97832	97836	97841	97845	97850	97855	97859
952	97864	97868	97873	97877	97882	97886	97891	97896	97900	97905
953	97909	97914	97918	97923	97928	97932	97937	97941	97946	97950
954	97955	97959	97964	97968	97973	97978	97982	97987	97991	97996
955	98000	98005	98009	98014	98019	98023	98028	98032	98037	98041
956	98046	98050	98055	98059	98064	98068	98073	98078	98082	98087
957	98091	98096	98100	98105	98109	98114	98118	98123	98127	98132
958	98137	98141	98146	98150	98155	98159	98164	98168	98173	98177
959	98182	98186	98191	98195	98200	98204	98209	98214	98218	98223
960	98227	98232	98236	98241	98245	98250	98254	98259	98263	98268
961	98272	98277	98281	98286	98290	98295	98299	98304	98308	98313
962	98318	98322	98327	98331	98336	98340	98345	98349	98354	98358
963	98363	98367	98372	98376	98381	98385	98390	98394	98399	98403
964	98408	98412	98417	98421	98426	98430	98435	98439	98444	98448
965	98453	98457	98462	98466	98471	98475	98480	98484	98489	98493
966	98498	98502	98507	98511	98516	98520	98525	98529	98534	98538
967	98543	98547	98552	98556	98561	98565	98570	98575	98579	98583
968	98588	98592	98597	98601	98605	98610	98614	98619	98623	98628
969	98632	98637	98641	98646	98650	98655	98659	98664	98668	98673
970	98677	98682	98686	98691	98695	98700	98704	98708	98713	98717
971	98722	98726	98731	98735	98740	98744	98749	98753	98758	98762
972	98767	98771	98776	98780	98784	98789	98793	98798	98802	98807
973	98811	98816	98820	98825	98829	98834	98838	98843	98847	98851
974	98856	98860	98865	98869	98874	98878	98883	98887	98892	98896
975	98900	98905	98909	98914	98918	98923	98927	98932	98936	98941
976	98945	98949	98954	98958	98963	98967	98972	98976	98981	98985
977	98989	98994	98998	99003	99007	99012	99016	99021	99025	99029
978	99034	99038	99043	99047	99052	99056	99061	99065	99069	99074
979	99078	99083	99087	99092	99096	99100	99106	99109	99114	99118
980	99123	99127	99131	99136	99140	99145	99149	99154	99158	99162
981	99167	99171	99176	99180	99185	99189	99193	99198	99202	99207
982	99211	99216	99220	99224	99229	99233	99238	99242	99247	99251
983	99255	99260	99264	99269	99273	99277	99282	99286	99291	99295
984	99300	99304	99308	99313	99317	99322	99326	99330	99335	99339
985	99344	99348	99352	99357	99361	99366	99370	99374	99379	99383
986	99388	99392	99396	99401	99405	99410	99414	99419	99423	99427
987	99432	99436	99441	99445	99449	99454	99458	99462	99467	99471
988	99476	99480	99484	99489	99493	99498	99502	99506	99511	99515
989	99520	99524	99528	99533	99537	99542	99546	99550	99555	99559
990	99564	99568	99572	99577	99581	99585	99590	99594	99599	99603
991	99607	99612	99616	99621	99625	99629	99634	99638	99642	99647
992	99651	99656	99660	99664	99669	99673	99677	99682	99686	99691
993	99695	99699	99704	99708	99712	99717	96721	99725	99730	99734
994	99739	99743	99747	99752	99756	99760	99765	99769	99774	99778
995	99782	99787	99791	99795	99800	99804	99808	99813	99817	99822
996	99826	99830	99835	99839	99843	99848	99852	99856	99861	99865
997	99870	99874	99878	99883	99887	99891	99896	99900	99904	99909
998	99913	99917	99922	99926	99930	99935	99939	99943	99948	99952
999	99957	99961	99965	99970	99974	99978	99983	99987	99991	99996
N°.	0	1	2	3	4	5	6	7	8	9

M	Sine	Co-ſine	Tang.	Co-tang.	Secant	Co-ſec.	V. Sine		
0	0.00000	10.00000	0.00000	Infinite	10.00000	Infinite		5.00000	60
1	6.46373	10.00000	6.46373	13.53627	10.00000	13.53627	7.62642	4.99987	59
2	6.76476	10.00000	6.76476	13.23524	10.00000	13.23524	8.22848	4.99975	58
3	6.94085	10.00000	6.94085	13.05915	10.00000	13.05915	8.58066	4.99962	57
4	7.06579	10.00000	7.06579	12.93421	10.00000	12.93421	8.83054	4.99949	56
5	7.16270	10.00000	7.16270	12.83730	10.00000	12.83730	9.02436	4.99937	55
6	7.24188	10.00000	7.24188	12.75812	10.00000	12.75812	9.18272	4.99924	54
7	7.30882	10.00000	7.30882	12.69117	10.00000	12.69118	9.31602	4.99911	53
8	7.36682	10.00000	7.36682	12.63318	10.00000	12.63318	9.43260	4.99899	52
9	7.41797	10.00000	7.41797	12.58203	10.00000	12.58203	9.53491	4.99886	51
10	7.46373	10.00000	7.46373	12.53627	10.00000	12.53627	9.62642	4.99873	50
11	7.50512	10.00000	7.50512	12.49488	10.00000	12.49488	9.70921	4.99861	49
12	7.54291	10.00000	7.54291	12.45709	10.00000	12.45709	9.78478	4.99848	48
13	7.57767	10.00000	7.57767	12.42233	10.00000	12.42233	9.85431	4.99835	47
14	7.60985	10.00000	7.60986	12.39014	10.00000	12.39015	9.91868	4.99823	46
15	7.63982	10.00000	7.63982	12.36018	10.00000	12.36018	9.97860	4.99810	45
16	7.66784	10.00000	7.66785	12.33215	10.00000	12.33216	0.03466	4.99797	44
17	7.69417	9.99999	7.69418	12.30582	10.00001	12.30583	0.08732	4.99785	43
18	7.71900	9.99999	7.71900	12.28100	10.00001	12,28100	0.13697	4.99772	42
19	7.74248	9.99999	7.74248	12.25752	10.00001	12.25752	0.18393	4.99759	41
20	7.76475	9.99999	7.76476	12.23524	10.00001	12.23525	0.22848	4.99747	40
21	7.78594	9.99999	7.78595	12.21405	10.00001	12.21406	0.27086	4.99734	39
22	7.80615	9.99999	7.80615	12.19384	10.00001	12.19385	0.31127	4.99721	38
23	7.82545	9.99999	7.82546	12.17454	10.00001	12.17455	0.34988	4.99708	37
24	7.84393	9.99999	7.84394	12.15606	10.00001	12.15607	0.38684	4.99696	36
25	7.86166	9.99999	7.86167	12.13833	10.00001	12.13834	0.42230	4.99683	35
26	7.87870	9.99999	7.87871	12.12129	10.00001	12.12130	0.45637	4.99670	34
27	7.89509	9.99999	7.89510	12.10490	10.00001	12.10491	0.48915	4.99658	33
28	7.91088	9.99999	7.91089	12.08911	10.00001	12.08912	0.52074	4.99645	32
29	7.92612	9.99998	7.92613	12.07387	10.00002	12.07388	0.55122	4.99632	31
30	7.94084	9.99998	7.94086	12.05914	10.00002	12.05916	0.58066	4.99619	30
31	7.95508	9.99998	7.95510	12.04490	10.00002	12.04492	0.00194	4.99607	29
32	7.96887	9.99998	7.96889	12.03111	10.00002	12.03113	0.63672	4.99594	28
33	7.98223	9.99998	7.98225	12.01775	10.00002	12.01777	0.66345	4.99581	27
34	7.99520	9.99998	7.99522	12.00478	10.00002	12.00480	0.68938	4.99568	26
35	8.00779	9.99998	8.00781	11.99219	10.00002	11.99221	0.71455	4.99556	25
36	8.02002	9.99998	8.02004	11.97995	10.00002	11.97998	0.73902	4.99543	24
37	8.03192	9.99997	8.03194	11.96805	10.00003	11.96808	0.76282	4.99530	23
38	8.04350	9.99997	8.04353	11.95647	10.00003	11.95650	0.78598	4.99517	22
39	8.05478	9.99997	8.05481	11.94519	10.00003	11.94522	0.80845	4.99504	21
40	8.06578	9.99997	8.06581	11.93419	10.00003	11.93422	0.83054	4.99492	20
41	8.07650	9.99997	8.07653	11.92347	10.00003	11.92350	0.85198	4.99479	19
42	8.08696	9.99997	8.08700	11.91300	10.00003	11.91303	0.87292	4.99466	18
43	8.09718	9.99997	8.09722	11.90278	10.00003	11.90282	0.89335	4.99453	17
44	8.10717	9.99996	8.10720	11.89280	10.00004	11.89283	0.91332	4.99441	16
45	8.11693	9.99996	8.11696	11.88304	10.00004	11.88307	0.93284	4.99428	15
46	8.12647	9.99996	8.12651	11.87349	10.00004	11.87353	0.95193	4.99415	14
47	8.13581	9.99996	8.13585	11.86415	10.00004	11.86419	0.97061	4.99402	13
48	8.14495	9.99996	8.14500	11.85500	10.00004	11.85505	0.95890	4.99389	12
49	8.15391	9.99996	8.15395	11.84605	10.00004	11.84609	1.00681	4.99377	11
50	8.16268	9.99995	8.16273	11.83727	10.00005	11.83732	1.02435	4.99364	10
51	8.17128	9.99995	8.17133	11.82867	10.00005	11.82872	1.04152	4.99351	9
52	8.17971	9.99995	8.17976	11.82024	10.00005	11.82029	1.05842	4.99338	8
53	8.18798	9.99995	8.18804	11.81196	10.00005	11.81202	1.07497	4.99325	7
54	8.19610	9.99995	8.19616	11.80384	10.00005	11.80390	1.09120	4.99312	6
55	8.20407	9.99994	8.20413	11.79587	10.00006	11.79593	1.10714	4.99300	5
56	8.21189	9.99994	8.21195	11.78805	10.00006	11.78811	1.12279	4.99287	4
57	8.21958	9.99994	8.21964	11.78035	10.00006	11.78042	1.13816	4.99274	3
58	8.22713	9.99994	8.22719	11.77280	10.00006	11.77287	1.15327	4.99261	2
59	8.23456	9.99994	8.23462	11.76538	10.00006	11.76544	1.16812	4.99248	1
60	8.24186	9.99993	8.24192	11.75808	10.00007	11.75814	1.18271	4.99235	0
	Co-ſine	Sine	Co-tan.	Tangent	Co-ſec.	Secant		. Sine	M

M	Sine	Co-ſine	Tang.	Co-tang	Secant	Co-ſec.	V. Sine		
0	8.24186	9.99993	8.24192	11.75808	10.00000	11.75814	1.18271	4.99235	60
1	8.24903	9.99993	8.24910	11.75090	10.00007	11.75097	1.19707	4.99222	59
2	8.25609	9.99993	8.25616	11.74384	10.00007	11.74391	1.21119	4.99210	58
3	8.26304	9.99993	8.26312	11.73688	10.00007	11.73696	1.22509	4.99197	57
4	8.26988	9.99992	8.26996	11.73004	10.00008	11.73012	1.23877	4.99184	56
5	8.27661	9.99992	8.27669	11.72331	10.00008	11.72339	1.25224	4.99171	55
6	8.28324	9.99992	8.28332	11.71668	10.00008	11.71676	1.26550	4.99158	54
7	8.28977	9.99992	8.28986	11.71014	10.00008	11.71023	1.27856	4.99145	53
8	8.29621	9.99991	8.29629	11.70371	10.00009	11.70379	1.29143	4.99132	52
9	8.30255	9.99991	8.30263	11.69737	10.00009	11.69745	1.30411	4.99120	51
10	8.30879	9.99991	8.30888	11.69112	10.00009	11.69121	1.31660	4.99107	50
11	8.31495	9.99991	8.31505	11.68495	10.00009	11.68505	1.32892	4.99094	49
12	8.32103	9.99990	8.32112	11.67888	10.00010	11.67897	1.34107	4.99081	48
13	8.32702	9.99990	8.32711	11.67289	10.00010	11.67298	1.35305	4.99068	47
14	8.33292	9.99990	8.33302	11.66698	10.00010	11.66708	1.36487	4.99055	46
15	8.33875	9.99990	8.33886	11.66114	10.00010	11.66125	1.37653	4.99042	45
16	8.34450	9.99989	8.34461	11.65539	10.00011	11.65550	1.38803	4.99029	44
17	8.35018	9.99989	8.35029	11.64971	10.00011	11.64982	1.39939	4.99016	43
18	8.35578	9.99989	8.35590	11.64410	10.00011	11.64422	1.41059	4.99003	42
19	8.36131	9.99989	8.36143	11.63857	10.00011	11.63869	1.42166	4.98990	41
20	8.36678	9.99988	8.36689	11.63311	10.00012	11.63322	1.43258	4.98977	40
21	8.37217	9.99988	8.37229	11.62771	10.00012	11.62783	1.44336	4.98965	39
22	8.37750	9.99988	8.37762	11.62238	20.00012	11.62250	1.45403	4.98952	38
23	8.38276	9.96987	8.38289	11.61711	10.00013	11.61724	1.46456	4.98939	37
24	8.38796	9.99987	8.38809	11.61191	10.00013	11.61204	1.47496	4.98926	36
25	8.39310	9.99987	8.39323	11.60677	10.00013	11.60690	1.48524	4.98913	35
26	8.39818	9.99986	8.39832	11.60168	10.00014	11.60182	1.49540	4.98900	34
27	8.40320	9.99986	8.40334	11.59666	10.00014	11.59680	1.50544	4.98887	33
28	8.40816	9.99986	8.40830	11.59170	10.00014	11.5918	1.51536	4.98874	32
29	8.41307	9.99985	8.41321	11.58679	10.00015	11.58693	1.52518	4.98861	31
30	8.41792	9.99985	8.41807	11.58193	10.0001	11.58208	1.53488	4.98848	30
31	8.42272	9.99985	8.42287	11.57713	10.00015	11.57728	1.54448	4.98835	29
32	8.42746	9.99984	8.42762	11.57238	10.00016	11.57254	1.55397	4.98822	28
33	8.43216	9.99984	8.43231	11.56768	10.00016	11.56784	1.56336	4.98809	27
34	8.43680	9.99984	8.43696	11.56304	10.00016	11.56320	1.57265	4.98796	26
35	8.44139	9.99983	8.44156	11.55844	10.00017	11.55861	1.58184	4.98783	25
36	8.44594	9.99983	8.44611	11.55389	10.00017	11.55406	1.59094	4.98770	24
37	8.45044	9.99983	8.45061	11.54939	10.00017	11.54956	1.59994	4.98757	23
38	8.45489	9.99982	8.45507	11.54493	10.00018	11.54511	1.60884	4.98744	22
39	8.45930	9.99982	8.45948	11.54052	10.00018	11.54070	1.61766	4.98731	21
40	8.46360	9.99982	8.46385	11.53615	10.00018	11.53634	1.62639	4.98718	20
41	8.46798	9.99981	8.46817	11.53183	10.00019	11.53202	1.63503	4.98705	19
42	8.47226	9.99981	8.47245	11.52755	10.00019	11.52774	1.64359	4.98692	18
43	8.47650	9.99980	8.47669	11.52331	10.00020	11.52350	1.65206	4.98679	17
44	8.48069	9.99980	8.48089	11.51911	10.00020	11.51931	1.66046	4.98666	16
45	8.48485	9.99680	8.48505	11.51495	10.00020	11.51515	1.66877	4.98653	15
46	8.48896	9.99979	8.48917	11.51083	10.00021	11.51104	1.67700	4.98640	14
47	8.49304	9.99979	8.49325	11.50675	10.00021	11.50696	1.68515	4.98627	13
48	8.49708	9.99979	8.49729	11.50271	10.00021	11.50292	1.69323	4.98614	12
49	8.50108	9.99978	8.50130	11.49870	10.00022	11.49892	1.70124	4.98601	11
50	8.50504	9.99978	8.50527	11.49473	10.00022	11.49496	1.70917	4.98588	10
51	8.50897	9.99977	8.50920	11.49080	10.00023	11.49103	1.71703	4.98575	9
52	8.51287	9.99977	8.51310	11,48690	10.00023	11.48713	1.72482	4.98562	8
53	8.51673	9.99977	8.51696	11.48304	10.00023	11.48327	1.73244	4.98549	7
54	8.52055	9.99976	8.52079	11.47921	10.9002	11.47975	1.74019	4.98536	6
55	8.52434	9.99976	8.52459	11.47541	10.00024	11.47566	1.74778	4.98523	5
56	8.52810	9.99975	8.52835	11.47165	10.00025	11.47190	1.75530	4.98510	4
57	8.53183	9.99975	8.53208	11.46792	10.00025	11.46817	1.76275	4.98496	3
58	8.53552	9.99974	8.53578	11.46422	10.00026	11.46448	1.77014	4.98483	2
59	8.53919	9.99974	8.53945	11.46055	10.00026	11.46081	1.77747	4.98470	1
60	8.54282	9.99974	8.54308	11.45692	10.00026	11.45718	1.78474	4.98457	0
	Co-ſine	Sine	Co-tan.	Tangent	Co-ſec.	Secant		V. Sine	M

M	Sine	Co-ſine	Tang	Co-tang.	Secant	Co-ſec.	V. Sine		
0	8.54282	9.99974	8.54308	11.45692	10.00026	11.45718	1.78474	4.98457	60
1	8.54642	9.99973	8.54669	11.45331	10.00027	11.45358	1.79195	4.98444	59
2	8.54999	9.99973	8.55027	11.44973	10.00027	11.45001	1.79910	4.98431	58
3	8.55354	9.99972	8.55382	11.44618	10.00028	11.44646	1.80619	4.98418	57
4	8.55705	9.99972	8.55734	11.44266	10.00028	11.44295	1.81322	4.98405	56
5	8.56054	9.99971	8.56083	11.43917	10.00029	11.43946	1.82019	4.98392	55
6	8.56400	9.99971	8.56429	11.43571	10.00029	11.43600	1.82711	4.98379	54
7	8.56743	9.99970	8.56773	11.43227	10.00030	11.43257	1.83398	4.98306	53
8	8.57084	9.99970	8.57114	11.42886	10.00030	11.42916	1.84079	4.98352	52
9	8.57421	9.99969	8.57452	11.42548	10.00031	11.42579	1.84755	4.98339	51
10	8.57757	9.99969	8.57788	11.42212	10.00031	11.42243	1.85426	4.98326	50
11	8.58089	9.99968	8.58121	11.41879	10.00032	11.41911	1.86091	4.98313	49
12	8.58419	9.99968	8.58451	11.41549	10.00032	11.41581	1.86752	4.98300	48
13	8.58747	9.99967	8.58779	11.41221	10.00033	11.41253	1.87407	4.98287	47
14	8.59072	9.99967	8.59105	11.40895	10.00033	11.40928	1.88058	4.98274	46
15	8.59395	9.99966	8.59428	11.40572	10.00034	11.40605	1.88703	4.98261	45
16	8.59715	9.99966	8.59749	11.40251	10.00034	11.40285	1.89344	4.98247	44
17	8.60033	9.99960	8.60068	11.39932	10.00035	11.39967	1.89981	4.98234	43
18	8.60349	9.99965	8.60384	11.39616	10.00035	11.39651	1.90612	4.98221	42
19	8.60662	9.99964	8.60698	11.39302	10.00036	11.39338	1.91239	4.98208	41
20	8.60973	9.99964	8.61009	11.38991	10.00036	11.39027	1.91862	4.98195	40
21	8.61282	9.99963	8.61319	11.38681	10.00037	11.38718	1.92480	4.98182	39
22	8.61589	9.99963	8.61626	11.38374	10.00037	11.38411	1.93094	4.98169	38
23	8.61894	9.99962	8.61931	11.38069	10.00038	11.38101	1.93703	4.98155	37
24	8.62196	9.99962	8.62234	11.37766	10.00038	11.37804	1.94308	4.98142	36
25	8.62497	9.99961	8.62535	11.37464	10.00039	11.37503	1.94909	4.98129	35
26	8.62795	9.99961	8.62834	11.37166	10.00039	11.37205	1.95506	4.98116	34
27	8.63091	9.99960	8.63131	11.36869	10.00040	11.36909	1.96099	4.98103	33
28	8.63385	9.99960	8.63426	11.36574	10.00040	11.36615	1.96688	4.98089	32
29	8.63678	9.99959	8.63718	11.36282	10.00041	11.36322	1.97273	4.98076	31
30	8.63968	9.99959	8.64009	11.35991	10.00041	11.36032	1.97854	4.98063	30
31	8.64256	9.99958	8.64298	11.35702	10.00042	11.35744	1.98431	4.98050	29
32	8.64543	9.99958	8.64585	11.35415	10.00042	11.35457	1.99004	4.98037	28
33	8.64827	9.99957	8.64870	11.35130	10.00043	11.35173	1.99573	4.98023	27
34	8.65110	9.99956	8.65154	11.34846	10.00044	11.34890	2.00139	4.98010	26
35	8.65391	9.99956	8.65435	11.34565	10.00044	11.34609	2.00699	4.97997	25
36	8.65670	9.99955	8.65715	11.34285	10.00045	11.34330	2.01260	4.97984	24
37	8.65947	9.99955	8.65993	11.34007	10.00045	11.34053	2.01825	4.97971	23
38	8.66223	9.99954	8.66269	11.33731	10.00046	11.33777	2.02366	4.97957	22
39	8.66497	9.99954	8.66543	11.33457	10.00046	11.33503	2.02914	4.97944	21
40	8.66769	9.99953	8.66816	11.33184	10.00047	11.33231	2.03458	4.97931	20
41	8.67039	9.99952	8.67087	11.32913	10.00048	11.32961	2.03999	4.97918	19
42	8.67308	9.99952	8.67356	11.32644	10.00048	11.32692	2.04537	4.97904	18
43	8.67575	9.99951	8.67624	11.32376	10.00049	11.32425	2.05072	4.97891	17
44	8.67841	9.99951	8.67890	11.32110	10.00049	11.32159	2.05603	4.97878	16
45	8.68104	9.99950	8.68154	11.31846	10.00050	11.31896	2.06131	4.97865	15
46	8.68367	9.99949	8.68417	11.31583	10.00051	11.31633	2.06655	4.97851	14
47	8.68627	9.99949	8.68678	11.31322	10.00051	11.31373	2.07177	4.97838	13
48	8.68885	9.99948	8.68938	11.31062	10.00052	11.31114	2.07695	4.97825	12
49	8.69144	9.99947	8.69196	11.30804	10.00053	11.30856	2.08211	4.97812	11
50	8.69400	9.99947	8.69453	11.30547	10.00053	11.30600	2.08723	4.97798	10
51	8.69654	9.99946	8.69708	11.30292	10.00054	11.30346	2.09232	4.97785	9
52	8.69907	9.99946	8.69962	11.30038	10.00054	11.30093	2.09739	4.97772	8
53	8.70159	9.99945	8.70214	11.29786	10.00055	11.29841	2.10242	4.97758	7
54	8.70409	9.99944	8.70465	11.29535	10.00056	11.29591	2.10743	4.97745	6
55	8.70658	9.99944	8.70714	11.29286	10.00056	11.29342	2.11240	4.97732	5
56	8.70905	9.99943	8.70962	11.29038	10.00057	11.29095	2.11735	4.97719	4
57	8.71151	9.99942	8.71208	11.28792	10.00058	11.28849	2.12227	4.97705	3
58	8.71395	9.99942	8.71453	11.28547	10.00058	11.28605	2.12717	4.97692	2
59	8.71638	9.99941	8.71697	11.28303	10.00059	11.28362	2.13203	4.97679	1
60	8.71880	9.99940	8.71940	11.28060	10.00060	11.28120	2.13687	4.97665	0
	Co-ſine	Sine	Co-tan.	Tangent.	Co-ſec.	Secant		V. Sine	M

M	Sine	Co-sine	Tang.	Co-tang.	Secant	Co-sec.	V. Sine		
0	8.71880	9.99940	8.71940	11.28060	10.00060	11.28120	2.13687	4.97665	60
1	8.72120	9.99940	8.72181	11.27819	10.00060	11.27880	2.14168	4.97652	59
2	8.72359	9.99939	8.72420	11.27580	10.00061	11.27641	2.14646	4.97639	58
3	8.72597	9.99938	8.72659	11.27341	10.00062	11.27403	2.15122	4.97625	57
4	8.72834	9.99938	8.72896	11.27104	10.00062	11.27166	2.15595	4.97612	56
5	8.73069	9.99937	8.73132	11.26868	10.00063	11.26931	2.16066	4.97599	55
6	8.73303	9.99936	8.73366	11.26634	10.00064	11.26697	2.16534	4.97585	54
7	8.73535	9.99936	8.73600	11.26400	10.00064	11.26465	2.17000	4.97572	53
8	8.73767	9.99935	8.73832	11.26168	10.00065	11.26233	2.17463	4.97559	52
9	8.73997	9.99434	8.74063	11.25937	10.00066	11.26003	2.17924	4.97545	51
10	8.74226	9.99934	8.74292	11.25708	10.00066	11.25774	2.18382	4.97532	50
11	8.74454	9.99933	8.74521	11.25479	10.00067	11.25546	2.18838	4.97519	49
12	8.74680	9.99932	8.74748	11.25252	10.00068	11.25320	2.19291	4.97505	48
13	8.74906	9.99932	8.74974	11.25026	10.00068	11.25094	2.19742	4.97492	47
14	8.75130	9.99931	8.75199	11.24801	10.00069	11.24870	2.20191	4.97479	46
15	8.75353	9.99930	8.75423	11.24577	10.00070	11.24647	2.20637	4.97465	45
16	8.75575	9.99929	8.75645	11.24355	10.00071	11.24425	2.21082	4.97452	44
17	8.75795	9.99929	8.75867	11.24133	10.00071	11.24205	2.21524	4.97439	43
18	8.76015	9.99928	8.76087	11.23913	10.00072	11.23985	2.21963	4.97425	42
19	8.76234	9.99927	8.76306	11.23694	10.00073	11.23766	2.22401	4.97412	41
20	8.76451	9.99926	8.76525	11.23475	10.00074	11.23549	2.22836	4.9739[illegible]	40
21	8.76667	9.99926	8.76742	11.23258	10.00074	11.23333	2.23269	4.9738[illegible]	39
22	8.76883	9.99925	8.76958	11.23042	10.00075	11.23117	2.23700	4.97372	38
23	8.77097	9.99924	8.77173	11.22827	10.00076	11.22903	2.24129	4.97358	37
24	8.77310	9.99923	8.77387	11.22613	10.00077	11.22690	2.24556	4.97345	36
25	8.77522	9.99923	8.77600	11.22400	10.00077	11.22478	2.24980	4.97331	35
26	8.77733	9.99922	8.77811	11.22189	10.00078	11.22267	2.25403	4.97318	34
27	8.77943	9.99921	8.78022	11.21978	10.00079	11.22057	2.25823	4.97305	33
28	8.78152	9.99920	8.78232	11.21768	10.00080	11.21848	2.26242	4.97291	32
29	8.78360	9.99920	8.78441	11.21559	10.00080	11.21640	2.26658	4.97278	31
30	8.78568	9.99910	8.78649	11.21351	10.00081	11.21432	2.27073	4.97264	30
31	8.78774	9.99918	8.78855	11.21145	10.00082	11.21226	2.27485	4.97251	29
32	8.78979	9.99917	8.79061	11.20939	10.00083	11.21021	2.27896	4.97237	28
33	8.79183	9.99917	8.79266	11.20734	10.00083	11.20817	2.28304	4.97224	27
34	8.79386	9.99916	8.79470	11.20530	10.00084	11.20614	2.28711	4.97211	26
35	8.79588	9.99915	8.79673	11.20327	10.00085	11.20412	2.29116	4.97197	25
36	8.79789	9.99914	8.79875	11.20125	10.00086	11.20211	2.29519	4.97184	24
37	8.79990	9.99913	8.80076	11.19924	10.00087	11.20010	2.29920	4.97170	23
38	8.80189	9.29913	8.80277	11.19723	10.00087	11.19811	2.30319	4.97157	22
39	8.80388	9.99912	8.80476	11.19524	10.00088	11.19612	2.30716	4.97143	21
40	8.80585	9.99911	8.80674	11.19326	10.00089	11.19415	2.31112	4.97130	20
41	8.80[illegible]32	9.99910	8.80872	11.19128	10.00090	11.19218	2.31506	4.97116	19
42	8.809[illegible]8	9.99909	8.81068	11.18932	10.00091	11.19022	2.31898	4.97103	18
43	8.81173	9.99909	8.81264	11.18736	10.00091	11.18828	2.32288	4.97089	17
44	8.81367	9.99908	8.81459	11.18541	10.00092	11.18633	2.32676	4.97076	16
45	8.81560	9.99907	8.81653	11.18347	10.00093	11.18440	2.33063	4.97062	15
46	8.81752	9.99906	8.81846	11.18154	10.00094	11.18248	2.33448	3.97049	14
47	8.81944	9.99905	8.82038	11.17962	10.00095	11.18056	2.33832	4.97035	13
48	8.82134	9.99904	8.82230	11.17770	10.00096	11.17866	2.34213	4.97022	12
49	8.82324	9.99904	8.82420	11.17580	10.00096	11.17676	2.34593	4.97008	11
50	8.82513	9.99903	8.82610	11.17390	10.00097	11.17487	2.34972	4.96995	10
51	8.82701	9.99902	8.82799	11.17201	10.00098	11.17299	2'35348	4.96981	9
52	8.82888	9.99901	8.82987	11.17013	10.00099	11.17112	2.35723	4.96968	8
53	8.83075	9.99900	8.83175	11.16825	10.00100	11.16925	2.36097	4.96954	7
54	8.83261	9.99899	8.83361	11.16639	10.00101	11.16739	2.36469	4.96941	6
55	8.83446	9.99898	8.83547	11.16453	10.00102	11.16554	2.36839	4.96927	5
56	8.83630	9.99898	8.83732	11.16268	10.00102	11.16370	2.37208	4.96914	4
57	8.83813	9.99897	8.83916	11.16084	10.00103	11.16187	2.37575	4.96900	3
58	8.83996	9.99896	8.84100	11.15900	10.00104	11.16004	2.37940	4.96887	2
59	8.84177	9.99895	8.84282	11.15718	10.00105	11.15823	2.38304	4.96873	1
60	8.84358	9.99894	8.84464	11.15536	10.00106	11.15642	2.38667	4.96860	0
	Co-sine	Sine	Co-tan.	Tangent	Co-sec.	Secant		V. Sine	M

M	Sine	Co-fine	Tang.	Co-tang.	Secant	Co-fec.	V. Sine		
0	8.84358	9.99894	8.84464	11.15536	10.00106	11.15642	2.38667	4.96860	60
1	8.84539	9.99893	8.84646	11.15354	10.00107	11.15461	2.39028	4.96846	59
2	8.84718	9.99892	8.84826	11.15174	10.00108	11 15282	2.39387	4.96833	58
3	8.84897	9.99891	8.85006	11.14994	10.00109	11.15103	2.39745	4.96819	57
4	8.85075	9.99891	8.85185	11.14815	10.00109	11.14923	2.40102	4.96805	56
5	8.85252	9.99890	8.85363	11.14637	10.00110	11.14748	2.40457	4.96792	55
6	8 85429	9.99889	8.85540	11.14460	10.00111	11.14571	2.40811	4.96778	54
7	8.85605	9.99888	8.85717	11.14283	10.00112	11.14395	2.41163	4.96765	53
8	8.85780	9.99887	8.85893	11.14107	10.00113	11.14220	2.41514	4.96751	52
9	8.85955	9.99886	8.86069	11.13931	10.00114	11.14045	2.41863	4.96738	51
10	8.86128	9.99885	8.86243	11.13757	10.00115	11.13872	2.42211	4.96724	50
11	8.86301	9.99884	8.86415	11.13583	10.00116	11.13699	2.42558	4.96710	49
12	8.86474	9.99883	8.86591	11.13409	10.00117	11.13526	2.42903	4.96697	48
13	8.86645	9.99882	8.86763	11.13237	10.00118	11.13355	2.43247	4.96683	47
14	8.86816	9.99881	8.86935	11.13065	10.00119	11.13184	2.43589	4.96670	46
15	8.86987	9.99880	8.87106	11.12894	10.00120	11.13013	2.43930	4.96656	45
16	8.87156	9.99879	8.87277	11.12723	10.00121	11.12844	2.44170	4.96642	44
17	8.87325	9.99879	8.87447	11.12553	10.00121	11.12675	2.44609	4.96629	43
18	8.87494	9.99878	8.87616	11.12384	10.00122	11.12506	2.44946	4.96615	42
19	8.87661	9.99877	8.87785	11.12215	10.00123	11.12339	2.45282	4.96602	41
20	8.87828	9.99876	8.87953	11.12047	10.00124	11.12171	2.45616	4.96588	40
21	8.87995	9.99875	8.88120	11.11880	10.00125	11.12005	2.45949	4.96574	39
22	8.88161	9.99874	8.88287	11.11713	10.00126	11.11839	2.46281	4.96561	38
23	8.88326	9.99873	8.88453	11.11547	10.00127	11.11674	2.46612	4.96547	37
24	8.88490	9.99872	8.88618	11.11381	10.00128	11.11510	2.46942	4.96533	36
25	8.88654	9.99871	8.88783	11.11217	10.00129	11.11346	2.47270	4.96520	35
26	8.88817	9.99870	8.88948	11.11052	10.00130	11.11183	2.47597	4.96506	34
27	8.88980	9.99869	8.89111	11.10889	10.00131	11.11020	2.47923	4.96492	33
28	8.89142	9.99868	8.89274	11.10726	10.00132	11.10858	2.48247	4.96479	31
29	8.89304	9.99867	8.89431	11.10563	10.00133	11.10696	2.48571	4.96465	32
30	8.89464	9.99866	8.89598	11.10402	10.00134	11.10536	2.48893	4.96451	30
31	8.89625	9.99865	8.89760	11.10240	10.00135	11.10375	2 49214	4.96438	29
32	8.89784	9.99864	8.89920	11.10080	10.00136	11.10216	2.49534	4.96424	28
33	8.89943	9.99863	8.90080	11.09920	10.00137	11.10057	2.49852	4.96410	27
34	8.90102	9.99862	8.90249	11.09760	10.00138	11.09898	2.50169	4.96397	26
35	8.90260	9.99861	8.90399	11.09601	10.00139	11.09740	2 50486	4.96383	25
36	8.90417	9.99860	8.90557	11.09443	10.00140	11.09583	2 50801	4.96369	24
37	8.90574	9.99859	8.90715	11.09285	10.00141	11 09426	2.51115	4.96356	23
38	8.90730	9.99858	8.90872	11.09128	10.00142	11.09270	2.51428	4.96342	22
39	8.90885	9.99857	8.91029	11.08971	10.00143	11.09115	2.51739	4.96328	21
40	8.91040	9.99856	8.91185	11.08815	10.00144	11.08960	2.52050	4.96315	20
41	8.91195	9.99855	8.91340	11.08660	10.00145	11.08805	2.52359	4.96301	19
42	8.91349	9.99854	8.91495	11.08505	10.00146	11.08651	2.52668	4.96287	18
43	8.91502	9.99853	8.91650	11.08350	10.00147	11.08498	2.52975	4.96273	17
44	8.91655	9.99852	8.91803	11.08197	10.00148	11.08345	2.53281	4.96260	16
45	8 91807	9.99851	8.91957	11.08043	10.00149	11.08193	2.53586	4.96246	15
46	8.91959	9.99850	8.92110	11 07890	10.00150	11.08041	2.53890	4.96232	14
47	8.92110	9.99848	8.92262	11.07738	10.00152	11.07890	2.54193	4.96219	13
48	8.92261	9.99847	8.92414	11.07586	10.00153	11.07739	2.54495	4.96205	12
49	8.92411	9.99846	8.92565	11.07435	10.00154	11.07589	2.54790	4.96191	11
50	8.92561	9.99845	8.92716	11.07284	10.00155	11.07439	2.55096	4.96177	10
51	8.92710	9.99844	8.92866	11.07134	10.00156	11.07290	2.55395	4.96164	9
52	8.92859	9.99843	8.93015	11 06984	10.00157	71.07141	2.55693	4.96150	8
53	8.93007	9.99842	8.93165	11.06835	10.00158	11.06993	2.55989	4.96136	7
54	8.93154	9.99841	8.93313	11.06687	10.00659	11.06846	2.56285	4.96122	6
55	8.93301	9.99840	8.93462	11.06538	10.00160	11.06699	2.56580	4.96109	5
56	8.93448	9.99839	8.93609	11.06391	10.00161	11.06552	2.56874	4.96095	4
57	8.93594	9.99838	8.93756	11.06243	10.00162	11.06406	2.57166	4.96081	3
58	8.93749	9.99837	8.93903	11.06097	10.00163	11.06260	2.57458	4.96067	2
59	8.93885	9.99836	8.94049	11.05951	10.00164	11.06115	2.57749	4.96053	1
60	8.94030	9.99834	8.94195	11.05805	10.00166	11.05970	2.58039	4.96040	0
	Co-fine	Sine	Co-tan.	Tangent	Co-fec.	Secant		V. Sine	M

M	Sine	Co-sine	Tang.	Co-tang.	Secant	Co-sec.	V. Sine		
0	8.94030	9.99834	8.94195	11.05805	10.00166	11.05970	2.58039	4.96040	60
1	8.94174	9.99833	8.94340	11.05660	10.00167	11.05826	2.58328	4.96026	59
2	8.94317	9.99832	8.94485	11.05515	10.00168	11.05683	2.58616	4.96012	58
3	8.94461	9.99831	8.94630	11.05370	10.00169	11.05539	2.58903	4.95998	57
4	8.94603	9.99830	8.94773	11.05227	10.00170	11.05397	2.59189	4.95984	56
5	8.94746	9.99829	8.94917	11.05083	10.00171	11.05254	2.59473	4.95971	55
6	8.94887	9.99828	8.95060	11.04940	10.00172	11.05113	2.59758	4.95957	54
7	8.95029	9.99827	8.95202	11.04798	10.00173	11.04971	2.60041	4.95943	53
8	8.95170	9.99825	8.95344	11.04656	10.00175	11.04830	2.60323	4.95929	52
9	8.95310	9.99824	8.95486	11.04514	10.00176	11.04690	2.60605	4.95915	51
10	8.95450	9.99823	8.95627	11.04373	10.00177	11.04550	2.60885	4.95902	50
11	8.95589	9.99822	8.95767	11.04233	10.00178	11.04411	2.61165	4.95888	49
12	8.95728	9.99821	8.95908	11.04092	10.00179	11.04272	2.61443	4.95874	48
13	8.95867	9.99820	8.96047	11.03953	10.00180	11.04133	2.61721	4.95860	47
14	8.96005	9.99819	8.96187	11.03813	10.00181	11.03995	2.61998	4.95846	46
15	8.96143	9.99817	8.96325	11.03675	10.00183	11.03857	2.62274	4.95832	45
16	8.96280	9.99816	8.96464	11.03536	10.00184	11.03720	2.62549	4.95819	44
17	8.96417	9.99815	8.96602	11.03398	10.00185	11.03583	2.62823	4.95805	43
18	8.96553	9.99814	8.96739	11.03261	10.00186	11.03447	2.63097	4.95791	42
19	8.96689	9.99813	8.96877	11.03123	10.00187	11.03311	2.63369	4.95777	41
20	8.96825	9.99812	8.97013	11.02987	10.00188	11.03175	2.63641	4.95763	40
21	8.96960	9.99810	8.97150	11.02850	10.00190	11.03040	2.63912	4.95749	39
22	8.97095	9.99809	8.97285	11.02715	10.00191	11.03905	2.64182	4.95735	38
23	8.97229	9.99808	8.97421	11.02579	10.00192	11.02771	2.64451	4.95722	37
24	8.97363	9.99807	8.97556	11.02444	10.00193	11.02637	2.64719	4.95708	36
25	8.97496	9.99806	8.97691	11.02309	10.00194	11.02504	2.64987	4.95694	35
26	8.97629	9.99804	8.97825	11.02175	10.00196	11.02371	2.65253	4.95680	34
27	8.97762	9.99803	8.97959	11.02041	10.00197	11.02238	2.65519	4.95666	33
28	8.97894	9.99802	8.98092	11.01908	10.00198	11.02106	2.65784	4.95652	32
29	8.98026	9.99801	8.98225	11.01775	10.00199	11.01974	2.66048	4.95638	31
30	8.98157	9.99800	8.98358	11.01642	10.00200	11.01843	2.66312	4.95624	30
31	8.98288	9.99798	8.98490	11.01510	10.00202	11.01712	2.66574	4.95610	29
32	8.98419	9.99797	8.98622	11.01378	10.00203	11.01581	2.66836	4.95596	28
33	8.98549	9.99796	8.98753	11.01247	10.00204	11.01451	2.67097	4.95583	27
34	8.98679	9.99795	8.98884	11.01116	10.00205	11.01321	2.67357	4.95569	26
35	8.98808	9.99793	8.99015	11.00985	10.00207	11.01192	2.67617	4.95555	25
36	8.98937	9.99792	8.99145	11.00855	10.00208	11.01063	2.67875	4.95541	24
37	8.99066	9.99791	8.99275	11.00725	10.00109	11.00934	2.68133	4.95527	23
38	8.99194	9.99790	8.99405	11.00595	10.00210	11.00806	2.68391	4.95513	22
39	8.99322	9.99788	8.99534	11.00466	10.00212	11.00678	2.68647	4.95499	21
40	8.99450	9.99787	8.99662	11.00338	10.00213	11.00550	2.68903	4.95485	20
41	8.99577	9.99786	8.99791	11.00209	10.00214	11.00423	2.69157	4.95471	19
42	8.99704	9.99785	9.99919	11.00081	10.00215	11.00296	2.69412	4.95457	18
43	8.99830	9.99783	9.00046	10.99954	10.00217	11.00170	2.69665	4.95443	17
44	8.99956	9.99782	9.00174	10.99826	10.00218	11.00044	2.69918	4.95429	16
45	9.00082	9.99781	9.00301	10.99699	10.00219	10.99918	2.70170	4.95415	15
46	9.00207	9.99780	9.00427	10.99573	10.00220	10.99793	2.70421	4.95401	14
47	9.00332	9.99778	9.40553	10.99447	10.00222	10.99668	2.70671	4.95387	13
48	9.00456	9.99777	9.00679	10.99321	10.00223	10.99544	2.70921	4.95373	12
49	9.00581	9.99776	9.00805	10.99195	10.00224	10.99419	2.71170	4.95359	11
50	9.00704	9.99775	9.00930	10.99070	10.00225	10.99296	2.71418	4.95345	10
51	9.00828	9.99773	9.01055	10.98945	10.00227	10.99172	2.71666	4.95331	9
52	9.00951	9.99772	9.01179	10.98821	10.00228	10.99049	2.71913	4.95317	8
53	9.01074	9.99771	9.01303	10.98697	10.00229	10.98926	2.72159	4.95303	7
54	9.01196	9.99769	9.01427	10.98573	10.00231	10.98804	2.72404	4.95289	6
55	9.01318	9.99768	9.01550	10.98450	10.00232	10.98682	2.72649	4.95275	5
56	9.01440	9.99767	9.01673	10.98327	10.00233	10.98560	2.72893	4.95261	4
57	9.01561	9.99765	9.01796	10.98204	10.00235	10.98439	2.73137	4.95247	3
58	9.01682	9.99764	9.01918	10.98082	10.00236	10.98318	2.73377	4.95233	2
59	9.01803	9.99763	9.02040	10.97960	10.00237	10.98197	2.73622	4.95219	1
60	9.01923	9.99761	9.02161	10.97838	10.00239	10.98077	2.73863	4.95205	0
	Co-sine	Sine	Co-tan.	Tangent	Co-sec.	Secant		V. Sine	M

M	Sine	Co-ſine	Tang.	Co-tang.	Secant	Co-ſec.	V. Sine		
0	9.01923	9.99761	9.02162	10.97838	10.00239	10.98077	2.73863	4.95205	60
1	9.02043	9.99760	9.02283	10.97717	10.0024	10.97957	2.74104	4 95191	59
2	9.02163	9.99759	9.02404	10.97596	10.00241	10.97837	2.74344	4.95177	58
3	9.02283	9.99757	9.02525	10.97475	10.00243	10.97717	2.74583	4.95163	57
4	9.02402	9.99756	9.02645	10.97355	10.00244	10.97598	2.74822	4.95149	56
5	9.02520	9.99755	9.02766	10.97234	10.00245	10.97480	2.75060	4.95135	55
6	9.02639	9.99753	9.02885	10.97115	10.00247	10.97362	2.75297	4.95121	54
7	9.02757	9.99752	9.03005	10.96995	10.00248	10.97243	2.75534	4.95107	53
8	9.02874	9.99751	9.03124	10.96876	10.00249	10.97126	2.75770	4.95093	52
9	9.02992	9.99749	9.03242	10.96758	10.00251	10.97008	2.76006	4.95079	51
10	9.03109	9.99748	9.03361	10.96639	10.00252	10.96891	2.76241	4.95065	50
11	9.03226	9.99747	9.03479	10.96521	10.00253	10.96774	2.76475	4.95051	49
12	9.03342	9.99745	9.03597	10.96403	10.00255	10.96658	2.76708	4.95037	48
13	9.03458	9.99744	9.03714	10.96286	10.00256	10.96542	2.76941	4.95024	47
14	6.03574	9.99742	9.03832	10.96168	10.00258	10.96423	2.77174	4.95008	46
15	9.03690	9.99741	9.03948	10.96052	10.00259	10.96310	2.77405	4.94994	45
16	9.03805	9.99740	9.04065	10.95935	10.00260	10.96195	2.77636	4.94980	44
17	9.03920	9.99738	9.04181	10.93819	10.00262	10.96080	2.77867	4 94966	43
18	9.04034	9.99737	9.04297	10.95703	10.00263	10.95966	2.78097	4.94952	42
19	9.04149	9.99736	9.04413	10.95587	10.00264	10.95851	2.78329	4.94938	41
20	9.04262	9.99734	9.04528	10.95472	10.00266	10.95738	2.78555	4.94924	40
21	9.04376	9.99733	9.04643	10.95357	10.00267	10.95624	2.78783	4.94910	39
22	9.04490	9.99731	9.04758	10.95242	10.00269	10.95510	2.79010	4.94896	38
23	9.04603	9.99730	9.04873	10.95127	10.00270	10.95397	2.79237	4.94881	37
24	9.04715	9.99728	9.04987	10.95013	10.00272	10.95285	2.79463	4.94867	36
25	9.04828	9.99727	9.05101	10.94899	10.00273	10.95172	2.79689	4.94853	35
26	9.04940	9.99726	9.05214	10.94786	10.00274	10.95060	2.79913	4.94839	34
27	9.05052	9.99724	9.05328	10.94672	10 00276	10.94948	2.80139	4.94825	33
28	9.05164	9.99723	9.05441	10.94559	10.00277	10.94836	2.80362	4.94811	32
29	9.05295	9.99721	9.05553	10.94447	10.00279	10.94725	2.80986	4.94797	31
30	9.05386	9.99720	9.05666	10.94334	10.00280	10.94614	2.80809	4.94782	30
31	9.05497	9.99718	9.05778	10.94222	10.00282	10.94503	2.81031	4.94768	29
32	9 05607	9.99717	9.05890	10.94110	10.00283	10.94393	2.81231	4.94754	28
33	9.05717	9.99716	9.06002	10.93998	10.00284	10 94283	2.81473	4.94740	27
34	9.05827	9.99714	9.06113	10.93887	10.00286	10.94173	2.81694	4.94726	26
35	9.05937	9.99713	9.06224	10.93776	10.00287	10.94063	2.81914	4.94712	25
36	9.06046	9.99711	9.06335	10.93665	10.00289	10.93954	2.82133	4.94697	24
37	9.06155	9.99710	9.06445	10.93555	10.00290	10.93845	2.92352	4.94683	23
38	9.06264	9.99708	9.06556	10.93445	10.00292	10.93736	2.82570	4.94669	22
39	9.06372	9.99707	9 06666	10.93334	10.00293	10.93628	2.82788	4.94655	21
40	9.06481	9.99705	9.06775	10.93225	10.00395	10.93519	2.83005	4.94641	20
41	9.06589	9.99704	9.06885	10.93115	10.00296	10.93411	2.83222	4 94626	19
42	9.06696	9.99702	9.06994	10.93006	10.00298	10.93304	2.83438	4.94611	18
43	9.06804	9.99701	9.07104	10.92897	10.00299	10.93196	2.83653	4.94598	17
44	9.06911	9.99699	9.07211	10.92789	10.00301	10.93089	2.83869	4.94584	16
45	9.07018	9.99698	9.07320	10.92680	10.00302	10.92982	2.84083	4.94570	15
46	9.07124	9.99696	9.07428	10.92572	10.00304	10.92876	2.84297	4.94555	14
47	9.07231	9.99695	9.07536	10.92464	10.00305	10.92769	2.84510	4.94541	13
48	9.07337	9.99693	9.07643	10.92357	10.00307	10.92663	2.84723	4.94527	12
49	9.07442	9.99692	9.07751	10.92249	10.00308	10.92558	2.84936	4.94513	11
50	9.07548	9.99690	9.07858	10.92142	10.00310	10.92452	2.85148	4 94498	10
51	9.07653	9.99689	9.07964	10.92036	10.00311	10.92347	2.85359	4.94484	9
52	9.07758	9.99687	9.08071	10.91929	10.00313	10.92242	2.85570	4.94470	8
53	9.07863	9.99686	9.08177	10.91823	10.00314	10.92137	2.85780	4.94456	7
54	9.07968	9.99684	9.08283	10.91717	10.00316	10.92032	2.85990	4.94442	6
55	9.08072	9.99683	9.08389	10.91611	10.00317	10.91928	2.86199	4.94427	5
56	9.08176	9.99681	9.08495	10.91505	10.00319	10.91824	2.86408	4.94413	4
57	9.08280	9.99680	9.08600	10.91400	10.00320	10.91720	2.89615	4.94399	3
58	9.08383	9.99678	9.08705	10.91295	10.00322	10.91617	2.86824	4.94384	2
59	9.08486	9.99677	9.08810	10.91190	10.00323	10.91514	2.87031	4.94370	1
60	9.08589	9.99675	9.08914	10.91086	10.00325	10.91411	2.87238	4.94356	0
	Co-ſine	Sine	Co-tan.	Tangent	Co-ſec.	Secant		V. Sine	M

M	Sine	Co-sine	Tang.	Co-tang.	Secant	Co-sec.	V'Sine		
0	9.08589	9.99675	9.08914	10.91086	10.00325	10.91411	2.87238	4.94356	60
1	9.08692	9.99674	9.09019	10.90981	10.00326	10.91308	2.87444	4.94342	59
2	9.08795	9.99672	9.09123	10.90877	10.00328	10.91205	2.87650	4.94327	58
3	9.08897	9.99670	9.09227	10.90773	10.00330	10.91103	2.88855	4.94313	57
4	9.08999	9.99669	9.09330	10.90670	10.00331	10.91001	2.88060	4.94299	56
5	9.09101	9.99667	9.09434	10.90566	10.00333	10.90899	2.88265	4.94284	55
6	9.09202	9.99666	9.09537	10.90463	10.00334	10.90798	2.88469	4.94270	54
7	9.09304	9.99664	9.09640	10.90360	10.00336	10.90696	2.88672	4.94256	53
8	9.09405	9.99663	9.09742	10.90258	10.00337	10.90595	2.88875	4.94242	52
9	9.09504	9.99661	9.09845	10.90155	10.00339	10.90494	2.89078	4.94227	51
10	9.09606	9.99659	9.09947	10.90053	10.00341	10.90394	2.89279	4.94213	50
11	9.09707	9.99658	9.00049	10.89951	10.00342	10.90293	2.89481	4.94199	49
12	9.09807	9.99656	9.10150	10.89850	10.00344	10.90193	2.89682	4.94184	48
13	9.09906	9.99655	9.10252	10.89748	10.00345	10.80093	2.89882	4.94170	47
14	9.10006	9.99653	9.10353	10.89647	10.00347	10.89994	2.90082	4.94156	46
15	9.10106	9.99651	9.10454	10.89545	10.00349	10.89894	2.90282	4.94141	45
16	9.10205	9.99650	9.10555	10.89444	10.00350	10.89795	2.90481	4.94127	44
17	9.10304	9.99648	9.10656	10.89344	10.00252	10.89696	2.90680	4.94113	43
18	9.10402	9.99647	9.10756	10.89244	10.00353	10.89598	2.90878	4.94098	42
19	9.10501	9.99645	9.10856	10.89144	10.00355	10.89499	2.91076	4.94084	41
20	9.10599	9.99643	9.10956	10.89044	10.00357	10.89401	2.91273	4.94069	40
21	9.10697	9.99642	9.11056	10.88944	10.00358	10.89303	2.91470	4.94055	39
22	9.10795	9.99640	9.11155	10.88845	10.00360	10.89205	2.91667	4.94041	38
23	9.10893	9.99638	9.11254	10.88746	10.00362	10.89107	2.91863	4.94026	37
24	9.10990	9.99637	9.11353	10.88647	10.00363	10.89010	2.92058	4.94012	36
25	9.11087	9.99636	9.11452	10.88548	10.00365	10.88913	2.92254	4.93998	35
26	9.11184	9.69633	9.11551	10.88449	10.00367	10.88816	2.92448	4.93983	34
27	9.11281	9.99632	9.11649	10.88341	10.00368	10.88719	2.92643	4.93969	33
28	9.11377	9.99630	9.11747	10.88243	10.00370	10.88623	2.92836	4.93954	32
29	9.11474	9.99629	9.11845	10.88155	10.00371	10.88526	2.93030	4.93940	31
30	9.11570	9.99627	9.11943	10.88057	10.00373	10.88430	2.93223	4.93926	30
31	9.11666	9.99626	9.12040	10.87960	10.00375	10.88334	2.93415	4.93912	29
32	9.11761	9.99624	9.12138	10.87862	10.00376	10.88239	2.93607	4.93897	28
33	9.11857	9.99622	9.12235	10.87765	10.00378	10.88143	2.93799	4.93882	27
34	9.12952	9.99620	9.12332	10.87668	10.00380	10.88048	2.93990	4.93868	26
35	9.12047	9.99618	9.12428	10.87572	10.00382	10.87953	2.94181	4.93854	25
36	9.12142	9.99617	9.12525	10.87475	10.00383	10.87858	2.94372	4.93839	24
37	9.12236	9.99615	9.12621	10.87379	10.00385	10.87764	2.94561	4.93824	23
38	9.12331	9.99613	9.12717	10.87282	10.00387	10.87669	2.94751	4.93810	22
39	9.12425	9.99612	9.12813	10.87187	10.00488	10.87575	2.94940	4.93796	21
40	9.12519	9.99610	9.12909	10.87091	10.00390	10.87481	2.95129	4.93781	20
41	9.12612	9.99608	9.13004	10.86996	10.00392	10.87388	2.95317	4.93797	19
42	9.12706	9.99607	9.13099	10.86901	10.00393	10.87294	2.95505	4.93752	18
43	9.12799	9.99605	9.13194	10.86806	10.00395	10.87201	2.95695	4.93738	17
44	9.12892	9.99603	9.13289	10.86711	10.00397	10.87108	2.95880	4.93724	16
45	9.12985	9.99601	9.13384	10.86616	10.00399	10.87015	2.96067	4.93709	15
46	9.13079	9.99600	9.13478	10.86522	10.00400	10.86922	2.96253	4.93695	14
47	9.13171	9.99598	9.13573	10.86427	10.00402	10.86829	2.96439	4.93680	13
48	9.13263	9.99596	9.13667	10.86333	10.00404	10.86737	2.96624	4.93666	12
49	9.13355	9.99595	9.13761	10.86239	10.00405	10.86644	2.96809	4.93651	11
50	9.13447	9.99593	9.13854	10.86146	10.00407	10.86553	2.96994	4.93637	10
51	9.13539	9.99591	9.13948	10.86052	10.00409	10.06461	2.97178	4.93623	9
52	9.13630	6.99589	9.14041	10.85959	10.00411	10.86370	2.97362	4.93608	8
53	9.13721	9.99588	9.14134	10.85866	10.00412	10.86278	2.97546	4.93593	7
54	9.13813	9.99586	9.14227	10.85773	10.00414	10.86187	2.97729	4.93579	6
55	9.13904	9.99584	9.14520	10.85680	10.00416	10.86096	2.97912	4.93564	5
56	9.13994	9.99582	9.14412	10.85588	10.00418	10.86006	2.98094	4.93550	4
57	9.14085	9.99581	9.14504	10.85496	10.00419	10.85915	2.98276	4.93532	3
58	9.14175	9.99579	9.14597	10.85403	10.00421	10.85825	2.98458	4.93521	2
59	9.14266	9.99577	9.14688	10.85312	10.00423	10.85734	2.98639	4.93506	1
60	9.14356	9.99575	9.14780	10.85220	10.00425	10.85644	2.98820	4.93492	0
	Co-sine	Sine	Co-tan.	Tangent	Co-sec.	Secant		V. Sine	M

M	Sine	Co-ſine	Tang.	Co-tan	Secant	Co-ſec.	V. Sine		
0	9.14356	9.99575	9.14780	10.85220	10.00425	10.85644	2.98820	4.93492	60
1	9.14445	9.99573	9.14872	10.85128	10.00427	10.85555	2.99000	4.93477	59
2	9.14535	9.99572	9.14963	10.85037	10.00428	10.85465	2.99180	4.93463	58
3	9.14624	9.99570	9.15054	10.84946	10.00430	10.85376	2.99360	4.93448	57
4	9.14714	9.99568	9.15145	10.84855	10.00432	10.85286	2.99540	4.93433	56
5	9.14803	9.99566	9.15236	10.84764	10.00434	10.85197	2.99719	4.93419	55
6	9.14891	9.99565	9.15327	10.84673	10.00435	10.85109	2.99897	4.93404	54
7	9.14980	9.99563	9.15417	10.84583	10.00437	10.85020	3.00075	4.93390	53
8	9.15069	9.99561	9.15508	10.84492	10.00439	10.84931	3.00253	4.93375	52
9	9.15157	9.99559	9.15598	10.84402	10.00441	10.84843	3.00431	4.93361	51
10	9.15245	9.99557	9.15688	10.84312	10.00443	10.84755	3.00608	4.93346	50
11	9.15333	9.99556	9.15777	10.84223	10.00444	10.84667	3.00785	4.93331	49
12	9.15421	9.99554	9.15867	10.84133	10.00446	10.84579	3.00961	4.93317	48
13	9.15508	9.99552	9.15956	10.84044	10.00448	10.84492	3.01137	4.93302	47
14	9.15596	9.99550	9.16046	10.83954	10.00450	10.84404	3.01313	4.93288	46
15	9.15683	9.99548	9.16135	10.83865	10.00452	10.84317	3.01488	4.93273	45
16	9.15770	9.99546	9.16224	10.83776	10.00454	10.84230	3.01663	4.93259	44
17	9.15857	9.99545	9.16312	10.83688	10.00455	10.84143	3.01838	4.93244	43
18	9.15944	9.99543	9.16401	10.83599	10.00457	10.84056	3.02012	4.93229	42
19	9.16030	9.99541	9.16489	10.83511	10.00459	10.83970	3.02186	4.93215	41
20	9.16116	9.99539	9.16577	10.83423	10.00461	10.83884	3.02360	4.93200	40
21	9.16203	9.99537	9.16665	10.83335	10.00463	10.83797	3.02533	4.93185	39
22	9.16289	9.99535	9.16753	10.83247	10.00465	10.83711	3.02706	4.93171	38
23	9.16374	9.99533	9.16841	10.83159	10.00467	10.83626	3.02878	4.93156	37
24	9.16460	9.99532	9.16928	10.83072	10.00468	10.83540	3.03051	4.93142	36
25	9.16545	9.99530	9.17016	10.82984	10.00470	10.83455	3.03222	4.93127	35
26	9.16631	9.99528	9.17103	10.82897	10.00472	10.83369	3.03394	4.93112	34
27	9.16716	9.99526	9.17190	10.82810	10.00474	10.83284	3.03565	4.93098	33
28	9.16801	9.99524	9.17277	10.82723	10.00476	10.83199	3.03736	4.93083	32
29	9.16886	9.99522	9.17363	10.82637	10.00478	10.83114	3.03906	4.93068	31
30	9.16970	9.99520	9.17450	10.82550	10.00480	10.83030	3.04077	4.93054	30
31	9.17055	9.99518	9.17536	10.82464	10.00482	10.82944	3.04246	4.93039	29
32	9.17139	9.99517	9.17622	10.82378	10.00483	10.82861	3.04416	4.93024	28
33	9.17223	9.99515	9.17708	10.82292	10.00485	10.82777	3.04585	4.93010	27
34	9.17307	9.99513	9.17794	10.82206	10.00487	10.82693	3.04754	4.92995	26
35	9.17391	9.99511	9.17880	10.82120	10.00489	10.82609	3.04922	4.92980	25
36	9.17474	9.99509	9.17965	10.82035	10.00491	10.82526	3.05091	4.92966	24
37	9.17558	9.99507	9.18051	10.81949	10.00493	10.82442	3.05258	4.92951	23
38	9.17641	9.99505	9.18136	10.81864	10.00495	10.82359	3.05426	4.92926	22
39	9.17724	9.99503	9.18221	10.81779	10.00497	10.82276	3.05593	4.92922	21
40	9.17807	9.99501	9.18306	10.81694	10.00499	10.82193	3.05760	4.92907	20
41	9.17890	9.99499	9.18391	10.81609	10.00501	10.82110	3.05927	4.92892	19
42	9.17973	9.99497	9.18475	10.81525	10.00503	10.82027	3.06093	1.92877	18
43	9.18055	9.99495	9.18500	10.81440	10.00505	10.81945	3.06259	4.92863	17
44	9.18137	9.99494	9.18644	10.81356	10.00506	10.81863	3.06414	4.92848	16
45	9.18220	9.99492	9.18728	10.81272	10.00508	10.81780	3.06590	4.92833	15
46	9.18302	9.99490	9.18812	10.81188	10.00510	10.81698	3.06755	4.92819	14
47	9.18383	9.99488	9.18896	10.81104	10.00512	10.81617	3.06919	4.92804	13
48	9.18465	9.99486	9.18979	10.81021	10.00514	10.81535	3.07084	4.92789	12
49	9.18547	9.99484	9.19063	10.80937	10.00516	10.81453	3.07248	4.92774	11
50	9.18628	9.99482	9.19146	10.80854	10.00518	10.81372	3.07411	4.92761	10
51	9.18709	9.99480	9.19229	10.80771	10.00520	10.81291	3.07575	4.92745	9
52	9.18790	9.99478	9.19312	10.80688	10.00522	10.81210	3.07738	4.92730	8
53	9.18871	9.99476	9.19395	10.80605	10.00524	10.81129	3.07901	4.92715	7
54	9.18952	9.99474	9.19478	10.80522	10.00526	10.81048	3.08063	4.92701	6
55	9.19033	9.99472	9.19561	10.80439	10.00528	10.80967	3.08225	4.92686	5
56	9.19113	9.99470	9.19643	10.80357	10.00530	10.80887	3.08387	4.92671	4
57	9.19193	9.99468	9.19725	10.80275	10.00532	10.80807	3.08550	4.92656	3
58	9.19273	9.99466	9.19807	10.80193	10.00534	10.80727	3.08710	4.92641	2
59	9.19353	9.99464	9.19889	10.80111	10.00536	10.80647	3.08871	4.92627	1
60	9.19433	9.99462	9.19971	10.80029	10.00538	10.80567	3.09032	4.92612	0
	Co-ſine	Sine	Co-tan.	Tangen.	Co-ſec.	Secant		V. Sine	M

(81 Deg.)

M	Sine	Co-sine	Tang.	Co-tang.	Secant	Co-sec.	V. Sine		
0	9.19433	9.99462	9.19971	10.80029	10.00538	10.80567	3.09032	4.92612	60
1	9.19513	9.99460	9.20053	10.79947	10.00540	10.80487	3.09192	4.92597	59
2	9.19592	9.99458	9.20134	10.79866	10.00542	10.80408	3.09352	4.92582	58
3	9.19672	9.99456	9.20216	10.79784	10.00544	10.80328	3.09512	4.92567	57
4	9.19751	9.99454	9.20297	10.79703	10.00546	10.80249	3.09671	4.92553	56
5	9.19830	9.99452	9.20398	10.59622	10.00548	10.80170	3.09831	4.92537	55
6	9.19909	9.99450	9.20459	10.79541	10.00550	10.80091	3.09989	4.92523	54
7	9.19988	9.99448	9.20540	10.79460	10.00552	10.80012	3.10148	4.92508	53
8	9.20067	9.99446	9.20621	10.79379	10.00554	10.79933	3.10306	4.92493	52
9	9.20145	9.99444	9.20701	10.79299	10.00556	10.79855	3.10464	4.92479	51
10	9.20223	9.99442	9.20782	10.79218	10.00558	10.79777	3.10622	4.92464	50
11	9.20302	9.99440	9.20862	10.79138	10.00560	10.79698	3.10780	4.92449	49
12	9.20380	9.99438	9.20942	10.79058	10.00562	10.79620	3.10937	4.92434	48
13	9.20458	9.99436	9.21022	10.78978	10.00564	10.79542	3.11094	4.92419	47
14	9.20535	9.99434	9.21102	10.78898	10.00566	10.79465	3.11250	4.92404	46
15	9.20613	9.99432	9.21182	10.78818	10.00568	10.79387	3.11406	4.92390	45
16	9.20691	9.99429	9.21261	10.78739	10.00571	10.79309	3.11562	4.92375	44
17	9.20768	9.99427	9.21340	10.78659	10.00572	10.79232	3.11718	4.92360	43
18	9.20845	9.99425	9.21420	10.78580	10.00575	10.79155	3.11874	4.92345	42
19	9.20922	9.99423	9.21499	10.78501	10.00577	10.79078	3.12029	4.92330	41
20	9.20999	9.99421	9.21578	10.78422	10.00579	10.79001	3.12184	4.92315	40
21	9.21076	9.99419	9.21657	10.78343	10.00581	10.78924	3.12338	4.92300	39
22	9.21153	9.99417	9.21736	10.78264	10.00583	10.78847	3.12493	4.92285	38
23	9.21229	9.99415	9.21814	10.78186	10.00585	10.78771	3.12647	4.92271	37
24	9.21306	9.99413	9.21893	10.78107	10.00587	10.78694	3.12801	4.92256	36
25	9.21382	9.99411	9.21971	10.78029	10.00589	10.78618	3.12954	4.92241	35
26	9.21458	9.99409	9.22049	10.77951	10.00591	10.78542	3.13107	4.92226	34
27	9.21534	9.99407	9.22127	10.77873	10.00593	10.78466	3.13260	4.92211	33
28	9.21610	9.99404	9.22205	10.77795	10.00596	10.78390	3.13413	4.92196	32
29	9.21685	9.99402	9.22283	10.77717	10.00598	10.78315	3.13566	4.92181	31
30	9.21761	9.99400	9.22361	10.77639	10.00600	10.78239	3.13718	4.92166	30
31	9.21836	9.99398	9.22438	10.77562	10.00602	10.78164	3.13870	4.92151	29
32	9.21912	9.99396	9.22516	10.77484	10.00604	10.78089	3.14021	4.92136	28
33	9.21987	9.99394	9.22593	10.77407	10.00606	10.78013	3.14173	4.92121	27
34	9.22062	9.99392	9.22670	10.77330	10.00608	10.77938	3.14324	4.92106	26
35	9.22137	9.99390	9.22747	10.77253	10.00610	10.77863	3.14475	4.92091	25
36	9.22211	9.99388	9.22824	10.77176	10.00612	10.77789	3.14625	4.92077	24
37	9.22286	9.99385	9.22901	10.77099	10.00615	10.77714	3.14775	4.92062	23
38	9.22361	9.99383	9.22977	10.77023	10.00617	10.77639	3.14925	4.92047	22
39	9.22435	9.99381	9.23054	10.76946	10.00619	10.77565	3.15075	4.92032	21
40	9.22509	9.99379	9.23130	10.76870	10.00621	10.77491	3.15225	4.92017	20
41	9.22583	9.99377	9.23206	10.76794	10.00623	10.77417	3.15374	4.92002	19
42	9.22657	9.99375	9.23283	10.76717	10.00625	10.77343	3.15523	4.91987	18
43	9.22731	9.99372	9.23359	10.76641	10.00628	10.77269	3.15672	4.91972	17
44	9.22805	9.99370	9.23435	10.76565	10.00630	10.77195	3.15820	4.91957	16
45	9.22878	9.99368	9.23511	10.76490	10.00632	10.77122	3.15969	4.91942	15
46	9.22952	9.99366	9.23586	10.76414	10.00634	10.77048	3.16117	4.91927	14
47	9.23025	9.99364	9.23661	10.76339	10.00636	10.77975	3.16264	4.91912	13
48	9.23098	9.99362	9.23737	10.76263	10.00638	10.76902	3.16412	4.91897	12
49	9.23171	9.99359	9.23812	10.76188	10.00641	10.76829	3.16559	4.91882	11
50	9.23244	9.99357	9.23887	10.76113	10.00643	10.76756	3.16706	4.91867	10
51	9.23317	9.99355	9.23962	10.76038	10.00645	10.76683	3.16853	4.91852	9
52	9.23390	9.99353	9.24037	10.75963	10.00647	10.76610	3.16999	4.91837	8
53	9.23462	9.99351	9.24112	10.75888	10.00649	10.76538	3.17145	4.91823	7
54	9.23535	9.99348	9.24186	10.75814	10.00652	10.76465	3.17291	4.91807	6
55	9.23607	9.99346	9.24261	10.75739	10.00654	10.76393	3.17437	4.91792	5
56	9.23679	9.99344	9.24335	10.75665	10.00656	10.76321	3.17583	4.91777	4
57	9.23751	9.99342	9.24410	10.75590	10.00658	10.76248	3.17728	4.91762	3
58	9.23823	9.99340	9.24484	10.75516	10.00660	10.76177	3.17873	4.91747	2
59	9.23895	9.99337	9.24558	10.75442	10.00663	10.76105	3.18018	4.91731	1
60	9.23967	9.99335	9.24632	10.75368	10.00665	10.76033	3.18162	4.91716	0
	Co-sine	Sine.	Co-tan.	Tang.	Co-sec.	Secant		V. sine	M

M	Sine	Co-fine	Tang.	Co-tang.	Secant	Co-fec.	V fine		
0	9.23967	9.99335	9.24632	10.75368	10.00665	10.76033	3.18162	4.91716	60
1	9.24039	9.99333	9.24706	10.75294	10.00667	10.75961	3.18306	4.91701	59
2	9.24110	9.99331	9.24779	10.75221	10.00669	10.75890	3.18451	4.91686	58
3	9.24181	9.99328	9.24853	10.75147	10.00672	10.75819	3.18594	4.91671	57
4	9.24253	9.99326	9.24926	10.75074	10.00674	10.75747	3.18738	4.91656	56
5	9.24324	9.99324	9.25000	10.75000	10.00676	10.75676	3.18881	4.91641	55
6	9.24395	9.99322	9.25073	10.74927	10.00678	10.75605	3.19024	4.91626	54
7	9.24466	9.99319	9.25146	10.74854	10.00681	10.75534	3.19167	4.91611	53
8	9.24536	9.99317	9.25219	10.74781	10.00683	10.75464	3.19310	4.91596	52
9	9.24607	9.99315	9.25292	10.74708	10.00685	10.75393	3.19452	4.91581	51
10	9.24677	9.99313	9.25365	10.74635	10.00687	10.75323	3.19594	4.91566	50
11	9.24748	9.99310	9.25437	10.74563	10.00690	10.75252	3.19736	4.91551	49
12	9.24818	9.99308	9.25510	10.74490	10.00692	10.75182	3.19878	4.91536	48
13	9.24888	9.99306	9.25582	10.74418	10.00694	10.75112	3.20019	4.91520	47
14	9.24958	9.99304	9.25655	10.74345	10.00696	10.75042	3.20160	4.91505	46
15	9.25028	9.99301	9.25727	10.74273	10.00699	10.74972	3.20301	4.91490	45
16	9.25098	9.99299	9.25799	10.74201	10.00701	10.74902	3.20442	4.91475	44
17	9.25168	9.99297	9.25871	10.74129	10.00703	10.74832	3.20583	4.91460	43
18	9.25237	9.99294	9.25943	10.74057	10.00706	10.74763	3.20723	4.91445	42
19	9.25307	9.99292	9.26015	10.73985	10.00708	10.74693	3.20863	4.91430	41
20	9.25376	9.99290	9.26086	10.73914	10.00710	10.74624	3.21003	4.91414	40
21	9.25445	9.99288	9.26158	10.73842	10.00712	10.74555	3.21142	4.91399	39
22	9.25514	9.99285	9.26229	10.73771	10.00715	10.74486	3.21282	4.91384	38
23	9.25583	9.99283	9.26301	10.73699	10.00717	10.74417	3.21421	4.91369	37
24	9.25652	9.99281	9.26372	10.73628	10.00719	10.74348	3.21560	4.91354	36
25	9.25721	9.99278	9.26443	10.73557	10.00722	10.74279	3.21699	4.91339	35
26	9.25790	9.99276	9.26514	10.73486	10.00724	10.74210	3.21837	4.91324	34
27	9.25858	9.99274	9.26585	10.73415	10.00726	10.74142	3.21975	4.91308	33
28	9.25927	9.99271	9.26655	10.73345	10.00729	10.74073	3.22113	4.91293	32
29	9.25995	9.99269	9.26726	10.73274	10.00731	10.74005	3.22251	4.91278	31
30	9.26063	9.99267	9.26797	10.73203	10.00733	10.73937	3.22389	4.91263	30
31	9.26131	9.99264	9.26867	10.73133	10.00736	10.73869	3.22526	4.91248	29
32	9.26199	9.99262	9.26937	10.73063	10.00738	10.73801	3.22663	4.91232	28
33	9.26267	9.99260	9.27008	10.72992	10.00740	10.73733	3.22800	4.91217	27
34	9.26335	9.99257	9.27078	10.72922	10.00743	10.73665	3.22937	4.91202	26
35	9.26403	9.99255	9.27148	10.72852	10.00745	10.73597	3.23073	4.91187	25
36	9.26470	9.99252	9.27218	10.72782	10.00748	10.73530	3.23210	4.91172	24
37	9.26538	9.99250	9.27288	10.72712	10.00750	10.73462	3.23346	4.91156	23
38	9.26605	9.99248	9.27357	10.72643	10.00752	10.73395	3.23482	4.91141	22
39	9.26672	9.99245	9.27427	10.72573	10.00755	10.73328	3.23617	4.91126	21
40	9.26739	9.99243	9.27496	10.72504	10.00757	10.73261	3.23753	4.91111	20
41	9.26806	9.99241	9.27566	10.72434	10.00759	10.73194	3.23888	4.91095	19
42	9.26873	9.99238	9.27635	10.72365	10.00762	10.73127	3.24023	4.91080	18
43	9.26940	9.99236	9.27704	10.72296	10.00764	10.73060	3.24157	4.91065	17
44	9.27007	9.99233	9.27773	10.72227	10.00767	10.72993	3.24292	4.91050	16
45	6.27073	9.99231	9.27842	10.72158	10.00769	10.72927	3.24427	4.91034	15
46	9.27140	9.99229	9.27911	10.72089	10.00771	10.72860	3.24561	4.91019	14
47	9.27206	9.99226	9.27980	10.72020	10.00774	10.72494	3.24695	4.91004	13
48	9.27273	9.99224	9.28049	10.71951	10.00776	10.72727	3.24829	4.90989	12
49	9.27339	9.99221	9.28117	10.71883	10.00779	10.72661	3.24962	4.90973	11
50	9.27405	9.99219	9.28186	10.71814	10.00781	10.72595	3.25095	4.90958	10
51	9.27471	9.99217	9.28254	10.71746	10.00783	10.72529	3.25229	4.90943	9
52	9.27537	9.99214	9.28323	10.71677	10.00786	10.72463	3.25362	4.90928	8
53	9.27602	9.99212	9.28391	10.71609	10.00788	10.72398	3.25494	4.90912	7
54	9.27668	9.99209	9.28459	10.71541	10.00791	10.72332	3.25627	4.90897	6
55	9.27734	9.99207	9.28527	10.71473	10.00793	10.72266	3.25759	4.90882	5
56	9.27799	9.99204	9.28595	10.71405	10.00796	10.72201	3.25891	4.90866	4
57	9.27864	9.99202	9.28662	10.71338	10.00798	10.72136	3.26023	4.90851	3
58	9.27930	9.99200	9.28730	10.71270	10.00800	10.72070	3.26155	4.90836	2
59	9.27995	9.99197	9.28798	10.71202	10.00803	10.72005	3.26286	4.90820	1
60	9.28060	9.99195	9.28865	10.71135	10.00805	10.71940	3.26418	4.90805	0
	Co-fine	Sine	Co-tan.	Tang.	Co-fec.	Secant		V. fine	M

M	Sine	Co-sine	Tang.	Co-tang.	Secant	Co-sec.	V. Sine		
0	9.28060	9.99195	9.28865	10.71135	10.00805	10.71940	3.26418	4.90805	60
1	9.28125	9.99192	9.28933	10.71067	10.00808	10.71875	3.26549	4.90790	59
2	9.28190	9.99190	9.29000	10.71000	10.00810	10.71810	3.26680	4.90774	58
3	9.28254	9.99187	9.29067	10.70933	10.00813	10.71746	3.26810	4.90759	57
4	9.28319	9.99185	9.29134	10.70866	10.00815	10.71681	3.26941	4.90744	56
5	9.28384	9.99182	9.29201	10.70799	10.00818	10.71616	3.27072	4.90728	55
6	9.28448	9.99180	9.29268	10.70732	10.00820	10.71552	3.27201	4.90713	54
7	9.28512	9.99177	9.29335	10.70665	10.00823	10.71488	3.27331	4.90698	53
8	9.28577	9.99175	9.29402	10.70598	10.00825	10.71423	3.27461	4.90682	52
9	9.28641	9.99172	9.29468	10.70532	10.00828	10.71359	3.27590	4.90667	51
10	9.28705	9.99170	9.29535	10.70465	10.00830	10.71295	3.27720	4.90652	50
11	9.28769	9.99167	9.29601	10.70399	10.00833	10.71231	3.27849	4.90636	49
12	9.28833	9.99165	9.29668	10.70332	10.00835	10.71167	3.27978	4.90621	48
13	9.28896	9.99162	9.29734	10.70266	10.00838	10.71104	3.28106	4.90605	47
14	9.28960	9.99160	9.29800	10.70200	10.00840	10.71040	3.28235	4.90590	46
15	9.29024	9.99157	9.29866	10.70134	10.00843	10.70976	3.28363	4.90575	45
16	9.29087	9.99155	9.29932	10.70068	10.00845	10.70913	3.28492	4.90559	44
17	9.29150	9.99152	9.29998	10.70002	10.00848	10.70850	3.28620	4.90544	43
18	9.29214	9.99150	9.30064	10.69936	10.00850	10.70786	3.28747	4.90529	42
19	9.29277	9.99147	9.30130	10.69870	10.00853	10.70723	3.28875	4.90513	41
20	9.29340	9.99145	9.30195	10.69805	10.00855	10.70660	3.29002	4.90498	40
21	9.29403	9.99142	9.30261	10.69739	10.00858	10.70597	3.29130	4.90482	39
22	9.29466	9.99140	9.30326	10.69674	10.00860	10.70534	3.29257	4.90467	38
23	9.29529	9.99137	9.30391	10.69609	10.00863	10.70471	3.29383	4.90451	37
24	9.29591	9.99135	9.30457	10.69543	10.00865	10.70409	3.29510	4.90436	36
25	9.29654	9.99132	9.30522	10.69478	10.00868	10.70346	3.29637	4.90421	35
26	9.29716	9.99130	9.30587	10.69413	10.00870	10.70284	3.29763	4.90405	34
27	9.29779	9.99127	9.30652	10.69348	10.00873	10.70221	3.29889	4.90390	33
28	9.29841	9.99124	9.30717	10.69283	10.00876	10.70159	3.30015	4.90374	32
29	9.29903	9.99122	9.30782	10.69218	10.00878	10.70097	3.30141	4.90359	31
30	9.29966	9.99119	9.30846	10.69154	10.00881	10.70034	3.30266	4.90343	30
31	9.30028	9.99117	9.30911	10.69089	10.00883	10.69972	3.30392	4.90328	29
32	9.30090	9.99114	9.30975	10.69025	10.00886	10.69910	3.30517	4.90312	28
33	9.30151	9.99112	9.31040	10.68960	10.00888	10.69849	3.30642	4.90297	27
34	9.30213	9.99109	9.31104	10.68896	10.00891	10.69787	3.30767	4.90281	26
35	9.30275	9.99106	9.31168	10.68832	10.00894	10.69725	3.30891	4.90266	25
36	9.30336	9.99104	9.31233	10.68767	10.00896	10.69664	3.31016	4.90250	24
37	9.30398	9.99101	9.31297	10.68703	10.00899	10.69602	3.31140	4.90235	23
38	9.30459	9.96099	9.31361	10.68639	10.00901	10.69541	3.31264	4.90219	22
39	9.30521	9.99096	9.31425	10.68575	10.00904	10.69479	3.31388	4.90204	21
40	9.30582	9.99093	9.31489	10.68511	10.00907	10.69418	3.31512	4.60188	20
41	9.30643	9.99091	9.31552	10.68448	10.00909	10.69357	3.31635	4.90173	19
42	9.30704	9.99088	9.31616	10.68384	10.00912	10.69296	3.31759	4.90157	18
43	9.30765	9.99086	9.31679	10.68321	10.00914	10.69235	3.31882	4.90142	17
44	9.30826	9.99083	9.31743	10.68257	10.00917	10.69174	3.32005	4.90126	16
45	9.30887	9.99080	9.31806	10.68194	10.00920	10.69113	3.32128	4.90111	15
46	9.30947	9.99078	9.31870	10.68130	10.00922	10.69053	3.32250	4.90095	14
47	9.31008	9.99075	9.31933	10.68067	10.00925	10.68992	3.32373	4.90080	13
48	9.31068	9.99072	9.31996	10.68004	10.00928	10.68932	3.32495	4.90064	12
49	9.31129	9.99070	9.32059	10.67941	10.00930	10.68871	3.32617	4.90049	11
50	9.31189	9.99067	9.32122	10.67878	10.00933	10.68811	3.32739	4.90033	10
51	9.31250	9.99064	9.32185	10.67815	10.00936	10.68750	3.32861	4.90018	9
52	9.31310	9,99062	9.32248	10.67752	10.00938	10.68690	3.32983	4.90002	8
53	9.31370	9.99059	9.32311	10.67689	10.00941	10.68630	3.33104	4.89986	7
54	9.31430	9.99056	9.32373	10.67627	10.00944	10.68570	3.33226	4.89971	6
55	9.31490	9.99054	9.32436	10.67564	10.00946	10.68510	3.33347	4.89955	5
56	9.31549	9.99051	9.32498	10.67502	10.00949	10.68451	3.33468	4.89940	4
57	9.31609	9.99048	9.32561	10.67439	10.00952	10.68391	3.33589	4.89924	3
58	9.31669	9.99046	9.32623	10.67377	10.00954	10.68331	3.33709	4.89909	2
59	9.31728	9.99043	9.32685	10.67315	10.00957	10.68272	3.33830	4.89893	1
60	9.31788	9.99040	9.32747	10.67253	10.00960	10.68212	3.33950	4.89877	0
	Co-sine	Sine	Co-tan.	Tang.	Co-sec.	Secant		V. Sine	M.

(78 Deg.)

M	Sine	Co-sine	Tang.	Co-tang.	Secant	Co-sec.	V Sine		
0	9.31788	9.99040	9.32747	10.67253	10.00960	10.68212	3.33950	4.89877	60
1	9.31847	9.99038	9.32810	10.67190	10.00962	10.68153	3.34070	4.89862	59
2	9.31907	9.99035	9.32872	10.67128	10.00965	10.68093	3.34190	4.89846	58
3	9.31966	9.99032	9.32933	10.67067	10.00968	10.68034	3.34310	4.89831	57
4	9.32025	9.99030	9.32995	10.67005	10.00970	10.67975	3.34429	4.89815	56
5	9.32084	9.99027	9.33057	10.66943	10.00973	10.67916	3.34549	4.89799	55
6	9.32143	9.99024	9.33119	10.66881	10.00976	10.67857	3.34668	4.89784	54
7	9.32202	9.99022	9.33180	10.66820	10.00978	10.67798	3.34787	4.89768	53
8	9.32261	9.99019	9.33242	10.66758	10.00981	10.67739	3.34906	4.89752	52
9	9.32319	9.99016	9.33303	10.66697	10.00984	10.67681	3.35025	4.89737	51
10	9.32378	9.99013	9.33365	10.66635	10.00987	10.67622	3.35144	4.89721	50
11	9.32437	9.99011	9.33426	10.66574	10.00989	10.67563	3.35262	4.89705	49
12	9.32495	9.99008	9.33487	10.66513	10.00992	10.67505	3.35380	4.89690	48
13	9.32553	9.99005	9.33548	10.66452	10.00995	10.67447	3.35498	4.89674	47
14	9.32612	9.99002	9.33609	10.66391	10.00998	10.67388	3.35616	4.89658	46
15	9.32670	9.99000	9.33670	10.66330	10.01000	10.67330	3.35734	4.89643	45
16	9.32728	9.98997	9.33731	10.66269	10.01003	10.67272	3.35852	4.89627	44
17	9.32786	9.98994	9.33792	10.66208	10.01006	10.67214	3.35969	4.89611	43
18	9.32844	9.98991	9.33853	10.66147	10.01009	10.67156	3.36087	4.89596	42
19	9.32902	9.98989	9.33913	10.66087	10.01011	10.67098	3.30204	4.89580	41
20	9.32960	9.98986	9.33974	10.66026	10.01014	10.67040	3.36321	4.89564	40
21	9.33018	9.98983	9.34034	10.65966	10.01017	10.66982	3.36438	4.89549	39
22	9.33075	9.98980	9.34095	10.65905	10.01020	10.66925	3.36554	4.89533	38
23	9.33133	9.98978	9.34155	10.65845	10.01022	10.66867	3.36671	4.89517	37
24	9.33190	9.98975	9.34215	10.65785	10.01025	10.66810	3.36787	4.89501	36
25	9.33248	9.98972	9.34270	10.65724	10.01028	10.66752	3.36903	4.89486	35
26	9.33305	9.98969	9.34336	10.65664	10.01031	10.66695	3.37019	4.89470	34
27	9.33362	9.98967	9.34396	10.65604	10.01033	10.66638	3.37135	4.89454	33
28	9.33420	9.98964	9.34456	10.65544	10.01036	10.66580	3.37251	4.89439	32
29	9.33477	9.98961	9.34516	10.65484	10.01039	10.66523	3.37367	4.89423	31
30	9.33534	9.98958	9.34576	10.65424	10.01042	10.66466	3.37482	4.89407	30
31	9.33591	9.98955	9.34636	10.65365	10.01045	10.66409	3.37597	4.89391	29
32	9.33647	9.98953	9.34695	10.65305	10.01047	10.66353	3.37713	4.89376	28
33	9.33704	9.98950	9.34755	10.65245	10.01050	10.66296	3.37828	4.89360	27
34	9.33761	9.98947	9.34814	10.65186	10.01053	10.66239	3.37942	4.89344	26
35	9.33818	9.98944	9.34874	10.65126	10.01056	10.66183	3.38057	4.89328	25
36	9.33874	9.98941	9.34933	10.65067	10.01059	10.66126	3.38171	4.89313	24
37	9.33931	9.98938	9.34992	10.65008	10.01062	10.66069	3.38286	4.89297	23
38	9.33987	9.98936	9.35051	10.64949	10.01064	10.66013	3.38400	4.89281	22
39	9.34043	9.98933	9.35111	10.64889	10.01067	10.65957	3.38514	4.89265	21
40	9.34100	9.98930	9.35170	10.64830	10.01070	10.65900	3.38628	4.89250	20
41	9.34156	9.98927	9.35229	10.64771	10.01073	10.65844	3.38742	4.89234	19
42	9.34212	9.98924	9.35288	10.64712	10.01076	10.65788	3.38855	4.89218	18
43	9.34268	9.98921	9.35346	10.64654	10.01079	10.65732	3.38969	4.89202	17
44	9.34324	9.98919	9.35405	10.64595	10.01081	10.65676	3.39082	4.89186	16
45	9.34380	9.98916	9.35464	10.64536	10.01084	10.65620	3.39195	4.89171	15
46	9.34436	9.98913	9.35523	10.64477	10.01087	10.65564	3.39308	4.89155	14
47	9.34491	9.98910	9.35581	10.64419	10.01090	10.65509	3.39421	4.89139	13
48	9.34547	9.98907	9.35640	10.64360	10.01093	10.65453	3.39534	4.89123	12
49	9.34602	9.98904	9.35698	10.64302	10.01096	10.65398	3.39646	4.89107	11
50	9.34658	9.98901	9.35757	10.64243	10.01099	10.65342	3.39759	4.89091	10
51	9.34713	9.98898	9.35815	10.64185	10.01102	10.65287	3.39871	4.89076	9
52	9.34769	9.98896	9.35873	10.64127	10.01104	10.65231	3.39983	4.89060	8
53	9.34824	9.98893	9.35931	10.64069	10.01107	10.65176	3.40095	4.89044	7
54	9.34879	9.98890	9.35989	10.64011	10.01110	10.65121	3.40207	4.89028	6
55	9.34934	9.98887	9.36047	10.63953	10.01113	10.65066	3.40319	4.89012	5
56	9.34989	9.98884	9.36105	10.63895	10.01116	10.65011	3.40430	4.88996	4
57	9.35044	9.98881	9.36163	10.63837	10.01119	10.64956	3.40541	4.88981	3
58	9.35099	9.98878	9.36221	10.63779	10.01122	10.64901	3.40653	4.88965	2
59	9.35154	9.98875	9.36279	10.63721	10.01125	10.64846	3.40764	4.88949	1
60	9.35209	9.98872	9.36336	10.63664	10.01128	10.64791	3.40875	4.88933	0
	Co-sine	Sine.	Co-tan.	Tangent	Co-sec.	Secant		V. Sine	M

M	Sine	Co-sine	Tang.	Co-tan.	Secant.	Co-sec.	V. Sine		
0	9.35209	9.98872	9.36336	10.63664	10.01128	10.64781	3.40875	4.88933	60
1	9.35263	9.98869	9.36394	10.63606	10.01131	10.64737	3.40986	4.88917	59
2	9.35318	9.98867	9.36452	10.63548	10.01133	10.64682	3.41096	4.88901	58
3	9.35373	9.98864	9.36509	10.63491	10.01136	10.64627	3.41207	4.88885	57
4	9.35427	9.98861	9.36566	10.63434	10.01139	10.64573	3.41317	4.88869	56
5	9.35481	9.98858	9.36624	10.63376	10.01142	10.64519	3.41427	4.88853	55
6	9.35536	9.98855	9.36681	10.63319	10.01145	10.64464	3.41537	4.88838	54
7	9.35590	9.98852	9.36738	10.63262	10.01148	10.64410	3.41647	4.88822	53
8	9.35644	9.98849	9.36795	10.63205	10.01151	10.64356	3.41757	4.88806	52
9	9.35698	9.98846	9.36852	10.63148	10.01154	10.64302	3.41867	4.88790	51
10	9.35752	9.98843	9.36909	10.63091	10.01157	10.64248	3.41976	4.88774	50
11	9.35806	9.98840	9.36966	10.63034	10.01160	10.64194	3.42086	4.88758	49
12	9.35860	9.98837	9.37023	10.62977	10.01163	10.64140	3.42195	4.88742	48
13	9.35914	9.98834	9.37080	10.62920	10.01166	10.64086	3.42304	4.88736	47
14	9.35968	9.98831	9.37187	10.62863	10.01169	10.64032	3.42413	4.88710	46
15	9.36022	9.98828	9.37193	10.62807	10.01172	10.63978	3.42523	4.88694	45
16	9.36075	9.98825	9.37250	10.62750	10.01175	10.63925	3.42631	4.88678	44
17	9.36129	9.98822	9.37309	10.62694	10.01178	10.63871	3.42759	4.88662	43
18	9.36182	9.98819	9.37363	10.62637	10.01181	10.63818	3.42848	4.88646	42
19	9.36236	9.98816	9.37419	10.62581	10.01184	10.63764	3.42956	4.88630	41
20	9.36289	9.98813	9.37476	10.62524	10.01187	10.63711	3.43064	4.88614	40
21	9.36342	9.98810	9.37532	10.62468	10.01190	10.63658	3.43172	4.88598	39
22	9.36395	9.98807	9.37588	10.62412	10.01193	10.63605	3.43280	4.88582	38
23	9.36449	9.98804	9.37644	10.62356	10.01196	10.63551	3.43388	4.88566	37
24	9.36502	9.98801	9.37700	10.62300	10.01199	10.63498	3.43495	4.88550	36
25	9.36555	9.98798	9.37756	10.62244	10.01202	10.63445	3.43603	4.88534	35
26	9.36607	9.98795	9.37812	10.62188	10.01205	10.63393	3.43710	4.88518	34
27	9.36660	9.98792	9.37868	10.62132	10.01208	10.63340	3.43817	4.88502	33
28	9.36713	9.98789	9.37924	10.62076	10.01211	10.63287	3.43924	4.88486	32
29	9.36766	9.98786	9.37980	10.62020	10.01214	10.63234	3.44031	4.88470	31
30	9.36819	9.98783	9.38035	10.61965	10.01217	10.63181	3.44138	4.88454	30
31	9.36871	9.98780	9.38091	10.61909	10.01220	10.63129	3.44245	4.88438	29
32	9.36924	9.98777	9.38147	10.61853	10.01223	10.63076	3.44351	4.88422	28
33	9.36976	9.98774	9.38202	10.61798	10.01226	10.63024	3.44457	4.88406	27
34	9.37028	9.98771	9.38257	10.61743	10.01229	10.62972	3.44564	4.88390	26
35	9.37081	9.98768	9.38313	10.61687	10.01232	10.62919	3.44670	4.88374	25
36	9.37133	9.98765	9.38368	10.61632	10.01235	10.62867	3.44776	4.88358	24
37	9.37185	9.98762	9.38423	10.61577	10.01238	10.62815	3.44882	4.88342	23
38	9.37237	9.98759	9.38479	10.61521	10.01241	10.62763	3.44988	4.88326	22
39	9.37289	9.98756	9.38534	10.61466	10.01244	10.62711	3.45093	4.88310	21
40	9.37341	9.98753	9.38589	10.61411	10.01247	10.62659	3.45199	4.88294	20
41	9.37393	9.98750	9.38644	10.61356	10.01250	10.62607	3.45304	4.88278	19
42	9.37445	9.98746	9.38699	10.61301	10.01254	10.62555	3.45410	4.88262	18
43	9.37497	9.98743	9.38754	10.61246	10.01257	10.62503	3.45515	4.88246	17
44	9.37549	9.98740	9.38808	10.61192	10.01260	10.62451	3.45620	4.88229	16
45	9.37600	9.98737	9.38863	10.61137	10.01263	10.62400	3.45724	4.88213	15
46	9.37652	9.98734	9.38918	10.61082	10.01266	10.62348	3.45829	4.88197	14
47	9.37703	9.98731	9.38972	10.61028	10.01269	10.62297	3.45934	4.88181	13
48	9.37755	9.98728	9.39027	10.60973	10.01272	10.62245	3.46038	4.88165	12
49	9.37806	9.98725	9.39082	10.60918	10.01275	10.62194	3.46143	4.88149	11
50	9.37858	9.98722	9.39136	10.60864	10.01278	10.62142	3.46247	4.88133	10
51	9.37909	9.98719	9.39190	10.60810	10.01281	10.62091	3.46351	4.88117	9
52	9.37960	9.98715	9.39245	10.60755	10.01285	10.62040	3.46455	4.88101	8
53	9.38011	9.98712	9.39299	10.60701	10.01288	10.61989	3.46559	4.88084	7
54	9.38062	9.98709	9.39353	10.60647	10.01291	10.61938	3.46662	4.88068	6
55	9.38113	9.98706	9.39407	10.60593	10.01294	10.61887	3.46765	4.88052	5
56	9.38164	9.98703	9.39461	10.60539	10.01297	10.61836	3.46869	4.88036	4
57	9.38215	9.98700	9.39515	10.60485	10.01300	10.61785	3.46973	4.88020	3
58	9.38266	9.98697	9.39569	10.60431	10.01303	10.61734	3.47076	4.88004	2
59	9.38317	9.98696	9.39623	10.60377	10.01306	10.61683	3.47179	4.87990	1
60	9.38368	9.98690	9.39677	10.60323	10.01310	10.61632	3.47282	4.87971	0
	Co-sine	Sine	Co-tan.	Tangent	Co-sec.	Secant		V. Sine	M

M	Sine	Co-sine	Tang.	Co-tan.	Secant	Co-sec.	V.Sine		
0	9.38368	9.98690	9.39677	10.60323	10.01310	10.61632	3.47282	4.87971	60
1	9.38418	9.98687	9.39731	10.60269	10.01313	10.61582	3.47385	4.87955	59
2	9.38469	9.98684	9.39785	10.60215	10.01316	10.61531	3.47487	4.87939	58
3	9.38519	9.98681	9.39838	10.60162	10.01319	10.61481	3.47590	4.87923	57
4	9.38570	9.98678	9.39892	10.60108	10.01322	10.61430	3.47692	4.87907	56
5	9.38620	9.98675	9.39945	10.60055	10.01325	10.61380	3.47795	4.87870	55
6	9.38670	9.98671	9.39999	10.60001	10.01329	10.61330	3.47897	4.87894	54
7	9.38721	9.98668	9.40052	10.59948	10.01332	10.61279	3.47999	4.87858	53
8	9.38771	9.98665	9.40106	10.59894	10.01335	10.61229	3.48101	4.87842	52
9	9.38821	9.98662	9.40159	10.59841	10.01338	10.61179	3.48203	4.87826	51
10	9.38871	9.98659	9.40212	10.59788	10.01341	10.61129	3.48305	4.87809	50
11	9.38921	9.98656	9.40266	10.59734	10.01344	10.61079	3.48406	4.87793	49
12	9.38971	9.98652	9.40319	10.59681	10.01348	10.61029	3.48508	4.87777	48
13	9.39021	9.98649	9.40372	10.59628	10.01351	10.60979	3.48609	4.87761	47
14	9.39071	9.98646	9.40425	10.59575	10.01354	10.60929	3.48710	4.87745	46
15	9.39121	9.98643	9.40478	10.59522	10.01357	10.60879	3.48811	4.87728	45
16	9.39170	9.98640	9.40531	10.59469	10.01360	10.60830	3.48912	4.87712	44
17	9.39220	9.98636	9.40584	10.59416	10.01364	10.60780	3.49013	4.87696	43
18	9.39270	9.98633	9.40636	10.59364	10.01367	10.60730	3.49114	4.87680	42
19	9.39319	9.98630	9.40689	10.59311	10.01370	10.60681	3.49215	4.87663	41
20	9.39369	9.98627	9.40742	10.59258	10.01373	10.60631	3.49315	4.87647	40
21	9.39418	9.98623	9.40795	10.59205	10.01377	10.60582	3.49416	4.87631	39
22	9.39467	9.98620	9.40847	10.59153	10.01380	10.60533	3.49516	4.87614	38
23	9.39517	9.98617	9.40900	10.59100	10.01383	10.60483	3.49616	4.87598	37
24	9.39566	9.98614	9.40952	10.59048	10.01386	10.60434	3.49716	4.87582	36
25	9.39615	9.98610	9.41005	10.58995	10.01390	10.60385	3.49816	4.87566	35
26	9.39664	9.98607	9.41057	10.58943	10.01393	10.60336	3.49916	4.87549	34
27	9.39713	9.98604	9.41109	10.58891	10.01396	10.60287	3.50016	4.87533	33
28	9.39762	9.98601	9.41161	10.58839	10.01399	10.60238	3.50115	4.87517	32
29	9.39811	9.98597	9.41214	10.58786	10.01403	10.60189	3.50215	4.87500	31
30	9.39860	9.98594	9.41266	10.58734	10.01406	10.60140	3.50314	4.87484	30
31	9.39909	9.98591	9.41318	10.58682	10.01409	10.60091	3.50413	4.87468	29
32	9.39958	9.98588	9.41370	10.58630	10.01412	10.60042	3.50513	4.8[illegible]51	28
33	9.40006	9.98584	9.41422	10.58578	10.01416	10.59994	3.50611	4.87435	27
34	9.40055	9.98581	9.41474	10.58526	10.01419	10.59945	3.50710	4.87419	26
35	9.40103	9.98578	9.41526	10.58474	10.01422	10.59897	3.50809	4.87402	25
36	9.40152	9.98574	9.41577	10.58422	10.01426	10.59848	3.50908	4.87386	24
37	9.40200	9.98571	9.41629	10.58371	10.01429	10.59800	3.51006	4.87370	23
38	9.40249	9.98568	9.41681	10.58319	10.01432	10.59751	3.51105	4.87353	22
39	9.40297	9.98565	9.41733	10.58267	10.01435	10.59703	3.51203	4.87337	21
40	9.40346	9.98561	9.41784	10.58216	10.01439	10.59654	3.51301	4.87321	20
41	9.40394	9.98558	9.41836	10.58164	10.01442	10.59606	3.51400	4.87304	19
42	9.40442	9.98555	9.41887	10.58113	10.01445	10.59558	3.51498	4.87288	18
43	9.40490	9.98551	9.41939	10.58061	10.01449	10.59510	3.51595	4.87272	17
44	9.40538	9.98548	9.41990	10.58010	10.01452	10.59462	3.51693	4.87255	16
45	9.40586	9.98545	9.42041	10.57959	10.01455	10.59414	3.51791	4.87239	15
46	9.40634	9.98541	9.42093	10.57907	10.01459	10.59366	3.51888	4.87222	14
47	9.40682	9.98538	9.42144	10.57856	10.01462	10.59318	3.51986	4.87206	13
48	9.40730	9.98535	9.42195	10.57805	10.01465	10.59270	3.52083	4.87190	12
49	9.40778	9.98531	9.42246	10.57754	10.01469	10.59222	3.52180	4.87173	11
50	9.40825	9.98528	9.42297	10.57703	10.01472	10.59175	3.52278	4.87157	10
51	9.40873	9.98525	9.42348	10.57652	10.01475	10.59127	3.52375	4.87140	9
52	9.40921	9.98521	9.42399	10.57601	10.01479	10.59079	3.52471	4.87124	8
53	9.40968	9.98518	9.42450	10.57550	10.01482	10.59032	3.52568	4.87108	7
54	9.41016	9.98515	9.42501	10.57499	10.01485	10.58984	3.52665	4.87091	6
55	9.41063	9.98511	9.42552	10.57448	10.01489	10.58937	3.52761	4.87075	5
56	9.41111	9.98508	9.42603	10.57397	10.01492	10.58889	3.52858	4.87058	4
57	9.41158	9.98505	9.42653	10.57347	10.01495	10.58842	3.52954	4.87042	3
58	9.41205	9.98501	9.42704	10.57296	10.01499	10.58795	3.53050	4.87025	2
59	9.41252	9.98498	9.42755	10.57245	10.01502	10.58748	3.53147	4.87009	1
60	9.41300	9.98494	9.4[illegible]805	10.57195	10.01506	10.58700	3.53243	4.86992	0
	Co-sine	Sine	Co-tan.	Tangent	Co-sec.	Secant		V. Sine	M

M	Sine	Co-fine	Tang.	Co-tang	Secant	Co-fec.	V. fine		
0	9.41300	9.98494	9.42805	10.57195	10.01506	10.58700	3.53243	4.86992	60
1	9.41347	9.98491	9.42856	10.57144	10.01509	10.58653	3.53338	4.86976	59
2	9.41394	9.98488	9.42906	10.57094	10.01512	10.58606	3.53434	4.86959	58
3	9.41441	9.98484	9.42957	10.57043	10.01516	10.58559	3.53530	4.86943	57
4	9.41488	9.98481	9.43007	10.56993	10.01519	10.58512	3.53626	4.86927	56
5	9.41535	9.98477	9.43057	10.56943	10.01523	10.58465	3.53721	4.86910	55
6	9.41582	9.98474	9.43108	10.56892	10.01526	10.58418	3.53816	4.86894	54
7	9.41628	9.98471	9.43158	10.56842	10.01529	10.58372	3.53912	4.86877	53
8	9.41675	9.98467	9.43208	10.56792	10.01533	10.58325	3.54007	4.86861	52
9	9.41722	9.98464	9.43258	10.56742	10.01536	10.58278	3.54102	4.86844	51
10	9.41768	9.98460	9.43308	10.56692	10.01540	10.58232	3.54197	4.86828	50
11	9.41815	9.98457	9.43358	10.56642	10.01543	10.58185	3.54292	4.86811	49
12	9.41861	9.98453	9.43408	10.56592	10.01547	10.58139	3.54386	4.86794	48
13	9.41908	9.98450	9.43458	10.56542	10.01550	10.58092	3.54481	4.86778	47
14	9.41954	9.98447	9.43508	10.56492	10.01553	10.58046	3.54575	4.86761	46
15	9.42001	9.98443	9.43558	10.56442	10.01557	10.57999	3.54670	4.86745	45
16	9.42047	9.98440	9.43607	10.56393	10.01560	10.57953	3.54764	4.86728	44
17	9.42093	9.98436	9.43657	10.56343	10.01564	10.57907	3.54858	4.86712	43
18	9.42139	9.98433	9.43707	10.56293	10.01567	10.57861	3.54953	4.86695	42
19	9.42186	9.98429	9.43756	10.56244	10.01571	10.57814	3.55047	4.86679	41
20	9.42232	9.98426	9.43806	10.56194	10.01574	10.57768	3.55140	4.86662	40
21	9.42278	9.98422	9.43855	10.56145	10.01578	10.57722	3.55234	4.86646	39
22	9.42324	9.98419	9.43905	10.56095	10.01581	10.57676	3.55328	4.86629	38
23	9.42370	9.98415	9.43954	10.56046	10.01585	10.57630	3.55422	4.86612	37
24	9.42416	9.98412	9.44004	10.55996	10.01588	10.57584	3.55515	4.86596	36
25	9.42461	9.98409	9.44053	10.55947	10.01591	10.57539	3.55608	4.86579	35
26	9.42507	9.98405	9.44102	10.55898	10.01595	10.57493	3.55702	4.86563	34
27	9.42553	9.98402	9.44151	10.55849	10.01598	10.57447	3.55795	4.86546	33
28	9.42599	9.98398	9.44201	10.55799	10.01602	10.57401	3.55888	4.86529	32
29	9.42644	9.98395	9.44250	10.55750	10.01605	10.57356	3.55981	4.86513	31
30	9.42690	9.98391	9.44299	10.55701	10.01609	10.57310	3.56074	4.86496	30
31	9.42735	9.98388	9.44348	10.55652	10.01612	10.57265	3.56167	4.86480	29
32	9.42781	9.98384	9.44397	10.55603	10.01616	10.57219	3.56259	4.86463	28
33	9.42826	9.98381	9.44446	10.55554	10.01619	10.57174	3.56352	4.86446	27
34	9.42872	9.98377	9.44495	10.55505	10.01623	10.57128	3.56444	4.86430	26
35	9.42917	9.98373	9.44544	10.55456	10.01627	10.37083	3.56537	4.86413	25
36	9.42962	9.98370	9.44592	10.55408	10.01630	10.57038	3.56629	4.86396	24
37	9.43007	9.98366	9.44640	10.55359	10.01634	10.56993	3.56721	4.86380	23
38	9.43053	9.98363	9.44690	10.55310	10.01637	10.56977	3.56813	4.86363	22
39	9.43098	9.98359	9.44738	10.55262	10.01641	10.56902	3.56905	4.86347	21
40	9.43143	9.98356	9.44787	10.55213	10.01644	10.56857	3.56997	4.86330	20
41	9.43188	9.98352	9.44836	10.55164	10.01648	10.56812	3.57089	4.86313	19
42	9.43233	9.98349	9.44884	10.55116	10.01651	10.56767	3.57180	4.86297	18
43	9.43278	9.98345	9.44933	10.55067	10.01655	10.56722	3.57272	4.86280	17
44	9.43323	9.98342	9.44981	10.55019	10.01658	10.56677	3.57364	4.86263	16
45	9.43367	9.98338	9.45029	10.54971	10.01662	10.56633	3.57455	4.86246	15
46	9.43412	9.98334	9.45078	10.54922	10.01666	10.56588	3.57546	4.86230	14
47	9.43457	9.98331	9.45126	10.54874	10.01669	10.56543	3.57637	4.86213	13
48	9.43502	9.98327	9.45174	10.54826	10.01673	10.56498	3.57728	4.86196	12
49	9.43546	9.98324	9.45222	10.54778	10.01676	10.56454	3.57819	4.86180	11
50	9.43591	9.98320	9.45271	10.54729	10.01680	10.56409	3.57910	4.86163	10
51	9.43635	9.98317	9.45319	10.54681	10.01683	10.56365	3.58001	4.86146	9
52	9.43680	9.98313	9.45367	10.54633	10.01687	10.56320	3.58092	4.86130	8
53	9.43724	9.98309	9.45415	10.54585	10.01691	10.56276	3.58182	4.86113	7
54	9.43769	9.98306	9.45463	10.54537	10.01694	10.56231	3.58273	4.86096	6
55	9.43813	9.98302	9.45511	10.54489	10.01698	10.56187	3.58363	4.86079	5
56	9.43857	9.98299	9.45559	10.54441	10.01701	10.56143	3.58454	4.86063	4
57	9.43901	9.98295	9.45606	10.54394	10.01705	10.56099	3.58554	4.86046	3
58	9.43946	9.98291	9.45654	10.54346	10.01709	10.56054	3.58634	4.86029	2
59	9.43990	9.98288	9.45702	10.54298	10.01712	10.56010	3.58724	4.86012	1
60	9.44034	9.98284	9.45750	10.54250	10.01716	10.56966	3.58814	4.85996	0
	Co-fine	Sine.	Co-tan.	Tangent	Co-fec.	Secant		V. Sine	M

(74 Deg.)

M	Sine	Co-sine	Tang.	Co-tang.	Secant	Co-sec.	V. S		M
0	9.44034	9.98284	9.45750	10.54250	10.01716	10.55966	3.58814	4.85996	60
1	9.44078	9.98281	9.45797	10.54203	10.01719	10.55922	3.58904	4.85979	59
2	9.44122	9.98277	9.45845	10.54155	10.01723	10.55878	3.58994	4.85962	58
3	9.44166	9.98273	9.45892	10.54108	10.01727	10.55834	3.59083	4.85945	57
4	9.44210	9.98270	9.45940	10.54060	10.01730	10.55790	3.59173	4.85929	56
5	9.44253	9.98266	9.45987	10.54013	10.01734	10.55747	3.59262	4.85912	55
6	9.44297	9.98262	9.46035	10.53965	10.01738	10.55703	3.59352	4.85895	54
7	9.44341	9.98259	9.46082	10.53918	10.01741	10.55659	3.59441	4.85878	53
8	9.44385	9.98255	9.46130	10.53870	10.01745	10.55615	3.59530	4.85861	52
9	9.44428	9.98251	9.46177	10.53823	10.01749	10.55572	3.59619	4.85845	51
10	9.44472	9.98248	9.46224	10.53776	10.01752	10.55528	3.59708	4.85828	50
11	9.44516	9.98244	9.46271	10.53729	10.01756	10.55484	3.59797	4.85811	49
12	9.44559	9.98240	9.46319	10.53681	10.01760	10.55441	3.59886	4.85794	48
13	9.44602	9.98237	9.46366	10.53634	10.01763	10.55398	3.59975	4.85777	47
14	9.44646	9.98233	9.46413	10.53587	10.01767	10.55354	3.60063	4.85760	46
15	9.44689	9.98229	9.46460	10.53540	10.01771	10.55311	3.60152	4.85744	45
16	9.44733	9.98226	9.46507	10.53493	10.01774	10.55267	3.60240	4.85727	44
17	9.44776	9.98222	9.46554	10.53446	10.01778	10.55224	3.60329	4.85710	43
18	9.44819	9.98218	9.46601	10.53399	10.01782	10.55181	3.60417	4.85693	42
19	9.44862	9.98215	9.46648	10.53352	10.01785	10.55138	3.60505	4.85676	41
20	9.44905	9.98211	9.46694	10.53306	10.01789	10.55095	3.60593	4.85659	40
21	9.44948	9.98207	9.46741	10.53259	10.01793	10.55052	3.60681	4.85642	39
22	9.44992	9.98204	9.46788	10.53212	10.01796	10.55008	3.60764	4.85626	38
23	9.45035	9.98200	9.46835	10.53165	10.01800	10.54965	3.60851	4.85609	37
24	9.45077	9.98196	9.46881	10.53119	10.01804	10.54923	3.60945	4.85592	36
25	9.45120	9.98192	9.46928	10.53072	10.01808	10.54880	3.61032	4.85575	35
26	9.45163	9.98189	9.46975	10.53025	10.01811	10.54837	3.61120	4.85558	34
27	9.45206	9.98185	9.47021	10.52979	10.01815	10.54794	3.61207	4.85541	33
28	9.45249	9.98181	9.47068	10.52932	10.01819	10.54751	3.61294	4.85524	32
29	9.45292	9.98177	9.47114	10.52886	10.01823	10.54708	3.61382	4.85507	31
30	9.45334	9.98174	9.47160	10.52840	10.01826	10.54666	3.61469	4.85490	30
31	9.45377	9.98170	9.47207	10.52793	10.01830	10.54623	3.61556	4.85473	29
32	9.45419	9.98166	9.47253	10.52747	10.01834	10.54581	3.61643	4.85457	28
33	9.45462	9.98162	9.47299	10.52701	10.01838	10.54538	3.61730	4.85440	27
34	9.45504	9.98159	9.47346	10.52654	10.01841	10.54496	3.61817	4.85423	26
35	9.45547	9.98155	9.47392	10.52608	10.01845	10.54453	3.61903	4.85406	25
36	9.45589	9.98151	9.47438	10.52562	10.01849	10.54411	3.61990	4.85389	24
37	9.45632	9.98147	9.47484	10.52516	10.01853	10.54368	3.62077	4.85372	23
38	9.45674	9.98144	9.47530	10.52470	10.01856	10.54326	3.62163	4.85355	22
39	9.45716	9.98140	9.47576	10.52424	10.01860	10.54284	3.62249	4.85338	21
40	9.45758	9.98136	9.47622	10.52378	10.01864	10.54242	3.62336	4.85321	20
41	9.45801	9.98132	9.47668	10.52332	10.01868	10.54199	3.62422	4.85304	19
42	9.45843	9.98128	9.47714	10.52286	10.01872	10.54157	3.62508	4.85287	18
43	9.45885	9.98125	9.47760	10.52240	10.01875	10.54115	3.62594	4.85270	17
44	9.45927	9.98121	9.47806	10.52194	10.01879	10.54073	3.62680	4.85253	16
45	9.45969	9.98117	9.47852	10.52148	10.01883	10.54031	3.62720	4.85236	15
46	9.46011	9.98113	9.47897	10.52103	10.01887	10.53989	3.62852	4.85219	14
47	9.46053	9.98109	9.47943	10.52057	10.01891	10.53947	3.62937	4.85202	13
48	9.46095	9.98106	9.47989	10.52011	10.01894	10.53905	3.63023	4.85185	12
49	9.46136	9.98102	9.48035	10.51965	10.01898	10.53864	3.63108	4.85168	11
50	9.46178	9.98098	9.48080	10.51920	10.01902	10.53822	3.63194	4.85151	10
51	9.46220	9.98094	9.48126	10.51874	10.01906	10.53780	3.63279	4.85134	9
52	9.46262	9.98090	9.48171	10.51829	10.01910	10.53738	3.63364	4.85117	8
53	9.46303	9.98087	9.48217	10.51783	10.01913	10.53697	3.63450	4.85100	7
54	9.46345	9.98083	9.48262	10.51738	10.01917	10.53655	3.63535	4.85083	6
55	9.46386	9.98079	9.48307	10.51693	10.01921	10.53614	3.63620	4.85066	5
56	9.46428	9.98075	9.48353	10.51647	10.01925	10.53572	3.63705	4.85049	4
57	9.46469	9.98071	9.48398	10.51602	10.01929	10.53531	3.63789	4.85032	3
58	9.46511	9.98067	9.48443	10.51557	10.01933	10.53489	3.63874	4.85015	2
59	9.46552	9.98063	9.48489	10.51511	10.01937	10.53448	3.63959	4.84998	1
60	9.46594	9.98060	9.48534	10.51466	10.01940	10.53406	3.64043	4.84981	0
	Co-sine	Sine	Co-tan.	Tangent	Co-sec.	Secant		V. Sine	

M (73 Deg.)

M	Sine	Co-sine	Tang.	Co-tang.	Secant	Co-sec.	V Sine		
0	9.46594	9.98060	9.48534	10.51466	10.01940	10.53406	3.64043	4.84981	60
1	9.46635	9.98056	9.48579	10.51421	10.01944	10.53365	3.64128	4.84963	59
2	9.46676	9.98052	9.48624	10.51376	10.01948	10.53324	3.64212	4.84946	58
3	9.46717	9.98048	9.48669	10.51331	10.01952	10.53283	3.64297	4.84929	57
4	9.46758	9.98044	9.4871[illegible]	10.51286	10.01956	10.53242	3.64382	4.84912	56
5	9.46800	9.9804	9.48759	10.51241	10.01960	10.53200	3.64465	4.84895	55
6	9.46841	9.98036	9.48804	10.51166	10.01964	10.53159	3.64549	4.84878	54
7	9.46882	9.98032	9.488[illegible]9	10.51151	10.01968	10.53118	3.64633	4.84861	53
8	9.46923	9.98029	9.4889	10.51106	10.01971	10.53077	3.64717	4.84844	52
9	9.46964	9.98025	9.48939	10.51061	10.01975	10.53036	3.64799	4.84827	51
10	9.4700	9.98021	9.4898[illegible]	10.51016	10.01979	10.52995	3.64885	4.84810	50
11	9.47045	9.98017	9.49029	10.50971	10.01983	10.52955	3.64968	4.84792	49
12	9.47086	9.98013	9.49073	10.50927	10.01[illegible]87	10.52914	3.65052	4.84775	48
13	9.47[illegible]27	9.98009	9.49118	10.50882	10.01991	10.52873	3.65135	4.84758	47
14	9.47168	9.9800	9.49163	10.50837	10.01995	10.52832	3.65219	4.84744	46
15	9.47209	9.98001	9.49207	10.50793	10.01999	10.52791	3.65302	4.84721	45
16	9.47249	9.97997	9.4925[illegible]	10.5074[illegible]	10.02003	10.52751	3.65385	4.84707	44
17	9.47290	9.97993	9.49296	10.50704	10.02007	10.52710	3.65468	4.84690	43
18	9.47330	9.97989	9.4[illegible]34[illegible]	10.50659	10.02011	10.52670	3.65551	4.84672	42
19	9.4737[illegible]	9.9798	9.49385	10.50615	10.02014	10.52629	3.65634	4.84655	41
20	9.47411	9.9798[illegible]	9.4943[illegible]	10.50570	10.02018	10.52589	3.65717	4.84638	40
21	9.47452	9.97978	9.49474	10.50526	10.02022	10.52548	3.65800	4.84621	39
22	9.47492	9.97974	9.4951[illegible]	10.50481	10.02026	10.52508	3.65883	4.84604	38
23	9.47533	9.97970	9.49563	10.50437	10.02030	10.52467	3.65966	4.84580	37
24	9.47573	9.97966	9.4960	10.50393	10.02034	10.52427	3.66048	4.84569	36
25	9.47613	9.9796[illegible]	9.49652	10.5034[illegible]	10.02038	10.52387	3.66131	4.84552	35
26	9.47654	9.97958	9.4969[illegible]	10.50304	10.02042	10.52346	3.66213	4.84535	34
27	9.4769[illegible]	9.97954	9.49740	10.50260	10.02046	10.52306	3.66296	4.84528	33
28	9.4773[illegible]	9.97950	9.4978[illegible]	10.50216	10.02050	10.52266	3.66378	4.84500	32
29	9.4777[illegible]	9.97946	9.49828	10.50172	10.02054	10.52226	3.66460	4.84483	31
30	9.478[illegible]4	9.97942	9.49872	10.50128	10.02058	10.52186	3.66[illegible]	4.84466	30
31	9.4785[illegible]	9.9793	9.499[illegible]	10.50084	10.02062	10.52146	3.66624	4.84449	29
32	9.47894	9.979[illegible]4	9.49960	10.50040	10.02066	10.52106	3.66706	4.84432	28
33	9.47934	9.97930	9.500[illegible]	10.49996	10.02070	10.52066	3.66788	4.84414	27
34	9.4797[illegible]	9.97926	9.5004[illegible]	10.49952	10.02074	10.52026	3.66870	4.84397	26
35	9.4801[illegible]	9.97922	9.5009	10.49908	10.02078	10.51986	3.66952	4.84380	25
36	9.480[illegible]	9.97918	9.501[illegible]	10.49864	10.02082	10.51946	3.67033	4.84363	24
37	9.480[illegible]	9.97914	9.50180	10.49820	10.02086	10.51906	3.67115	4.84345	23
38	9.48133	9.97910	9.50223	10.49777	10.02090	10.51867	3.67196	4.84328	22
39	9.48173	9.97906	9.50267	10.49733	10.02094	10.51827	3.67278	4.84311	21
40	9.48213	9.97902	9.50311	10.49689	10.02098	10.51787	3.67359	4.84293	20
41	9.48252	9.97898	9.50355	10.49645	10.02102	10.51747	3.67440	4.84276	19
42	9.48292	9.97894	9.50398	10.49602	10.02106	10.51708	3.67521	4.84259	18
43	9.4833	9.97890	9.5044[illegible]	10.49558	10.02110	10.51668	3.67603	4.84242	17
44	9.4837[illegible]	9.97886	9.50485	10.4951[illegible]	10.02114	10.51629	3.67684	4.84224	10
45	9.48411	9.97882	9.50529	10.49471	10.02118	10.51589	3.67765	4.84207	15
46	9.48450	9.97878	9.50572	10.49428	10.02122	10.51550	3.67845	4.84190	14
47	9.48490	9.97874	9.50616	10.49394	10.02126	10.51510	3.67926	4.84172	13
48	9.4852[illegible]	9.97870	9.50659	10.49341	10.02130	10.51471	3.68007	4.84155	12
49	9.48568	9.97866	9.50703	10.49297	10.02134	10.51432	3.68088	4.84138	11
50	9.48607	9.97861	9.50746	10.49254	10.02139	10.51393	3.68168	4.84120	10
51	9.48647	9.97857	9.50789	10.49211	10.02143	10.51353	3.68249	4.84103	9
52	9.48686	9.97853	9.50833	10.49167	10.02147	10.51314	3.68329	4.84086	8
53	9.48725	9.97849	9.50876	10.49124	10.02151	10.51275	3.68409	4.84068	7
54	9.4876[illegible]	9.97845	9.50919	10.49081	10.021[illegible]	10.51236	3.68490	4.84051	6
55	9.48803	9.97841	9.50962	10.49038	10.02159	10.51197	3.68570	4.84034	5
56	9.48842	9.97837	9.51005	10.48995	10.02163	10.51158	3.68650	4.84016	4
57	9.48881	9.97833	9.51048	10.48952	10.02167	10.51119	3.68710	4.83999	3
58	9.48920	9.97829	9.51092	10.48908	10.02171	10.51080	3.68810	4.83981	2
59	9.48959	9.97825	9.51135	10.48865	10.02175	10.51041	3.68890	4.83964	1
60	9.48998	9.97821	9.51178	10.48822	10.02179	10.51002	3.68969	4.83947	0
	Co-sine	Sine	Co-tan.	Tangent	Co-sec.	Secant		V. Sine	M

M	Sine	Co-sin	Tang.	Co-tan.	Secant.	Co-sec.	V. Sine		
0	9.48998	9.97821	9.51170	10.48822	10.02179	10.51002	3.[illegible]8969	4.83947	60
1	9.49037	9.97817	9.51221	10.48779	10.02183	10.50963	3.69049	4.83929	59
2	9.49076	9.97812	9.51264	10.48736	10.02188	10.50924	3.69129	4.83912	58
3	9.49115	9.97808	9.51306	10.48694	10.02192	10.50885	3.69208	4.83895	57
4	9.49153	9.97804	9.51349	10.48651	10.02196	10.50847	3.69288	4.83877	56
5	9.49192	9.97800	9.51392	10.48608	10.02200	10.50808	3.69367	4.83860	55
6	9.49231	9.97796	9.514[illegible]	10.48565	10.02204	10.50769	3.69447	4.83842	54
7	9.49269	9.97792	9.51478	10.4852[illegible]	10.02208	10.50731	3.69[illegible]0	4.83825	53
8	9.49308	9.97788	9.51520	10.484[illegible]0	10.09212	10.50692	3.69605	4.83807	52
9	9.49347	9.97784	9.51563	10.48437	10.02216	10.50653	3.69684	4.83790	51
10	9.49385	9.97779	9.51606	10.48394	10.02221	10.50615	3.69763	4.83773	50
11	9.49424	9.97775	9.51648	10.48352	10.0222[illegible]	10.50576	3.69842	4.83755	49
12	9.49462	9.97771	9.516[illegible]	10.48309	10.0222[illegible]	10.50538	3.69921	4.83738	48
13	9.49500	9.97767	9.51734	10.48266	10.0223[illegible]	10.50500	3.70000	4.83720	47
14	9.49539	9.97763	9.51776	10.48224	10.02237	10.50461	3.70079	4.83703	46
15	9.49577	9.97758	9.51819	10.48181	10.0224[illegible]	10.50423	3.70158	4.83685	45
16	9.49615	9.97754	9.51861	10.48139	10.02246	10.50385	3.70236	4.83668	44
17	9.49654	9.97750	9.51903	10.4809[illegible]	10.02250	10.50346	3.7031[illegible]	4.83650	43
18	9.49692	9.97746	9.51946	10.48054	10.02[illegible]54	10.50308	3.70393	4.8363[illegible]	42
19	9.49730	9.97742	9.51988	10.48012	10.02258	10.50270	3.70472	4.83615	41
20	9.49768	9.97738	9.52031	10.47969	10.02262	10.50232	3.7055[illegible]	4.8359[illegible]	40
21	9.498[illegible]6	9.97734	9.52073	10.47927	10.02266	[illegible]0.50194	3.70628	4.8358[illegible]	39
22	9.49844	9.97729	9.52115	10.47885	[illegible]0.02271	[illegible]0.50156	3.70706	4.8356[illegible]	38
23	9.498[illegible]	9.97725	9.52157	10.47843	10.0227[illegible]	10.50118	3.70784	4.83545	37
24	9.49920	9.97721	9.52199	10.47801	10.0227[illegible]	[illegible]0.50080	3.7[illegible]8[illegible]	4.8352[illegible]	36
25	9.49958	9.97717	9.52242	10.47758	10.0228[illegible]	10.5004[illegible]	3.70940	4.83510	35
26	9.40996	9.97713	9.52284	10.47716	10.02287	10.5000[illegible]	3.71010	4.83493	34
27	9.50034	9.97708	9.5232[illegible]	10.47674	10.02292	10.49966	3.71096	4.83475	33
28	9.50072	9.97704	9.52368	10.47632	10.02296	10.49928	3.71174	4.83458	32
29	9.50110	9.97700	9.52410	10.47590	[illegible]0.0230[illegible]	10.49890	3.71252	4.83440	31
30	9.50148	9.976[illegible]6	9.52452	10.47548	10.02304	10.49852	3.71329	4.83423	30
31	9.50185	9.97691	9.52494	10.47506	10.02309	10.49815	3.71407	4.83405	29
32	9.50223	9.97687	9.52536	10.47464	10.02313	10.49777	3.7158[illegible]	4.83388	28
33	9.50261	9.97683	9.52578	10.47422	10.02317	10.49739	3.71562	4.83370	27
34	9.50298	9.97679	9.52620	10.47380	10.02321	10.49702	3.71639	4.83352	26
35	9.50336	9.97674	9.52661	10.47339	10.02326	10.49664	3.71716	4.83335	25
36	9.50374	9.97670	9.52703	10.4[illegible]297	10.02330	10.49026	3.71793	4.83317	24
37	9.50411	9.97666	9.52745	10.47255	10.02334	10.49589	[illegible].71870	4.83300	23
38	9.50449	9.97662	9.52[illegible]8[illegible]	10.47213	10.02338	10.49551	3.71947	4.83282	22
39	9.50486	9.97657	9.52828	10.47171	10.02343	10.49514	3.7202[illegible]	4.83265	21
40	9.50523	9.97653	9.528[illegible]0	10.47130	[illegible]0.02347	10.49477	[illegible].72101	4.83247	20
41	9.50561	9.97649	9.52912	10.47088	10.02351	10.49439	3.72178	4.83229	19
42	9.50598	9.97645	9.52953	10.47147	10.02355	10.49402	3.72255	4.83212	18
43	9.50035	9.97640	9.52995	10.47005	10.02360	10.4936[illegible]	3.72332	4.83194	17
44	9.50673	9.97636	9.5303[illegible]	10.46963	10.02364	10.4932[illegible]	3.72408	4.8317[illegible]	16
45	9.50710	9.97632	9.53078	10.46922	10.02368	10.49290	3.72485	4.83159	15
46	9.50747	9.97627	9.53120	10.46880	10.02373	10.49253	3.72561	4.83151	14
47	9.50784	9.97623	9.53161	10.46839	10.02377	10.49216	3.72638	4.83124	13
48	9.50821	9.9761[illegible]	9.53202	10.46798	10.0238[illegible]	10.49179	3.7271[illegible]	4.83106	12
49	9.50858	9.97615	9.53244	10.46756	10.02385	10.49142	[illegible].7279[illegible]	4.83088	11
50	9.50896	9.97610	9.53285	10.46715	10.02390	10.4910[illegible]	3.72867	4.83071	10
51	9.50933	9.97606	9.53327	10.46673	10.02394	10.4906[illegible]	3.72943	4.83053	9
52	9.50970	9.97602	9.53368	10.46632	10.02398	10.49030	3.73019	4.83025	8
53	9.51007	9.97597	9.53409	10.46591	10.02403	10.4899[illegible]	[illegible].73095	4.83018	7
54	9.51043	9.97593	9.53450	10.46550	10.02407	10.4895[illegible]	[illegible].73171	4.8[illegible]	6
55	9.51080	9.97589	9.53[illegible]92	10.46508	10.02411	10.48920	3.73247	[illegible]	5
56	9.51117	9.97584	9.5353[illegible]	10.4646[illegible]	10.02416	1[illegible].4888[illegible]	3.73322	4.82964	4
57	9.51154	9.97580	9.53574	10.46426	10.02420	10.48846	3.73398	4.82[illegible]4	3
58	9.51191	9.97576	9.53615	10.46385	10.02424	10.48809	3.73474	4.82929	2
59	9.51227	9.9757[illegible]	9.53656	10.46344	10.02429	10.48773	3.73549	4.82912	1
60	9.51264	9.97567	9.53697	10.46303	10.02433	10.48736	3.73625	4.82894	0
	Co-sine	Sine	Co-tan.	Tang	Co-sec.	Secant		V. Sine	M

M	Sine	Co-sine	Tang.	Co-tang.	Secant	Co-sec.	V. Sine		
0	9.51264	9.97567	9.53697	10.46303	10.02433	10.48736	3.73625	4.82894	60
1	9.51301	9.97563	9.53738	10.46262	10.02437	10.48699	3.73700	4.82876	59
2	9.51337	9.97558	9.53779	10.46221	10.02442	10.48663	3.73776	4.82858	58
3	9.51374	9.97554	9.53820	10.46180	10.02446	10.48626	3.73851	4.82841	57
4	9.51411	9.97550	9.53861	10.46139	10.02450	10.48589	3.73926	4.82823	56
5	9.51447	9.97545	9.53902	10.46098	10.02455	10.48553	3.74001	4.82805	55
6	9.51484	9.97541	9.53943	10.46057	10.02459	10.48516	3.74077	4.82787	54
7	9.51520	9.97536	9.53984	10.46016	10.02464	10.48480	3.74152	4.82770	53
8	9.51552	9.97532	9.54025	10.45975	10.02468	10.48443	3.74227	4.82752	52
9	9.51593	9.97528	9.54065	10.45935	10.02472	10.48407	3.74302	4.82734	51
10	9.51629	9.97523	9.54106	10.45894	10.02477	10.48371	3.74376	4.82716	50
11	9.51666	9.97519	9.54147	10.45853	10.02481	10.48334	3.74451	4.82699	49
12	9.51702	9.97515	9.54187	10.45813	10.02485	10.48298	3.74526	4.82681	48
13	9.51738	9.97510	9.54228	10.45772	10.02490	10.48262	3.74601	4.82663	47
14	9.51774	9.97506	9.53269	10.45741	10.02494	10.48226	3.74675	4.82645	46
15	9.51811	9.97501	9.54309	10.45691	10.02599	10.48189	3.74750	4.82628	45
16	9.51847	9.97497	9.54350	10.45650	10.02503	10.48153	3.74824	4.82610	44
17	9.51883	9.97492	9.54390	10.45610	10.02508	10.48117	3.74899	4.82592	43
18	9.51919	9.97488	9.54431	10.45569	10.02512	10.48081	3.74973	4.82574	42
19	9.51955	9.97485	9.54471	10.45529	10.02516	10.48045	3.75047	4.82556	41
20	9.51991	9.97479	9.54512	10.45488	10.02521	10.48009	3.75121	4.82538	40
21	9.52027	9.97475	9.54552	10.45448	10.02525	10.47973	3.75195	4.82521	39
22	9.52063	9.97470	9.54593	10.45407	10.02530	10.47937	3.75270	4.82503	38
23	9.52099	9.97466	9.54633	10.45367	10.02534	10.47901	3.75344	4.82485	37
24	9.52135	9.97461	9.54673	10.45327	10.02539	10.47865	3.75418	4.82467	36
25	9.52171	9.97457	9.54714	10.45286	10.02543	10.47829	3.75491	4.82449	35
26	9.52207	9.97453	9.54754	10.45246	10.02547	10.47793	3.75565	4.82431	34
27	9.52242	9.97448	9.54794	10.45206	10.02552	10.47758	3.75639	4.82414	33
28	9.52278	9.97444	9.54835	10.45165	10.02556	10.47722	3.75713	4.82396	32
29	9.52314	9.97439	9.54875	10.45125	10.02561	10.47686	3.75786	4.82378	31
30	9.52350	9.97435	9.54915	10.45085	10.02565	10.47650	3.75860	4.82360	30
31	9.52385	9.97430	9.54955	10.45045	10.02570	10.47615	3.75933	4.82342	29
32	9.52421	9.97426	9.54995	10.45005	10.02574	10.47579	3.76007	4.82324	28
33	9.52456	9.97421	9.55035	10.44965	10.02579	10.47544	3.76080	4.82306	27
34	9.52492	9.97417	9.55075	10.44925	10.02583	10.47508	3.76153	4.82288	26
35	9.52527	9.97412	9.55115	10.44885	10.02588	10.47473	3.76227	4.82271	25
36	9.52563	9.97408	9.55155	10.44845	10.02592	10.47437	3.76300	4.82253	24
37	9.52598	9.97403	9.55195	10.44805	10.02597	10.47402	3.76373	4.82235	23
38	9.52634	9.97399	9.55235	10.44765	10.03601	10.47366	3.76446	4.82217	22
39	9.52669	9.97394	9.55275	10.44724	10.02606	10.47331	3.76519	4.82199	21
40	9.52705	9.97390	9.55315	10.44685	10.02610	10.47295	3.76592	4.82181	20
41	9.52740	9.97385	9.55355	10.44645	10.02615	10.47260	3.76665	4.82163	19
42	9.52775	9.97381	9.55395	10.44605	10.02619	10.47225	3.76737	4.82145	18
43	9.52811	9.97376	9.55434	10.44566	10.02624	10.47189	3.76810	4.82127	17
44	9.52846	9.97372	9.55474	10.44526	10.02628	10.47154	3.76883	4.82109	16
45	9.52881	9.97367	9.55514	10.44486	10.02633	10.47119	3.76955	4.82091	15
46	9.52916	9.97363	9.55554	10.44446	10.02637	10.47084	3.77028	4.82073	14
47	9.52951	9.97358	9.55593	10.44407	10.02642	10.47049	3.77100	4.82055	13
48	9.52986	9.97353	9.55633	10.44367	10.02647	10.47014	3.77173	4.82037	12
49	9.53021	9.97349	9.55673	10.44327	10.02651	10.46979	3.77245	4.82019	11
50	9.53056	9.97344	9.55712	10.44288	10.02656	10.46944	3.77318	4.82001	10
51	9.53092	9.97340	9.55752	10.44248	10.02660	10.46908	3.77390	4.81983	9
52	9.53126	9.97335	9.55791	10.44209	10.02665	10.46874	3.77462	4.81965	8
53	9.53161	9.97331	9.55831	10.44169	10.02669	10.46839	3.77534	4.81947	7
54	9.53196	9.97326	9.55870	10.44130	10.02674	10.46804	3.77606	4.81929	6
55	9.53231	9.97322	9.55910	10.44090	10.02678	10.46769	3.77678	4.81911	5
56	9.53266	9.97317	9.55949	10.44051	10.02683	10.46734	3.77750	4.81893	4
57	9.53301	9.97312	9.55989	10.44011	10.02688	10.46699	3.77822	4.81875	3
58	9.53336	9.97308	9.56028	10.43972	10.02692	10.46664	3.77894	4.81857	2
59	9.53370	9.97303	9.56067	10.43933	10.02697	10.46630	3.77965	4.81839	1
60	9.53405	9.97299	9.56107	10.43893	10.02701	10.46595	3.78037	4.81821	0
	Co-sine	Sine	Co-tan.	Tang.	Co-sec.	Secant		V. Sine	M

M	Sine	Co-sine	Tang.	Co-tan.	Secant	Co-sec.	V.Sine		
0	9.53405	9.97209	9.56107	10.43893	10.02701	10.46595	3.78037	4.81821	60
1	9.53440	9.97294	9.56146	10.43854	10.02706	10.46560	3.78109	4.81803	59
2	9.53475	9.97289	9.56185	10.43815	10.02711	10.46525	3.78180	4.81785	58
3	9.53509	9.97285	9.56224	10.43776	10.02715	10.46491	3.78252	4.81767	57
4	9.53544	9.97280	9.56264	10.43736	10.02720	10.46456	3.78323	4.81749	56
5	9.53578	9.97276	9.56303	10.43697	10.02724	10.46422	3.78395	4.81731	55
6	9.53613	9.97271	9.56342	10.43658	10.02729	10.46387	3.78466	4.81713	54
7	9.53647	9.97266	9.56381	10.43619	10.02733	10.46353	3.78537	4.81695	53
8	9.53682	9.97262	9.56420	10.43580	10.02738	10.46318	3.78608	4.81677	52
9	9.53716	9.97257	9.56459	10.43541	10.02743	10.46284	3.78679	4.81659	51
10	9.53751	9.97252	9.56498	10.43502	10.02748	10.46249	3.78750	4.81641	50
11	9.53785	9.97248	9.56537	10.43463	10.02752	10.46215	3.78821	4.81622	49
12	9.53819	9.97243	9.56576	10.43424	10.02757	10.46181	3.78892	4.81604	48
13	9.53854	9.97238	9.56615	10.43385	10.02762	10.46146	3.78963	4.81586	47
14	9.53888	9.97234	9.56654	10.43346	10.02766	10.46112	3.79034	4.81568	46
15	9.53922	9.97229	9.56693	10.43307	10.02771	10.46078	3.79105	4.81550	45
16	9.53957	9.97224	9.56732	10.43268	10.02776	10.46043	3.79176	4.81532	44
17	9.53991	9.97220	9.56771	10.43229	10.0278	10.46009	3.79246	4.81514	43
18	9.54025	9.97215	9.56810	10.43190	10.02785	10.45975	3.79317	4.81496	42
19	9.54059	9.97210	9.56849	10.43151	10.02790	10.45941	3.79387	4.81477	41
20	9.54093	9.97206	9.56887	10.43113	10.02794	10.45907	3.79458	4.81459	40
21	9.54127	9.97201	9.56026	10.43074	10.02799	10.45873	3.79528	4.81441	39
22	9.54161	9.97196	9.56965	10.43035	10.02804	10.45839	3.79599	4.81423	38
23	9.54195	9.97192	9.57004	10.42996	10.02808	10.45805	3.79669	4.81405	37
24	9.54229	9.97187	9.57042	10.42958	10.02813	10.45771	3.79739	4.81387	36
25	9.54263	9.97182	9.57081	10.42919	10.02818	10.45737	3.79809	4.81368	35
26	9.54297	9.97178	9.57120	10.42880	10.02822	10.45703	3.79880	4.81350	34
27	9.54331	9.97173	9.57158	10.42842	10.02827	10.45669	3.79950	4.81332	33
28	9.54365	9.97168	9.57197	10.42803	10.02832	10.45635	3.80020	4.81314	32
29	9.54399	9.97163	9.57235	10.42765	10.02837	10.45601	3.80090	4.81296	31
30	9.54433	9.97159	9.57274	10.42726	10.02841	10.45567	3.80159	4.81277	30
31	9.54466	9.97154	9.57312	10.42688	10.02846	10.45534	3.80229	4.81259	29
32	9.54500	9.97149	9.57351	10.42649	10.02851	10.45500	3.80299	4.81241	28
33	9.54534	9.97145	9.57389	10.42611	10.02855	10.45466	3.80369	4.81223	27
34	9.54567	9.97140	9.57428	10.42572	10.02860	10.45433	3.80438	4.81205	26
35	9.54601	9.97135	9.57466	10.42534	10.02865	10.45399	3.80508	4.81186	25
36	9.54635	9.97130	9.57504	10.42496	10.02870	10.45365	3.80578	4.81168	24
37	9.54668	9.97126	9.57543	10.42457	10.02874	10.45332	3.80647	4.81150	23
38	9.54702	9.97121	9.57581	10.42419	10.02879	10.45298	3.80716	4.81132	22
39	9.54735	9.97116	9.57619	10.42381	10.02884	10.45265	3.80786	4.81113	21
40	9.54769	9.97111	9.57658	10.42342	10.02889	10.45231	3.80855	4.81095	20
41	9.54802	9.97107	9.57696	10.42304	10.02893	10.45198	3.80924	4.81077	19
42	9.54836	9.97102	9.57734	10.42266	10.02898	10.45164	3.80994	4.81059	18
43	9.54869	9.97097	9.57772	10.42228	10.02903	10.45131	3.81063	4.81040	17
44	9.54903	9.97092	9.57810	10.42190	10.02908	10.45097	3.81132	4.81022	16
45	9.54936	9.97087	9.57849	10.42151	10.02913	10.45064	3.81201	4.81004	15
46	9.54969	9.97083	9.57887	10.42113	10.02917	10.45041	3.81270	4.80985	14
47	9.55003	9.97078	9.57925	10.42075	10.02922	10.44997	3.81339	4.80967	13
48	9.55036	9.97073	9.57963	10.42037	10.02927	10.44964	3.81408	4.80949	12
49	9.55069	9.97068	9.58001	10.41999	10.02932	10.44931	3.81476	4.80930	11
50	9.55102	9.97063	9.58039	10.41961	10.02937	10.44898	3.81545	4.80912	10
51	9.55136	9.97059	9.58077	10.41923	10.02941	10.44864	3.81614	4.80894	9
52	9.55169	9.97054	9.58115	10.41885	10.02946	10.44831	3.81683	4.80875	8
53	9.55202	9.97049	9.58153	10.41847	10.02951	10.44798	3.81751	4.80857	7
54	9.55235	9.97044	9.58191	10.41809	10.02956	10.44765	3.81820	4.80839	6
55	9.55268	9.97039	9.58229	10.41771	10.02961	10.44732	3.81888	4.80820	5
56	9.55301	9.97035	9.58267	10.41733	10.02965	10.44699	3.81957	4.80802	4
57	9.55334	9.97030	9.58304	10.41696	10.02970	10.44666	3.82025	4.80784	3
58	9.55367	9.97025	9.58342	10.41658	10.02975	10.44633	3.82093	4.80765	2
59	9.55400	9.97020	9.58380	10.41620	10.02980	10.44600	3.82161	4.80747	1
60	9.55433	9.97015	9.58418	10.41582	10.02985	10.44567	3.82230	4.80729	0
	Co-sine	Sine	Co tan	Tangent	Co-sec.	Secant		V. Sine	M

(69 Deg.)

M.	Sine	Co-sine	Tang.	Co-tang.	Secant	Co-sec.	V. Sine		
0	9.55433	9.97015	9.58418	10.41582	10.02985	10.44567	3.82230	4.80729	60
1	9.55466	9.97010	9.58455	10.41545	10.02990	10.44534	3.82298	4.80810	59
2	9.55499	9.97005	9.58493	10.41507	10.02995	10.44501	3.82366	4.80692	58
3	9.55532	9.97001	9.58531	10.41469	10.02999	10.44468	3.82434	4.80673	57
4	9.55564	9.96996	9.58569	10.41431	10.03004	10.44436	3.82502	4.80655	56
5	9.55597	9.96991	9.58606	10.41494	10.03009	10.44403	3.82570	4.80637	55
6	9.55630	9.96986	9.58644	10.41356	10.03014	10.44370	3.82638	4.80618	54
7	9.55663	9.96981	9.58681	10.41319	10.03019	10.44337	3.82705	4.80600	53
8	9.55695	9.96976	9.58719	10.41281	10.03024	10.44305	3.82773	4.80581	52
9	9.55728	9.96971	9.58757	10.41243	10.03029	10.44272	3.82841	4.80563	51
10	9.55761	9.96966	9.58794	10.41206	10.03034	10.44239	3.82908	4.80545	50
11	9.55793	9.96962	9.58832	10.41168	10.03038	10.44207	3.82976	4.80526	49
12	9.55826	9.96957	9.58869	10.41131	10.03043	10.44174	3.83044	4.80508	48
13	9.55858	9.96952	9.58907	10.41093	10.03048	10.44142	3.83111	4.80489	47
14	9.55891	9.96947	9.58944	10.41056	10.03053	10.44109	3.83178	4.80471	46
15	9.55923	9.96942	9.58981	10.41019	10.03058	10.44077	3.83246	4.80452	45
16	9.55956	9.96937	9.59019	10.40981	10.03063	10.44044	3.83313	4.80434	44
17	9.55988	9.96932	9.59056	10.40944	10.03068	10.44012	3.83380	4.80415	43
18	9.56021	9.96927	9.59094	10.40906	10.03073	10.43979	3.83448	4.80397	42
19	9.56053	9.96922	9.59131	10.40869	10.030[illegible]	10.43947	3.835[illegible]	4.80378	41
20	9.56085	9.96917	9.59168	10.40832	10.03083	10.43915	3.83582	4.80360	40
21	9.56118	9.96912	9.59205	10.40795	10.03088	10.43882	3.83649	4.80341	39
22	9.56150	9.96907	9.59243	10.40757	10.03093	10.43850	3.83716	4.80323	38
23	9.56182	9.96903	9.59280	10.40720	10.03097	10.43818	3.83783	4.80304	37
24	9.56215	9.96898	9.59317	10.40683	10.03102	10.43785	3.83850	4.80286	36
25	9.56247	9.96893	9.59354	10.40646	10.03107	10.43753	3.83917	4.80267	35
26	9.56279	9.96888	9.59391	10.40609	10.03112	10.43721	3.8398	4.80249	34
27	9.56311	9.96883	9.59429	10.40571	10.03117	10.43[illegible]8[illegible]	3.84050	4.80230	33
28	9.56343	9.96878	9.59466	10.40534	10.03122	10.43657	3.84117	4.80212	32
29	9.56375	9.96873	9.59503	10.40497	10.03127	10.43625	3.8418	4.80193	31
30	9.56408	9.96868	9.59540	10.40460	10.03132	10.43592	3.84[illegible]50	4.80175	30
31	9.56440	9.96863	9.59577	10.40423	10.03137	10.43560	3.84316	4.80156	29
32	9.56472	9.96858	9.59614	10.40386	10.03142	10.43528	3.84383	4.8013[illegible]	28
33	9.56504	9.96853	9.59651	10.40349	10.03147	10.43496	3.84449	4.80119	27
34	9.56536	9.96848	9.59688	10.40312	10.03152	10.43464	3.84516	4.80100	26
35	9.56568	9.96843	9.59725	10.40275	10.03157	10.43432	3.84582	4.80082	25
36	9.56599	9.96838	9.59762	10.40238	10.03162	10.43401	3.8464	4.80063	24
37	9.56631	9.96833	9.59799	10.40201	10.03167	10.43369	3.84714	4.8[illegible]45	23
38	9.56663	9.96828	9.59835	10.[illegible]0165	10.03172	10.43337	3.84781	4.80026	22
39	9.56695	9.96823	9.59872	10.40128	10.03177	10.43305	3.84847	4.80007	21
40	9.56727	9.96818	9.59909	10.40091	10.03182	10.43273	3.84913	4.79989	20
41	9.56759	9.96813	9.59946	10.40054	10.03187	10.43241	3.84979	4.79970	19
42	9.56790	9.96808	9.59983	10.40017	10.03192	10.43210	3.85045	4.79952	18
43	9.56822	9.96803	9.60019	10.39981	10.03197	10.43178	3.85111	4.79933	17
44	9.56854	9.96798	9.60056	10.39944	10.03202	10.43146	3.85176	4.79914	16
45	9.56886	9.96793	9.60093	10.39907	10.03207	10.43114	3.85242	4.79896	15
46	9.56917	9.96788	9.60130	10.39870	10.03212	10.43083	3.85308	4.79877	14
47	9.56949	9.96783	9.60166	10.39834	10.03217	10.43051	3.85374	4.79858	13
48	9.56980	9.96778	9.60203	10.39797	10.03222	10.43020	3.85430	4.79840	12
49	9.57012	9.96772	9.60240	10.39760	10.03228	10.42988	3.85505	4.79821	11
50	9.57044	9.96767	9.60276	10.39724	10.03233	10.42956	3.85570	4.79802	10
51	9.57075	9.96762	9.60313	10.39687	10.03238	10.42925	3.85636	4.79784	9
52	9.57107	9.96757	9.60349	10.39651	10.03243	10.42893	3.85701	4.79765	8
53	9.57138	9.96752	9.60386	10.39614	10.03248	10.42862	3.85767	4.79746	7
54	9.57169	9.96747	9.60422	10.39578	10.03253	10.42831	3.85832	4.79728	6
55	9.57201	9.96742	9.60459	10.39541	10.03258	10.42799	3.85897	4.79709	5
56	9.57232	9.96737	9.60495	10.39505	10.03263	10.42768	3.85962	4.79690	4
57	9.57264	9.96732	9.60532	10.39468	10.03268	10.42736	3.86028	4.79671	3
58	9.57295	9.96727	9.60568	10.39432	10.03273	10.42705	3.86093	4.79653	2
59	9.57326	9.96722	9.60605	10.39395	10.03278	10.42674	3.86158	4.79634	1
60	9.57358	9.96717	9.60641	10.39359	10.03283	10.42642	3.86223	4.79615	0
	Co-sine	Sine.	Co-tan.	Tangent	Co-sec.	Secant		V. Sine	M

(68 Deg.)

M	Sine	Co-sine	Tang.	Co-tang.	Secant	Co-sec.	V. Sine		
0	9.57358	9.96717	9.60641	10.39359	10.03283	10.4264[illegible]	3.8622[illegible]	4.7961[illegible]	[illegible]
1	9.57389	9.967[illegible]1	9.60677	10.39323	10.03289	1[illegible]4261[illegible]	3.86288	4.7959[illegible]	59
2	9.57420	9.96706	9.60714	10 39286	10.03294	10.4258	3.86333	4.79578	58
3	9.57451	9.96701	9.60750	10.39250	10.03299	10.4254[illegible]	3 86417	4.79559	57
4	9.57482	9.96696	9.6078[illegible]	10.39214	10.03304	10.42518	[illegible].86482	4.79540	56
5	9.57514	9.96691	9.60823	10.39177	10.03309	10.42486	3.8654[illegible]	4.79522	55
6	9.57545	9 96686	9.60859	10.39141	10.0331[illegible]	10.4245[illegible]	3.8661[illegible]	4 79503	54
7	9.57576	9.96681	9.60[illegible]95	10.39[illegible]0[illegible]	10.03319	10.42424	3.866[illegible]0	4.79484	53
8	9.57607	9.96676	9.60[illegible]31	10.39069	10.03324	10.4239[illegible]	3.867[illegible]1	4. [illegible]9465	52
9	9.57638	9.966[illegible]0	9.60967	10.39033	10.03330	1[illegible].42362	3.86806	4.79447	51
10	9.57669	9.96665	9.61004	10.389[illegible]6	10.03335	1[illegible].42331	3.86870	4.79498	50
11	9.57700	9.96660	9.61040	10.38960	10 03340	10.4230[illegible]	3.86935	4.[illegible]94[illegible]	49
12	9 57731	9.96655	[illegible].61076	1[illegible].38924	10.03[illegible]45	10.42269	3 86990	4[illegible]	48
13	9.[illegible]7762	9 96650	[illegible].61112	10.388[illegible]8	10.033[illegible]	10.4[illegible]238	3.8706[illegible]	4.[illegible]	47
14	9.57793	9.96645	9.61148	10 38852	10.03355	10.4[illegible]207	3.87128	4.79353	46
15	9.57824	9.96640	9.61184	10.38816	10.03360	10.42176	3.87192	4.79334	45
16	9.57854	9.96634	9.61220	10.38780	10.03366	1[illegible].42146	3.87256	4.79315	44
17	9.57885	9.96629	9.61256	1[illegible].38744	1[illegible]3371	10.42115	3.87320	4.79296	43
18	9.57916	9.96624	9.61292	10.38708	1[illegible].03376	[illegible]0.42084	3.8738[illegible]	4.79277	42
19	9.57947	9 96619	9.[illegible]1328	10 38672	10[illegible]3381	[illegible]0.42053	3.87449	4 7925[illegible]	41
20	9.57978	9.96614	9.61364	1[illegible].38636	10.03386	10.42[illegible]22	3.87513	4.79240	40
21	9.58008	9.96608	9 61400	10.38600	10.03[illegible]92	10.41992	3.87577	4.7[illegible]221	39
22	9.58039	9.96603	9.61436	10.38564	10.03397	10.41[illegible]61	3.87641	4.79202	38
23	9.58070	9.96598	9.61472	10.38528	10.[illegible]3402	10.[illegible]1930	3.8770[illegible]	[illegible].79183	37
24	9.58101	9.96593	9.61508	10.38492	1[illegible].03407	1[illegible].418[illegible]	3.8[illegible]768	4.79164	36
25	9.58131	9.96588	9.61544	10.38456	10.03412	10.41[illegible]	3.8783[illegible]	[illegible].79145	35
26	9.58162	9.96582	9.61579	10.38421	10.03418	1[illegible].41838	3.87[illegible]96	[illegible].79126	34
27	9.58192	9.96577	9.61615	10.38385	10.03423	1[illegible].418[illegible]	3.87959	4.19107	33
28	9.58223	9.96572	9.61651	1[illegible].38349	10.03428	10.41777	3.8802[illegible]	4.79089	32
29	9.58253	9.96567	9.6[illegible]687	10.383[illegible]3	1[illegible].0343[illegible]	10.41[illegible]47	3.8808[illegible]	[illegible].79[illegible]70	31
30	9.58284	9.96562	9.6172[illegible]	10.38278	10.[illegible]3438	1[illegible].41716	3.88150	4.79051	30
31	9.58314	[illegible].96556	9.62[illegible]5[illegible]	10.38242	10.03444	10.4168[illegible]	3.8821[illegible]	4.79032	29
32	9.58345	9.96551	9.61794	10.38206	1[illegible].03449	10.41655	3.88277	4.7901[illegible]	28
33	9.58375	9.96546	9.61830	10.38170	10.0345[illegible]	[illegible]0.41625	3.88[illegible]4[illegible]	[illegible].78994	27
34	9.58406	9.96541	9.61865	10.38135	10.0345[illegible]	[illegible].41594	3.88[illegible]	[illegible].78975	26
35	9.58436	9.96535	9.61901	1[illegible].38099	10.03465	10.41[illegible]6[illegible]	3.88467	[illegible].78956	25
36	9.58467	9.96530	9.6193[illegible]	10.3806[illegible]	10.03470	10.41[illegible]	3.88530	[illegible].78937	24
37	9.58497	9.96525	9.61972	10.3[illegible]02[illegible]	10.03475	10.41503	3.8859[illegible]	[illegible].78918	23
38	9.58527	9.96520	9.62008	10.37992	10.[illegible]3480	10.41473	3.88[illegible]5[illegible]	4.[illegible]8899	22
39	9.58557	9.96514	9.6204[illegible]	10.3795[illegible]	1[illegible].03486	10 41443	3.887[illegible]	4.78[illegible]0	21
40	9.58588	9.96509	9.620[illegible]	10.37921	10.03491	10.41412	3.8878[illegible]	4.[illegible]8861	20
41	9.58618	9.96504	9.62114	10 37886	10.03496	10.41382	3.88[illegible]46	4.[illegible]8842	19
42	9.58648	9.96498	9.621[illegible]	10.37850	10.03502	[illegible]0.4135[illegible]	3.8[illegible]	[illegible].78823	18
43	9.586[illegible]8	9.96493	9.[illegible]	10.37815	10.03507	10.413[illegible]	3.88[illegible]	[illegible].78804	17
44	9.58708	9.96488	9.62221	10.37779	1[illegible].03512	[illegible]0.4129[illegible]	3.89035	4.7[illegible]786	16
45	9.58739	9.96483	[illegible].62256	[illegible]0.3774[illegible]	[illegible]0.03517	10.41261	3.8909[illegible]	4.78767	15
46	9.58769	9.96477	9.62[illegible]91	10.37709	[illegible]0.03523	10.41231	3.89[illegible]60	4.78[illegible]48	14
47	9.58799	9.96472	9.6232[illegible]	10 3767[illegible]	[illegible]0.03528	10.41[illegible]01	3 89223	[illegible].78729	13
48	9.5882[illegible]	9.96466	9.6236[illegible]	10.3763[illegible]	10.03533	10.41171	[illegible].89286	4.[illegible]8710	12
49	9.58859	9.96461	[illegible] 62398	10.37602	[illegible]0.03539	10.4114[illegible]	[illegible].89348	4.78690	11
50	9.58889	9.96456	9.62433	10.37567	10.03544	10 411[illegible]	3.89411	4.7[illegible]671	10
51	9.58919	9.96451	9.62468	[illegible]0.37532	10.0354[illegible]	[illegible]0.41081	3.89473	4.78652	9
52	9.58949	9.96445	9.62504	10.37496	10.03555	[illegible]0.41051	[illegible].89536	4.78633	8
53	9.58979	9.96440	9.62539	10.3746[illegible]	10.03560	10.41021	[illegible].89598	4.78614	7
54	9.59909	9.96435	[illegible].62574	10.3742[illegible]	10.03565	10.40991	3.89661	4.785[illegible]	6
55	9.59039	9 96429	9.62609	10.37391	10.03571	10.4[illegible]961	3.8[illegible]723	4.78576	5
56	9.59069	9.96424	9.62645	10.37355	10.03576	10 40931	3.8[illegible]785	4.7855[illegible]	4
57	9.59098	9.96419	9.62680	10.37320	10.03581	10.40902	3.89[illegible]48	4.7853[illegible]	3
58	9.59128	9.96413	9.62715	10.37285	10.03587	10.4087[illegible]	3.89910	4.78[illegible]1[illegible]	2
59	9.59158	9.96408	9.62750	10.37250	10.03592	10.40842	3.89972	4.7850[illegible]	1
60	9.59188	9.96403	9.62785	10.37215	10.03597	10.40812	3.90034	4.7848[illegible]	0
	Co-sine	Sine.	Co-tan	Tangent	Co-sec.	Secant			

M	Sine	Co-sine	Tang.	Co-tan.	Secant	Co-sec.	V. Sine		
0	9.59188	9.96403	9.62785	10.37215	10.03597	10.40812	3.90034	4.78481	60
1	9.59218	9.96397	9.62820	10.37180	10.03603	10.40782	3.90096	4.78462	59
2	9.59247	9.96392	9.62855	10.37145	10.03608	10.40753	3.90158	4.78443	58
3	9.59277	9.96386	9.62890	10.37110	10.03614	10.40723	3.90220	4.78424	57
4	9.59307	9.96381	9.62926	10.37074	10.03619	10.40693	3.90282	4.78404	56
5	9.59336	9.96376	9.62961	10.37039	10.03624	10.41664	3.90344	4.78385	55
6	9.59366	9.96370	9.62996	10.37004	10.03630	10.40634	3.90406	4.78366	54
7	9.59396	9.96365	9.63031	10.36969	10.03635	10.40604	3.90468	4.78347	53
8	9.59425	9.96360	9.63066	10.36934	10.03640	10.40575	3.90529	4.78328	52
9	9.59455	9.96354	9.63101	10.36899	10.03646	10.40545	3.90591	4.78309	51
10	9.59484	9.96349	9.63135	10.36865	10.03651	10.40516	3.90653	4.78290	50
11	9.59514	9.96343	9.63170	10.36830	10.03657	10.40486	3.90714	4.78271	49
12	9.59543	9.96338	9.63205	10.36795	10.03662	10.40457	3.90776	4.78251	48
13	9.59573	9.96333	9.63240	10.36760	10.03667	10.40427	3.90837	4.78232	47
14	9.59602	9.96327	9.63275	10.36725	10.03673	10.40398	3.90899	4.78213	46
15	9.59632	9.96322	9.63310	10.36690	10.03678	10.40368	3.90960	4.78194	45
16	9.59661	9.96316	9.63345	10.36655	10.03684	10.40339	3.91022	4.78175	44
17	9.59690	9.96311	9.63379	10.36621	10.03689	10.40310	3.91083	4.78156	43
18	9.59720	9.96305	9.63414	10.36586	10.03695	10.40280	3.91144	4.78136	42
19	9.59749	9.96300	9.63449	10.36551	10.03700	10.40251	3.91206	4.78117	41
20	9.59778	9.96294	9.63484	10.36516	10.03706	10.40222	3.91267	4.78098	40
21	9.59808	9.96289	9.63518	10.36482	10.03711	10.40192	3.91328	4.78079	39
22	9.59837	9.96284	9.63553	10.36447	10.03716	10.40163	3.91389	4.78060	38
23	9.59866	9.96278	9.63588	10.36412	10.03722	10.40134	3.91450	4.78090	37
24	9.59895	9.96273	9.63623	10.36377	10.03727	10.40105	3.91511	4.78021	36
25	9.59924	9.96267	9.63657	10.36343	10.03733	10.40076	3.91572	4.78002	35
26	9.59954	9.96262	9.63692	10.36308	10.03738	10.40046	3.91633	4.77983	34
27	9.59983	9.96256	9.63726	10.36274	10.03744	10.40017	3.91694	4.77963	33
28	9.60012	9.96251	9.63761	10.36239	10.03749	10.39988	3.91755	4.77944	32
29	9.60041	9.96245	9.63796	10.36204	10.03755	10.39959	3.91816	4.77935	31
30	9.60070	9.96240	9.63830	10.36170	10.03760	10.39930	3.91876	4.77906	30
31	9.60099	9.96234	9.63865	10.36135	10.03766	10.39901	3.91937	4.77886	29
32	9.60128	9.96229	9.63899	10.36101	10.03771	10.39872	3.91998	4.77867	28
33	9.60157	9.96223	9.63934	10.36066	10.03777	10.39843	3.92058	4.77848	27
34	9.60186	9.96218	9.63968	10.36032	10.03782	10.39814	3.92119	4.77828	26
35	9.60215	9.96212	9.64003	10.35997	10.03788	10.39785	3.92179	4.77809	25
36	9.60244	9.96207	9.64037	10.35963	10.03793	10.39756	3.92240	4.77790	24
37	9.60273	9.96201	9.64072	10.35928	10.03799	10.39727	3.92300	4.77771	23
38	9.60302	9.96196	9.64106	10.35894	10.03804	10.39698	3.92361	4.77751	22
39	9.60331	9.96190	9.64140	10.35860	10.03810	10.39669	3.92421	4.77732	21
40	9.60359	9.96185	9.64175	10.35825	10.03815	10.39641	3.92482	4.77713	20
41	9.60388	9.96179	9.64209	10.35791	10.03821	10.39612	3.92542	4.77693	19
42	9.60417	9.96174	9.64243	10.35757	10.03826	10.39583	3.92602	4.77674	18
43	9.60446	9.96168	9.64278	10.35722	10.03832	10.39554	3.92662	4.77655	17
44	9.60474	9.96162	9.64312	10.35688	10.03838	10.39526	3.92722	4.77635	16
45	9.60503	9.96157	9.64346	10.35654	10.03843	10.39497	3.92782	4.77616	15
46	9.60532	9.96151	9.64381	10.35619	10.03849	10.39468	3.92843	4.77596	14
47	9.60561	9.96146	9.64415	10.35585	10.03854	10.39439	3.92902	4.77577	13
48	9.60589	9.96140	9.64449	10.35551	10.03860	10.39411	3.92962	4.77558	12
49	9.60618	9.96135	9.64483	10.35517	10.03865	10.39382	3.93022	4.77538	11
50	9.60646	9.96129	9.64515	10.35483	10.03871	10.39354	3.93082	4.77519	10
51	9.60675	9.96123	9.64552	10.35448	10.03877	10.39325	3.93142	4.77500	9
52	9.60704	9.96118	9.64586	10.35414	10.03882	10.39296	3.93202	4.77480	8
53	9.60732	9.96112	9.64620	10.35380	10.03888	10.39268	3.93262	4.77461	7
54	9.60761	9.96107	9.64654	10.35346	10.03893	10.39239	3.93321	4.77441	6
55	9.60789	9.96101	9.64688	10.35312	10.03899	10.39211	3.93381	4.77422	5
56	9.60818	9.96095	9.64722	10.35278	10.03905	10.39182	3.93441	4.77403	4
57	9.60846	9.96090	9.64756	10.35244	10.03915	10.39154	3.93500	4.77383	3
58	9.60875	9.96084	9.64790	10.35210	10.03910	10.39125	3.93560	4.77364	2
59	9.60903	9.96079	9.64824	10.35176	10.03921	10.39097	3.93619	4.77344	1
60	9.60931	9.96073	9.64858	10.35142	10.03927	10.39069	3.93679	4.77325	0
	Co-sine	Sine	Co-tan.	Tangent	Co-sec.	Secant		V. Sine	M

M	Sine	Co-fine	Tang.	Co-tan.	Secant	Co-fec.	V.Sine		
0	9.60931	9.96073	9.64858	10.35142	10.03927	10.39069	3.93679	4.77325	60
1	9.60960	9.96067	9.64892	10.35108	10.03933	10.39040	3.93738	4.77305	59
2	9.60988	9.96062	9.64926	10.35074	10.03938	10.39012	3.93798	4.77286	58
3	9.61016	9.96056	9.64960	10.35040	10.03944	10.38984	3.93857	4.77266	57
4	9.61045	9.96050	9.64994	10.35006	10.03950	10.38955	3.93916	4.77247	56
5	9.61073	9.96045	9.65028	10.34972	10.03955	10.38927	3.93975	4.77227	55
6	9.61101	9.96039	9.65062	10.34938	10.03961	10.38899	3.94035	4.77208	54
7	9.61129	9.96034	9.65096	10.34904	10.03966	10.38871	3.94094	4.77188	53
8	9.61158	9.96028	9.65130	10.34870	10.03972	10.38842	3.94153	4.77169	52
9	9.61186	9.96022	9.65164	10.34836	10.03978	10.38814	3.94212	4.77149	51
10	9.61214	9.96017	9.65197	10.34803	10.03983	10.38786	3.94271	4.77130	50
11	9.61242	9.96011	9.65231	10.34769	10.03989	10.38758	3.94330	4.77110	49
12	9.61270	9.96005	9.65265	10.34735	10.03995	10.38730	3.94389	4.77091	48
13	9.61298	9.96000	9.65299	10.34701	10.04000	10.38702	3.94448	4.77071	47
14	9.61326	9.95994	9.65333	10.34667	10.04006	10.38674	3.94507	4.77052	46
15	9.61354	9.95988	9.65366	10.34634	10.04012	10.38646	3.94566	4.77032	45
16	9.61382	9.95982	9.65400	10.34600	10.04018	10.38618	3.94624	4.77013	44
17	9.61411	9.95977	9.65434	10.34566	10.04023	10.38589	3.94683	4.76993	43
18	9.61438	9.95971	9.65467	10.34533	10.04029	10.38562	3.94742	4.76974	42
19	9.61466	9.95965	9.05501	10.34499	10.04035	10.38534	3.94800	4.76954	41
20	9.61494	9.95960	9.65535	10.34465	10.04040	10.38506	3.94859	4.76934	40
21	9.61522	9.95954	9.65568	10.34432	10.04046	10.38478	3.94918	4.76915	39
22	9.61550	9.95948	9.65602	10.34398	10.04052	10.38450	3.94976	4.76895	38
23	9.61578	9.95942	9.65636	10.34364	10.04058	10.38422	3.95035	4.76876	37
24	9.61606	9.95937	9.65669	10.34331	10.04063	10.38394	3.95093	4.76856	36
25	9.61634	9.95931	9.65703	10.34297	10.04069	10.38366	3.95151	4.76836	35
26	9.61662	9.95925	9.65736	10.34264	10.04075	10.38338	3.95210	4.76817	34
27	9.61689	9.95920	9.65770	10.34230	10.04080	10.38311	3.95268	4.76797	33
28	9.61717	9.95914	9.65803	10.34197	10.04086	10.38283	3.95326	4.76778	32
29	9.61745	9.95908	9.65837	10.34163	10.04092	10.38255	3.95385	4.76758	31
30	9.61773	9.95902	9.65870	10.34130	10.04098	10.38227	3.95443	4.76738	30
31	9.61800	9.95897	9.65904	10.34096	10.04103	10.38200	3.95501	4.76719	29
32	9.61828	9.95891	9.65937	10.34063	10.04109	10.38172	3.95559	4.76699	28
33	9.61856	9.95885	9.65971	10.34029	10.04115	10.38144	3.95617	4.76679	27
34	9.61883	9.95879	9.66004	10.33996	10.04121	10.38117	3.95675	4.76660	26
35	9.61911	9.95873	9.66038	10.33962	10.04127	10.38089	3.95733	4.76640	25
36	9.61938	9.95868	9.66071	10.33929	10.04132	10.38061	3.95791	4.76620	24
37	9.61966	9.95862	9.66104	10.33896	10.04138	10.38034	3.95849	4.76601	23
38	9.61994	9.95856	9.66138	10.33862	10.04144	10.38006	3.95907	4.76581	22
39	9.62021	9.95850	9.66171	10.33829	10.04150	10.37979	3.95965	4.76561	21
40	9.62049	9.95844	9.66204	10.33796	10.04156	10.37951	3.96023	4.76542	20
41	9.62076	9.95839	9.66238	10.33762	10.04161	10.37924	3.96080	4.76522	19
42	9.62104	9.95833	9.66271	10.33729	10.04167	10.37896	3.96138	4.76502	18
43	9.62131	9.95827	9.66304	10.33696	10.04173	10.37869	3.96196	4.76482	17
44	9.62159	9.95821	9.66337	10.33663	10.05179	10.37841	3.96254	4.76463	16
45	9.62186	9.95815	9.66371	10.33629	10.04185	10.37814	3.96311	4.76443	15
46	9.62214	9.95810	9.66404	10.33596	10.04190	10.37786	3.96329	4.76423	14
47	9.62241	9.95804	9.66437	10.33563	10.04196	10.37759	3.96426	4.76404	13
48	9.62268	9.95798	9.66470	10.33530	10.04202	10.37732	3.96484	4.76384	12
49	9.62296	9.95792	9.66503	10.33497	10.04208	10.37704	3.96541	4.76364	11
50	9.62323	9.95786	9.66537	10.33463	10.04214	10.37677	3.96599	4.76344	10
51	9.62350	9.95780	9.66570	10.33430	10.04220	10.37650	3.96656	4.76325	9
52	9.62377	9.95775	9.66603	10.33397	10.04225	10.37623	3.96713	4.76305	8
53	9.62405	9.95769	9.66636	10.33364	10.04231	10.37595	3.96770	4.76285	7
54	9.62432	9.95763	9.66669	10.33331	10.04237	10.37568	3.96828	4.76265	6
55	9.62459	9.95757	9.66702	10.33298	10.04243	10.37541	3.96885	4.76245	5
56	9.62486	9.95751	9.66735	10.33265	10.04249	10.37514	3.96942	4.76226	4
57	9.62513	9.95745	9.66768	10.33232	10.04255	10.37487	3.96999	4.76206	3
58	9.62541	9.95739	9.66801	10.33199	10.04261	10.37459	3.97056	4.76186	2
59	9.62568	9.95733	9.66834	10.33166	10.04267	10.37432	3.97113	4.76166	1
60	9.62595	9.95728	9.66867	10.33133	10.04272	10.37405	3.97170	4.76146	0
	Co-fine	[illegible]	Co-tan	Tangent	Co-fec.	Secant		V. Sine	M

(65 Deg.)

M	Sine	Co-sine	Tang.	Co-tang.	Secant	Co-sec.	V. Sine		
0	9.62595	9.95728	9.66867	10.33133	10.04272	10.37405	3.97170	4.76146	60
1	9.62622	9.95722	9.66900	10.33100	10.04278	10.37378	3.97227	4.76126	59
2	9.62649	9.95716	9.66933	10.33067	10.04284	10.37351	3.97284	4.76107	58
3	9.62676	9.95710	9.66966	10.33034	10.04290	10.37324	3.97341	4.76087	57
4	9.62703	9.95704	9.66999	10.33001	10.04296	10.37297	3.97398	4.76067	56
5	9.62730	9.95698	9.67032	10.32968	10.04302	10.37270	3.97455	4.76047	55
6	9.62757	9.95692	9.67065	10.32935	10.04308	10.37243	3.97512	4.76027	54
7	9.62784	9.95686	9.67098	10.32902	10.04314	10.37216	3.97568	4.76007	53
8	9.62811	9.95680	9.67131	10.32869	10.04320	10.37189	3.97625	4.75987	52
9	9.62838	9.95674	9.67163	10.32837	10.04326	10.37162	3.97682	4.75968	51
10	9.62865	9.95668	9.67196	10.32804	10.04332	10.37135	3.97738	4.75948	50
11	9.62892	9.95662	9.67229	10.32771	10.04338	10.37108	3.97795	4.75928	49
12	9.62918	9.95657	9.67262	10.32738	10.04343	10.37082	3.97851	4.75908	48
13	9.62945	9.95651	9.67295	10.32705	10.04349	10.37055	3.97908	4.75888	47
14	9.62972	9.95645	9.67327	10.32673	10.04355	10.37028	3.97964	4.75868	46
15	9.62999	9.95639	9.67360	10.32640	10.04361	10.37001	3.98021	4.75848	45
16	9.63026	9.95633	9.67393	10.32607	10.04367	10.36974	3.98077	4.75828	44
17	9.63052	9.95627	9.67426	10.32574	10.04373	10.36948	3.98133	4.75808	43
18	9.63079	9.95621	9.67458	10.32542	10.04379	10.36921	3.98190	4.75788	42
19	9.63106	9.95615	9.67491	10.32509	10.04385	10.36894	3.98246	4.75768	41
20	9.63133	9.95609	9.67524	10.32476	10.04391	10.36867	3.98302	4.75748	40
21	9.63159	9.95603	9.67556	10.32444	10.04397	10.36841	3.98358	4.75728	39
22	9.63186	9.95597	9.67589	10.32411	10.04403	10.36814	3.98415	4.75708	38
23	9.63213	9.95591	9.67622	10.32378	10.04409	10.36787	3.98471	4.75689	37
24	9.63239	9.95585	9.67654	10.32346	10.04415	10.36761	3.98527	4.75769	36
25	9.63266	9.95579	9.67687	10.32313	10.04421	10.36734	3.98583	4.75649	35
26	9.63292	9.95573	9.67719	10.32281	10.04427	10.36708	3.98639	4.75629	34
27	9.63319	9.95567	9.67752	10.32248	10.04433	10.36681	3.98695	4.75609	33
28	9.63345	9.95561	9.67785	10.32215	10.04439	10.36655	3.98751	4.75589	32
29	9.63372	9.95555	9.67817	10.32183	10.04445	10.36628	3.98807	4.75569	31
30	9.63398	9.95549	9.67850	10.32150	10.04451	10.36602	3.98862	4.75549	30
31	9.63425	9.95543	9.67882	10.32118	10.04457	10.36575	3.98918	4.75528	29
32	9.63451	9.95537	9.67915	10.32085	10.04463	10.36549	3.98974	4.75508	28
33	9.63478	9.95531	9.67947	10.32953	10.04469	10.36522	3.99030	4.75488	27
34	9.63504	9.95525	9.67980	10.32020	10.04475	10.36496	3.99085	4.75468	26
35	9.63531	9.95519	9.68012	10.31988	10.04481	10.36469	3.99141	4.75448	25
36	9.63557	9.95513	9.68044	10.31956	10.04487	10.36443	3.99197	4.75428	24
37	9.63583	9.95507	9.68077	10.31923	10.04493	10.36417	3.99252	4.75408	23
38	9.63610	9.95500	9.68109	10.31891	10.04500	10.36390	3.99308	4.75388	22
39	9.63636	9.95494	9.68142	10.31858	10.04506	10.36364	3.99363	4.75368	21
40	9.63662	9.95488	9.68174	10.31826	10.04512	10.36337	3.99419	4.75348	20
41	9.63689	9.95482	9.68206	10.31794	10.04518	10.36311	3.99474	4.75328	19
42	9.63715	9.95476	9.68239	10.31761	10.04524	10.36285	3.99530	4.75308	18
43	9.63741	9.95470	9.68271	10.31729	10.04530	10.36259	3.99585	4.75288	17
44	9.63767	9.95464	9.68303	10.31697	10.04536	10.36233	3.99640	4.75268	16
45	9.63794	9.95458	9.68336	10.31664	10.04542	10.36206	3.99696	4.75247	15
46	9.63820	9.95452	9.68368	10.31632	10.04548	10.36180	3.99751	4.75227	14
47	9.63846	9.95446	9.68400	10.31600	10.04554	10.36154	3.99806	4.75207	13
48	9.63872	9.95440	9.68432	10.31568	10.04560	10.36128	3.99861	4.75187	12
49	9.63898	9.95434	9.68465	10.31535	10.04566	10.36102	3.99916	4.75167	11
50	9.63924	9.95427	9.68497	10.31503	10.04573	10.36076	3.99972	4.73147	10
51	9.63950	9.95421	9.68529	10.31471	10.04579	10.36050	4.00027	4.75127	9
52	9.63976	9.95415	9.68561	10.31439	10.04585	10.36024	4.00082	4.75106	8
53	9.64002	9.95409	9.68593	10.31407	10.04591	10.35998	4.00137	4.75086	7
54	9.64028	9.95403	9.68626	10.31374	10.04597	10.35972	4.00192	4.75066	6
55	9.64054	9.95397	9.68658	10.31342	10.04603	10.35946	4.00247	4.75047	5
56	9.64080	9.95391	9.68690	10.31310	10.04609	10.35920	4.00301	4.75026	4
57	9.64106	9.95384	9.68722	10.31278	10.04016	10.35894	4.00356	4.75006	3
58	9.64132	9.95378	9.68754	10.31246	10.04622	10.35868	4.00411	4.74985	2
59	9.64158	9.95372	9.68786	10.31214	10.04628	10.35842	4.00466	4.74965	1
60	9.64184	9.95366	9.68818	10.31182	10.04634	10.35816	4.00521	4.74945	0
	Co-sine	Sine	Co-tan.	Tang.	Co-sec.	Secant		V. Sine	M

M	Sine	Co-fine	Tang.	Co-tang.	Secant	Co-fec.	V. Sine		
0	9.64184	9.95366	9.68818	10.31182	10.04634	10.35816	4.00521	4.74945	60
1	9.64210	9.95360	9.68850	10.31150	10.04640	10.35790	4.00575	4.74925	59
2	9.64236	9.95354	9.68882	10.31118	10.04646	10.35764	4.00630	4.74904	58
3	9.64262	9.95348	9.68914	10.31086	10.04652	10.35738	4.00685	4.74884	57
4	9.64288	9.95341	9.68946	10.31054	10.04659	10.35712	4.00739	4.74864	56
5	9.64313	9.95335	9.68978	10.31022	10.04665	10.35687	4.00793	4.74844	55
6	9.64339	9.95329	9.69010	10.30990	10.04671	10.35661	4.00848	4.74824	54
7	9.64365	9.95323	9.69042	10.30958	10.04677	10.35635	4.00903	4.74803	53
8	9.64391	9.95317	9.69074	10.30926	10.04683	10.35609	4.00957	4.74783	52
9	9.64417	9.95310	9.69105	10.30894	10.04690	10.35583	4.01012	4.74763	51
10	9.64442	9.95304	9.69136	10.30862	10.04696	10.35558	4.01066	4.74742	50
11	9.64468	9.95298	9.69170	10.30830	10.04702	10.35532	4.01120	4.74722	49
12	9.64494	9.95292	9.69202	10.30798	10.04708	10.35506	4.01175	4.74702	48
13	9.64519	9.95286	9.69234	10.30766	10.04714	10.35481	4.01229	4.74682	47
14	9.64545	9.95279	9.69266	10.30734	10.04721	10.35455	4.01283	4.74661	46
15	9.64571	9.95273	9.69298	10.30703	10 04727	10.35429	4.01337	4.74641	45
16	9.64596	9.95267	9 69329	10.30671	10.04733	10.35404	4.01392	4.74621	44
17	9.64622	9.95261	9.69361	10.30639	10.04739	10.35378	4.01446	4.74600	43
18	9.64647	9.95254	9.69393	10.30607	10.04746	10.35353	4.01500	4.74580	42
19	9.64673	9.95248	9.69425	10.30575	10.04752	10.35327	4.01554	4.74560	41
20	9.64698	9.95242	9.69457	10.30543	10.04758	10.35302	4.01608	4.74539	40
21	9.64724	9.95236	9.69488	10.30512	10.04764	10.35276	4.01662	4 74519	39
22	9.64749	9.95229	9.69520	10.30480	10.04771	10.35251	4.01716	4.74499	38
23	9.64775	9.95223	9.69552	10.30448	10.04777	10.35225	4.01770	4.74478	37
24	9.64800	9.95217	9.69584	10.30416	10.04783	10.35200	4.01824	4.74458	36
25	9 64826	9.95211	9.69615	10.30385	10.04789	10.35174	4.01877	4.74437	35
26	9.64851	9.95204	9.69647	10.30353	10.04796	10.35149	4.01931	4.74417	34
27	9.64877	9 95198	9.69679	10.30321	10.04802	10.35123	4.01985	4.74397	33
28	9.64902	9.95192	9.69710	10.30290	10.04800	10.35098	4.02039	4.74376	32
29	9.64927	9.95185	9.69742	10 30258	10.04815	10.35073	4.02092	4.74356	31
30	9.64953	9 95179	9.69774	10.30226	10.04821	10.35047	4.02146	4.74335	30
31	9.64978	9.95173	9.69805	10.30195	10.04827	10.35022	4.02200	4.74315	29
32	9.65003	9.95167	9.69837	10.30163	10.04833	10.34997	4.02253	4.74295	28
33	9.65029	9.95160	9.69868	10.30132	10.04840	10.34971	4.02307	4.74274	27
34	9.65054	9.95154	9.69900	10.30100	10.04846	10.34946	4.02360	4.74254	26
35	9.65079	9.95148	9.69932	10.30068	10.04852	10.34921	4.02414	4.74233	25
36	9.65104	9.95141	9.69963	10.30037	10.04859	10.34896	4.02467	4.74213	24
37	9.65130	9.95135	9.69995	10.30005	10.04865	10.34870	4.02521	4.74192	23
38	9.65155	9.95129	9.70026	10.29974	10.04872	10.34845	4.02574	4.74172	22
39	9.65180	9.95122	9.70058	10.29942	10.04878	10.34820	4.02627	4.74151	21
40	9.65205	9.95116	9.70089	10.29911	10.04884	10.34795	4.02681	4.74131	20
41	9.65230	9.95110	9.70121	10.29879	10.04890	10.34770	4.02734	4.74110	19
42	9.65255	9.95103	9.70152	10.29848	10.04897	10.34745	4.02787	4.74090	18
43	9.65281	9.95097	9.70184	10.29816	10.04903	10.34719	4.02641	4.74070	17
44	9.65306	9.95090	9.70215	10.29785	10.04910	10.34694	4.02894	4.74049	16
45	9.65331	9.95084	9.70247	10.29753	10.04916	10.34669	4.02947	4.74028	15
46	9.65356	9.95078	9.70278	10.29722	10.04922	10.34644	4.03000	4.74008	14
47	9.65381	9.95071	9.70309	10 29691	10.04929	10 34619	4.03053	4.73987	13
48	9.65406	9.95065	9.70341	10.29659	10.04935	10.34594	4.03106	4.73967	12
49	9.65431	9.95059	9.70372	10.29628	10.04941	10.34569	4.03159	4.73946	11
50	9.65456	9.95052	9.70404	10.29596	10.04948	10.34544	4.03212	4.73926	10
51	9.65481	9.95046	9.70435	10.29565	10.04954	10.34519	4.03265	4.73905	9
52	9.65506	9.95039	9.70466	10.29534	10.94961	10.34494	4.03318	4.73885	8
53	9.65531	9 95033	9.70498	10 29502	10.04967	10.34469	4.03371	4.73864	7
54	9.65556	9.95027	9.70529	10.29471	10.04973	10.34444	4.03424	4.73844	6
55	9.65580	9.95020	9.70560	10.29440	10.04980	10.34420	4.03477	4.73823	5
56	9.65605	9.95014	9.70592	10.29408	10.04986	10.34395	4.03529	4.73802	4
57	9.65630	9.95007	9.70623	10.29377	10.04993	10.34370	4.03582	4.73782	3
58	9.65655	9.95001	9.70654	10.29346	10.04999	10.34345	4.03635	4.73761	2
59	9.65680	9.94995	9.70685	10.29315	10.05005	10.34320	4.03687	4.73741	1
60	9.65705	9.94988	9.70717	10.29283	10.05012	10.34295	4.03740	4.73720	0
	Co-fine	Sine	Co-tan.	Tangent	Co-fec.	Secant		V. Sine	M

M.	Sine	Co-ſine	Tang.	Co-tang.	Secant	Co-ſec.	V. Sine		
0	9.65705	9.94988	9.70717	10.29283	10.05012	10.34295	4.03740	4.73720	60
1	9.65729	9.94982	9.70748	10.29252	10.05018	10.34271	4.03743	4.73699	59
2	9.65754	9.94975	9.70779	10.29221	10.05025	10.34246	4.03845	4.73679	58
3	9.65779	9.94969	9.70810	10.29190	10.05031	10.34221	4.03898	4.73658	57
4	9.65804	9.94962	9.70841	10.29159	10.05038	10.34196	4.03950	4.73637	56
5	9.65828	9.94956	9.70873	10.29127	10.05044	10.34172	4.04003	4.73617	55
6	9.65853	9.94949	9.70904	10.29096	10.05051	10.34147	4.04055	4.73596	54
7	9.65878	9.94943	9.70935	10.29065	10.05057	10.34122	4.04108	4.73576	53
8	9.65902	9.94936	9.70966	10.29034	10.05064	10.34098	4.04159	4.73555	52
9	9.65927	9.94930	9.70997	10.29003	10.05070	10.34073	4.04212	4.73534	51
10	9.65952	9.94923	9.71028	10.28972	10.05077	10.34048	4.04265	4.73514	50
11	9.65976	9.94917	9.71059	10.28941	10.05083	10.34024	4.04317	4.73493	49
12	9.66001	9.94911	9.71090	10.28910	10.05089	10.33999	4.04369	4.73472	48
13	9.66025	9.94904	9.71121	10.28879	10.05096	10.33975	4.04421	4.73451	47
14	9.66050	9.94898	9.71153	10.28847	10.05102	10.33950	4.04473	4.73431	46
15	9.66075	9.94891	9.71184	10.28816	10.05109	10.33925	4.04526	4.73410	45
16	9.66099	9.94884	9.71215	10.28785	10.05116	10.33901	4.04578	4.73389	44
17	9.66124	9.94878	9.71246	10.28754	10.05122	10.33876	4.046 0	4.73369	43
18	9.66148	9.94871	9.71277	10.28723	10.05129	10.33852	4.04682	4.73348	42
19	9.66173	9.94865	9.71308	10.28692	10.05135	10.33827	4.04734	4.73327	41
20	9.66197	9.94858	9.71339	10.28661	10.05142	10.33803	4.04786	4.73306	40
21	9.66221	9.94852	9.71370	10.28630	10.05148	10.33779	4.04838	4.73286	39
22	9.66246	9.94845	9.71401	10.28599	10.05155	10.33754	4.04890	4.73265	38
23	9.66270	9.94839	9.71431	10.28569	10.05161	10.33730	4.04941	4.73244	37
24	9.66295	9.94832	9.71462	10.28538	10.05168	10.33705	4.04993	4.73223	36
25	9.66319	9.94826	9.71493	10.28507	10.05174	10.33681	4.05045	4.73203	35
26	9.66343	9.94819	9.71524	10.28476	10.05181	10.33657	4.05097	4.73182	34
27	9.66368	9.94813	9.71555	10.28445	10.05187	10.33632	4.05149	4.73161	33
28	9.66392	9.94806	9.71586	10.28414	10.05194	10.33608	4.05200	4.73140	32
29	9.66416	9.94799	9.71617	10.28383	10.05201	10.33584	4.05252	4.73119	31
30	9.66441	9.94793	9.71648	10.28352	10.05207	10.33559	4.05304	4.73099	30
31	9.66465	9.94786	9.71679	10.28321	10.05214	10.33535	4.05355	4.73078	29
32	9.66489	9.94780	9.71709	10.28291	10.05220	10.33511	4.05407	4.73057	28
33	9.66513	9.94773	9.71740	10.28260	10.05227	10.33487	4.05458	4.73036	27
34	9.66537	9.94767	9.71771	10.28229	10.05233	10.33463	4.05510	4.73015	26
35	9.66562	9.94760	9.71802	10.28198	10.05240	10.33438	4.05561	4.72994	25
36	9.66586	9.94753	9.71833	10.28167	10.05247	10.33414	4.05613	4.72973	24
37	9.66610	9.94747	9.71863	10.28137	10.05253	10.33390	4.05664	4.72953	23
38	9.66634	9.04740	9.71894	10.28106	10.05260	10.33366	4.05716	4.72932	22
39	9.66658	9.94734	9.71925	10.28075	10.05266	10.33342	4.05767	4.72911	21
40	9.66682	9.94727	9.71955	10.28045	10.05273	10.33318	4.05818	4.72890	20
41	9.66706	9.94720	9.71986	10.28014	10.05280	10.33294	4.05870	4.72869	19
42	9.66731	9.9471[illegible]	9.72017	10.27983	10.05286	10.33269	4.05921	4.72848	18
43	9.66755	9.94707	9.72048	10.27952	10.05393	10.33245	4.05972	4.72827	17
44	9.66779	9.94700	9.72078	10.27922	10.05300	10.33221	4.06023	4.72806	16
45	9.66803	9.94694	9.72109	10.27891	10.05306	10.33197	4.06074	4.72785	15
46	9.66827	9.94687	9.72140	10.27860	10.05313	10.33173	4.06125	4.72765	14
47	9.66851	9.94680	9.72170	10.27830	10.05320	10.33149	4.06177	4.72744	13
48	9.66875	9.94674	9.72201	10.27799	10.05326	10.33125	4.06228	4.72723	12
49	9.66899	9.94667	9.72231	10.27769	10.05333	10.33101	4.06279	4.72702	11
50	9.66922	9.94660	9.72262	10.27738	10.05340	10.33078	4.06330	4.72681	10
51	9.66946	9.94654	9.72293	10.27707	10.05347	10.33054	4.06381	4.72660	9
52	9.66970	9.94647	9.72323	10.27677	10.05353	10.33030	4.06432	4.72639	8
53	9.66994	9.94640	9.72354	10.27646	10.05360	10.33006	4.06483	4.72618	7
54	9.67018	9.94634	9.72384	10.27616	10.05366	10.32982	4.06533	4.72597	6
55	9.67042	9.94627	9.72415	10.27585	10.05373	10.32958	4.06584	4.72576	5
56	9.67066	9.94620	9.72445	10.27555	10.05380	10.32934	4.06635	4.72555	4
57	9.67090	9.94614	9.72476	10.27524	10.05386	10.32910	4.06686	4.72534	3
58	9.67113	9.94607	9.72506	10.27494	10.05393	10.32887	4.06737	4.72513	2
59	9.67137	9.94600	9.72537	10.27463	10.05400	10.32863	4.06787	4.72492	1
60	9.67161	9.94593	9.72567	10.27433	10.05407	10.32839	4.06838	4.72471	0
	Co-ſine	Sine.	Co-tan.	Tangent	Co-ſec.	Secant		V. Sine	M.

(62 Deg.)

M	Sine	Co-sine	Tan.	Co-tan.	Secant	Co-sec.	V. Sine		
0	9.67161	9.94593	9.72567	10.27433	10.05407	10.32839	4.06838	4.72471	60
1	9.67185	9.94587	9.72598	10.27402	10.05413	10.32815	4.06889	4.72450	59
2	9.67208	9.94580	9.72628	10.27372	10.05420	10.32792	4.06939	4.72429	58
3	9.67232	9.94573	9.72659	10.27341	10.05427	10.32768	4.06990	4.72408	57
4	9.67256	9.94567	9.72689	10.27311	10.05433	10.32744	4.07040	4.72387	56
5	9.67280	9.94560	9.72720	10.27280	10.05440	10.32720	4.07091	4.72366	55
6	9.67303	9.94553	9.72750	10.27250	10.05447	10.32697	4.07141	4.72345	54
7	9.67327	9.94546	9.72780	10.27220	10.05454	10.32673	4.07192	4.72324	53
8	9.67350	9.94540	9.72811	10.27189	10.05460	10.32650	4.07242	4.72302	52
9	9.67374	9.94533	9.72841	10.27159	10.05467	10.32626	4.07292	4.72281	51
10	9.67398	9.94526	9.72872	10.27128	10.05474	10.32602	4.07343	4.72260	50
11	9.67421	9.94519	9.72902	10.27098	10.05481	10.32579	4.07393	4.72239	49
12	9.67445	9.94513	9.72932	10.27068	10.05487	10.32555	4.07444	4.72218	48
13	9.67468	9.94506	9.72963	10.27037	10.05494	10.32532	4.07494	4.72197	47
14	9.67492	9.94499	9.72993	10.27007	10.05501	10.32508	4.07544	4.72176	46
15	9.67515	9.94492	9.73023	10.26977	10.05508	10.32485	4.07595	4.72155	45
16	9.67539	9.94485	9.73054	10.26946	10.05515	10.32461	4.07645	4.72134	44
17	9.67562	9.94479	9.73084	10.26916	10.05521	10.32438	4.07695	4.72112	43
18	9.67586	9.94472	9.73114	10.26886	10.05528	10.32414	4.07745	4.72091	42
19	9.67609	9.94465	9.73144	10.26856	10.05535	10.32391	4.07795	4.72070	41
20	9.67633	9.94458	9.73175	10.26825	10.05542	10.32367	4.07845	4.72049	40
21	9.67656	9.94451	9.73205	10.26795	10.05549	10.32344	4.07895	4.72028	39
22	9.67680	9.94445	9.73235	10.26765	10.05555	10.32320	4.07945	4.72007	38
23	9.67703	9.94438	9.73265	10.26735	10.05562	10.32297	4.07995	4.71985	37
24	9.67726	9.94431	9.73295	10.26705	10.05569	10.32274	4.08045	4.71964	36
25	9.67750	9.94424	9.73326	10.26674	10.05576	10.32250	4.08095	4.71943	35
26	9.67773	9.94417	9.73356	10.26644	10.05583	10.32227	4.08145	4.71922	34
27	9.67796	9.94410	9.73386	10.26614	10.05590	10.32204	4.08195	4.71901	33
28	9.67820	9.94404	9.73416	10.26584	10.05596	10.32180	4.08245	4.71879	32
29	9.67843	9.94397	9.73446	10.26554	10.05603	10.32157	4.08294	4.71858	31
30	9.67866	9.94390	9.73476	10.26524	10.05610	10.32134	4.08344	4.71837	30
31	9.67890	9.94383	9.73507	10.26493	10.05617	10.32110	4.08394	4.71816	29
32	9.67913	9.94376	9.73537	10.26463	10.05624	10.32087	4.08444	4.71794	28
33	9.67936	9.94369	9.73567	10.26433	10.05631	10.32064	4.08493	4.71773	27
34	9.67959	9.94362	9.73597	10.26403	10.05638	10.32041	4.08543	4.71752	26
35	9.67982	9.94355	9.73627	10.26373	10.05645	10.32018	4.08592	4.71731	25
36	9.68006	9.94349	9.73657	10.26343	10.05651	10.31994	4.08642	4.71709	24
37	9.68029	9.94342	9.73687	10.26313	10.05658	10.31971	4.08692	4.71688	23
38	9.68052	9.94335	9.73717	10.26283	10.05665	10.31948	4.08741	4.71667	22
39	9.68075	9.94328	9.73747	10.26253	10.05672	10.31925	4.08791	4.71646	21
40	9.68098	9.94321	9.73777	10.26223	10.05679	10.31902	4.08840	4.71624	20
41	9.68121	9.94314	9.73807	10.26193	10.05686	10.31879	4.08889	4.71603	19
42	9.68144	9.94307	9.73837	10.26163	10.05693	10.31856	4.08939	4.71582	18
43	9.68167	9.94300	9.73867	10.26133	10.05700	10.31833	4.08988	4.71560	17
44	9.68190	9.94293	9.73897	10.26103	10.05707	10.31810	4.09038	4.71539	16
45	9.68213	9.94286	9.73927	10.26073	10.05714	10.31787	4.09087	4.71518	15
46	9.68237	9.94279	9.73957	10.26043	10.05721	10.31763	4.09136	4.71496	14
47	9.68260	9.94273	9.73987	10.26013	10.05727	10.31740	4.09185	4.71475	13
48	9.68282	9.94266	9.74017	10.25983	10.05734	10.31718	4.09235	4.71454	12
49	9.68305	9.94259	9.74047	10.25953	10.05741	10.31695	4.09284	4.71432	11
50	9.68328	9.94252	9.74077	10.25923	10.05748	10.31672	4.09333	4.71411	10
51	9.68351	9.94245	9.74107	10.25893	10.05755	10.31649	4.09382	4.71389	9
52	9.68374	9.94238	9.74136	10.25864	10.05762	10.31626	4.09431	4.71368	8
53	9.68397	9.94231	9.74166	10.25834	10.05769	10.31603	4.09480	4.71347	7
54	9.68420	9.94224	9.74196	10.25804	10.05776	10.31580	4.09529	4.71325	6
55	9.68443	9.94217	9.74226	10.25774	10.05783	10.31557	4.09578	4.71304	5
56	9.68466	9.94210	9.74256	10.25744	10.05790	10.31534	4.09627	4.71282	4
57	9.68489	9.94203	9.74286	10.25714	10.05797	10.31511	4.09676	4.71261	3
58	9.68512	9.94196	9.74316	10.25684	10.05804	10.31488	4.09725	4.71240	2
59	9.68534	9.94189	9.74345	10.25655	10.05811	10.31466	4.09774	4.71218	1
60	9.68557	9.94182	9.74375	10.25625	10.05818	10.31443	4.09823	4.71197	0
	Co-sine	Sine	Co-tan.	Tang.	Co-sec.	Secant		V. Sine	M

M.	Sine	Co-fine	Tang.	Co-tang.	Secant	Co-fec.	V. Sine		
0	9.68557	9.94182	9.74375	10.25625	10.05818	10.31443	4.09823	4.71197	60
1	9.68580	9.94175	9.74405	10.25595	10.05825	10.31420	4.10872	4.71175	59
2	9.68603	9.94168	9.74435	10.25565	10.05832	10.31397	4.10921	4.71154	58
3	9.68625	9.94161	9.74465	10.25535	10.05839	10.31375	4.10969	4.71132	57
4	9.68648	9.94154	9.74494	10.25506	10.05846	10.31352	4.10018	4.71111	56
5	9.68671	9.94147	9.74524	10.25476	10.05853	10.31329	4.10067	4.71089	55
6	9.68694	9.94140	9.74554	10.25446	10.05860	10.31306	4.10115	4.71068	54
7	9.68716	9.94133	9.74583	10.25417	10.05867	10.31284	4.10164	4.71046	53
8	9.68739	9.94126	9.74613	10.25387	10.05874	10.31261	4.10213	4.71025	52
9	9.68762	9.94119	9.74643	10.25357	10.05881	10.31238	4.10261	4.71003	51
10	9.68784	9.94112	9.74673	10.25327	10.05888	10.31216	4.10210	4.70982	50
11	9.68807	9.94105	9.74702	10.25298	10.05895	10.31193	4.10358	4.70960	49
12	9.68829	9.94098	9.74732	10.25268	10.05902	10.31171	4.10407	4.70939	48
13	9.68852	9.94090	9.74762	10.25238	10.05910	10.31148	4.10455	4.70917	47
14	9.68875	9.94083	9.74791	10.24209	10.05917	10.31125	4.10504	4.70896	46
15	9.68897	9.94076	9.74821	10.25179	10.05924	10.31103	4.10552	4.70874	45
16	9.68920	9.94069	9.74851	10.25149	10.05931	10.31080	4.10601	4.70853	44
17	9.68942	9.94062	9.74880	10.25120	10.05938	10.31058	4.10649	4.70831	43
18	9.68965	9.94055	9.74910	10.25090	10.05945	10.31035	4.10697	4.70810	42
19	9.68987	9.94048	9.74939	10.25061	10.05952	10.31013	4.10746	4.70788	41
20	9.69010	9.94041	9.74969	10.25031	10.05959	10.30990	4.10794	4.70766	40
21	9.69032	9.94034	9.74998	10.25002	10.05966	10.30968	4.10842	4.70745	39
22	9.69055	9.94027	9.75028	10.24972	10.05973	10.30945	4.10891	4.70723	38
23	9.69077	9.94020	9.75038	10.24942	10.05980	10.30923	4.10939	4.70702	37
24	9.69100	9.94012	9.75087	10.24913	10.05988	10.30900	4.10987	4.70680	36
25	9.69122	9.94005	9.75117	10.24883	30.05995	10.30878	4.11035	4.70658	35
26	9.69144	9.93998	9.75146	10.24854	10.06002	10.30856	4.11083	4.70637	34
27	9.69167	9.93991	9.75176	10.24824	10.06009	10.30833	4.11131	4.70615	33
28	9.69189	9.93984	9.75205	10.24795	10.06016	10.30811	4.11179	4.70593	32
29	9.69212	9.93977	9.75235	10.24765	10.06023	10.30788	4.11227	4.70572	31
30	9.69234	9.93970	9.75264	10.24736	10.06030	10.30766	4.11275	4.70550	30
31	9.69256	9.93963	9.75294	10.24706	10.06037	10.30734	4.11323	4.70528	29
32	9.69279	9.93955	9.75323	10.24677	10.06045	10.30721	4.11371	4.70507	28
33	9.69301	9.93948	9.75353	10.24647	10.06052	10.30699	4.11419	4.70485	27
34	9.69323	9.93941	9.75382	10.24618	10.06059	10.30677	4.11467	4.70463	26
35	9.69345	9.93934	9.75411	10.24589	10.06066	10.30655	4.11515	4.70442	25
36	9.69368	9.93927	9.75441	10.24559	10.06073	10.30632	4.11563	4.70420	24
37	9.69390	9.93920	9.75470	10.24530	10.06080	10.30610	4.11611	4.70398	23
38	9.69412	9.93912	9.75500	10.24500	10.06088	10.30588	4.11658	4.70377	22
39	9.69434	9.93905	9.75529	10.24471	10.06095	10.30566	4.11706	4.70355	21
40	9.69456	9.93898	9.75558	10.24442	10.06002	10.30544	4.11754	4.70333	20
41	9.69479	9.93891	9.75588	10.24412	10.06109	10.30521	4.11801	4.70317	19
42	9.69501	9.93884	9.75617	10.24383	10.06116	10.30499	4.11849	4.70290	18
43	9.69523	9.93876	9.75647	10.24353	10.06124	10.30477	4.11897	4.70268	17
44	9.69545	9.93869	9.75676	10.24324	10.06131	10.30455	4.11944	4.70246	10
45	9.69567	9.93862	9.75705	10.24295	10.06138	10.30433	4.11992	4.70224	15
46	9.69589	9.93855	9.75735	10.24265	10.06145	10.30411	4.12039	4.70203	14
47	9.69611	9.93847	9.75764	10.24236	10.06153	10.30389	4.12087	4.70181	13
48	9.69633	9.93840	9.75793	10.24207	10.06160	10.30367	4.12134	4.70159	12
49	9.69655	9.93833	9.75822	10.24178	10.06167	10.30345	4.12182	4.70137	11
50	9.69677	9.93826	9.75852	10.24148	10.06174	10.30323	4.12229	4.70115	10
51	9.69699	9.93819	9.75881	10.24119	10.06181	10.30301	4.12277	4.70094	9
52	9.69721	9.93811	9.75910	10.24090	10.06189	10.30279	4.12324	4.70072	8
53	9.69743	9.93804	9.75939	10.24061	10.06196	10.30257	4.12372	4.70050	7
54	9.69765	9.93797	9.75969	10.24031	10.06203	10.30235	4.12419	4.70028	6
55	9.69787	9.93789	9.75998	10.24002	10.06211	10.30213	4.12466	4.70006	5
56	9.69809	9.93782	9.76027	10.23973	10.06218	10.30191	4.12513	4.69984	4
57	9.69831	9.93775	9.76056	10.23944	10.06225	10.30169	4.12561	4.69963	3
58	9.69853	9.93768	9.76086	10.23914	10.06232	10.30147	4.12608	4.69941	2
59	9.69875	9.93760	9.76115	10.23885	10.06240	10.30125	4.12655	4.69919	1
60	9.69897	9.93753	9.76144	10.23856	10.06247	10.30103	4.12702	4.69897	0
	Co-fine	Sine	Co-tan	Tangent	Co-fec.	Secant		V. Sine	M.

(60 Deg.)

M	Sine	Co-ſine	Tang.	Co-tang.	Secant	Co-ſec.	V Sine		
0	9.69897	9.93753	9.76144	10.23856	10.06247	10.30103	4.12702	4.69897	60
1	9.69919	9.93746	9.76173	10.23827	10.06254	10.30081	4.12749	4.69875	59
2	9.69941	9.93738	9.76202	10 23798	10.06262	10.30059	4.12796	4.69853	58
3	9.69963	9.93731	9.76231	10.23769	10.06269	10 30037	4.12844	4.69831	57
4	9.69984	9.93724	9.76261	10.23739	10.06276	10.30016	4.12891	4.69809	56
5	9.700 6	9.93717	9.76290	10 23710	10.06283	10.29994	4.12938	4 69787	55
6	9.70028	9.93709	9.76319	10.23681	10.06291	10.29972	4.12985	4.69766	54
7	9.70050	9.93702	9.76348	10.23652	10.06298	10.29950	4.13032	4.69744	53
8	9.70072	9 93695	9.76377	10.23623	10.06305	10.29928	4.13079	4.69722	52
9	9.70093	9.93687	9.76406	10.23594	10.06313	10.29907	4.13125	4.69700	51
10	9.70115	9.93680	9.76435	10.23565	10.06320	10.29885	4.13172	4.69678	50
11	9.70137	9.93673	9 76464	10.23536	10.06327	10.29863	4.13219	4.69656	49
12	9.70159	9.93665	9.76493	10.23507	10.06335	10.29841	4.13266	4.69634	48
13	9.70180	9.93658	9.76522	10.23478	10.06342	10.29820	4.13313	4.69612	47
14	9 70202	9.93650	9.76551	10.23449	10.06350	10.29798	4.13360	4.69590	46
15	9.70224	9.93643	9.76580	10.23420	10.06357	10.29776	4.13406	4.69568	45
16	9.70245	9.93636	9.76609	10.23391	10.06364	10.29755	4.13453	4.69546	44
17	9.70267	9.93628	9.76639	10.23361	10 06372	10.29733	4.13500	4.69524	43
18	9.70288	9.93621	9.76668	10.23332	10.06379	10.29712	4.13546	4.69502	42
19	9.70310	9.93614	9.76697	10.23303	10.06386	10.29690	4.13593	4.69480	41
20	9.70332	9 93606	9.76725	10.23275	10.06394	10.29668	4.13640	4.69458	40
21	9.70353	9.93599	9.76754	10.23246	10.06401	10.29647	4.13686	4.69436	39
22	9.70375	9.93591	9.76783	10.23217	10.06409	10.29625	4.13733	4.69414	38
23	9.70396	9.93584	9.76812	10.23188	10 06416	10.29604	4.13779	4.69392	37
24	9.70418	9.93577	9.76841	10.23159	10.06423	10.29582	4.13826	4.69370	36
25	9.70439	9.93569	9.76870	10.23130	10.06431	10.29561	4.13872	4.69348	35
26	9.70461	9.93562	9.76899	10.23101	10.06438	10.29539	4.13919	4.69326	34
27	9.70482	9.93554	9.76928	10.23072	10.06446	10.29518	4.13965	4.69304	33
28	9.70504	9.93547	9.76957	10.23043	10.06453	10.29496	4.14012	4.69281	32
29	9.70525	9.93539	9.76986	10.23014	10.06461	10.29475	4.14058	4 69259	31
30	9.70547	9.93532	9.77015	10.22985	10.06468	10.29453	4.14104	4.69237	30
31	9.70568	9.93525	9.77044	10.22956	10.06475	10.29432	4.14151	4.69215	29
32	9.70590	9.93517	9.77073	10.22927	10.06483	10.29410	4.14197	4.69193	28
33	9.70611	9.93510	9.77101	10.22899	10.06490	10.29389	4.14243	4.69171	27
34	9.70633	9.93502	9.77130	10.22870	10.06498	10.29367	4.14290	4.69149	26
35	9.70654	9.93495	9.77159	10.22841	10.06505	10.29346	4.14336	4.69127	25
36	9.70675	9.93487	9 77188	10.22812	10.06513	10.29325	4.14382	4.69104	24
37	9 70697	9.93480	9.77217	10.22783	10.06520	10.29303	4.14428	4.69082	23
38	9.70718	9.93472	9.77246	10 22754	10.06528	10.29282	4.14474	4.69060	22
39	9 70739	9.93465	9.77274	10.22726	10.06535	10.29261	4.14520	4.69038	21
40	9.70761	9.93457	9.77303	10.22697	10.06543	10.29239	4.14567	4.69016	20
41	9.70782	9.93450	9.77332	10.22668	10.06550	10.29218	4.14613	4.68994	19
42	9.70803	9.93442	9.77361	10.22639	10.06558	10.29197	4.14659	4.68971	18
43	9.70824	9.93435	9.77390	10.22610	10.06565	10.29176	4.14705	4.68949	17
44	9.70846	9.93427	9 77418	10.22582	10.06573	10.29154	4.14751	4.68927	16
45	9.70867	9.93420	9.77447	10.22553	10.06580	10.29133	4.14897	4.68905	15
46	9.70888	9.93412	9.77476	10.22524	10.06588	10.29112	4.14842	4.68883	14
47	9.70909	9.93405	9.77505	10.22495	10.06595	10.29091	4.14988	4.68860	13
48	9.70931	9.93397	9.77533	10.22467	10 06603	10.29069	4.14934	4.68838	12
49	9.70952	9.93390	9.77562	10.22438	10.06610	10.29048	4.14080	4.66816	11
50	9.70973	9.93382	9.77591	10.22409	10.06618	10.29027	4.15026	4.68794	10
51	9 70994	9 93375	9.77619	10.22381	10.06625	10.29006	4.15172	4.68771	9
52	9.71015	9.93367	9.77648	10.22352	10.06633	10.28985	4.15118	4.68749	8
53	9.71036	9.93360	9.77677	10.22323	10.06640	10.28964	4.15263	4.68727	7
54	9.71058	9.93352	9.77706	10.22294	10.06648	10.28942	4-15209	4.68705	6
55	9.71079	9.93344	9.77734	10.22266	10.06656	10.28921	4.15355	4.68682	5
56	9.71100	9.93337	9.77763	10.22237	10.06663	10.28900	4.15300	4.68660	4
57	9.71121	9.93329	9.77791	10.22209	10.06671	10.28879	4.15446	4.68638	3
58	9.71142	9.93322	9.77820	10.22180	10.06678	10.28858	4.15492	4.68615	2
59	9.71163	9.93314	9.77849	1 .22151	10.06686	10.28837	4.15537	4.68593	1
	9.71184	9.93307	9.77877	10.22123	10.06693	10.28816	4.15583	4.68571	0
	Co-ſine	Sine	Co-tan.	Tangen	Co-ſec.	Secant		V. Sine	M

M	Sine	Co-sine	Tang.	Co-tang.	Secant	Co-sec.	V. Sine		M
0	9.71184	9.93307	9.77877	10.22123	10.06693	10.28816	4.15483	4.68571	60
1	9.71205	9.93299	9.77906	10.22094	10.06701	10.28795	4.15528	4.68548	59
2	9.71226	9.93291	9.77935	10.22065	10.06709	10.28774	4.15574	4.68526	58
3	9.71247	9.93284	9.77963	10.22037	10.06716	10.28753	4.15619	4.68504	57
4	9.71268	9.93276	9.77992	10.22908	10.06724	10.28732	4.15665	4.68481	56
5	9.71289	9.93269	9.78020	10.21980	10.06731	10.28711	4.15710	4.68459	55
6	9.71310	9.93261	9.78049	10.21951	10.06739	10.28690	4.15756	4.68437	54
7	9.71331	9.93253	9.78077	10.21923	10.06747	10.28669	4.15801	4.68414	53
8	9.71352	9.93246	9.78106	10.21894	10.06754	10.28648	4.15846	4.68392	52
9	9.71373	9.93238	9.78135	10.21865	10.06762	10.28627	4.15892	4.68369	51
10	9.71393	9.93230	9.78163	10.21837	10.06770	10.28607	4.15937	4.68347	50
11	9.71414	9.93223	9.78192	10.21808	10.06777	10.28586	4.15982	4.68325	49
12	9.71435	9.93215	9.78220	10.21780	10.06785	10.28565	4.16028	4.68302	48
13	9.71456	9.93207	9.78249	10.21751	10.06793	10.28544	4.16073	4.68280	47
14	9.71477	9.93200	9.78277	10.21723	10.06800	10.28523	4.16118	4.68257	46
15	9.71498	9.93192	9.78306	10.21694	10.06808	10.28502	4.16163	4.68235	45
16	9.71519	9.93184	9.78334	10.21666	10.06816	10.28481	4.16208	4.68213	44
17	9.71539	9.93177	9.78363	10.21637	10.06823	10.28461	4.16253	4.68190	43
18	9.71560	9.93169	9.78391	10.21609	10.06831	10.28440	4.16299	4.68168	42
19	9.71581	9.93161	9.78419	10.21581	10.06839	10.28419	4.16344	4.68145	41
20	9.71602	9.93154	9.78448	10.21552	10.06846	10.28398	4.16389	4.68123	40
21	9.71622	9.93146	9.78476	10.21524	10.06854	10.28378	4.16434	4.68100	39
22	9.71643	9.93138	9.78505	10.21495	10.06862	10.28357	4.16479	4.68078	38
23	9.71664	9.93131	9.78533	10.21467	10.06869	10.28336	4.16524	4.68055	37
24	9.71685	9.93123	9.78562	10.21438	10.06877	10.28315	4.16569	4.68033	36
25	9.71705	9.93115	9.78590	10.21410	10.06885	10.28295	4.16614	4.68010	35
26	9.71726	9.93107	9.78618	10.21382	10.06893	10.28274	4.16659	4.67988	34
27	9.71747	9.93100	9.78647	10.21353	10.06900	10.28253	4.16703	4.67965	33
28	9.71767	9.93092	9.78675	10.21325	10.06908	10.28233	4.16748	4.67943	32
29	9.71788	9.93084	9.78704	10.21296	10.06916	10.28212	4.16793	4.67920	31
30	9.71809	9.93077	9.78732	10.21268	10.06923	10.28191	4.16838	4.67897	30
31	9.71829	9.93069	9.78760	10.21240	10.06931	10.28171	4.16883	4.67875	29
32	9.71850	9.93061	9.78789	10.21211	10.06939	10.28150	4.16927	4.67852	28
33	9.71870	9.93053	9.78817	10.21183	10.06947	10.28130	4.16972	4.67830	27
34	9.71891	9.93046	9.78845	10.21155	10.06954	10.28109	4.17017	4.67807	26
35	9.71911	9.93038	9.78874	10.21126	10.06962	10.28089	4.17061	4.67785	25
36	9.71932	9.93030	9.78902	10.21098	10.06970	10.28068	4.17106	4.67762	24
37	9.71952	9.93022	9.78930	10.21070	10.06978	10.28048	4.17151	4.07739	23
38	9.71973	9.93014	9.78959	10.21041	10.06986	10.28027	4.17195	4.67717	22
39	9.71993	9.93007	9.78987	10.21013	10.06993	10.28007	4.17240	4.67694	21
40	9.72014	9.92999	9.79015	10.20985	10.07001	10.27986	4.17285	4.67671	20
41	9.72034	9.92991	9.79043	10.20957	10.07009	10.27966	4.17329	4.67649	19
42	9.72055	9.92983	9.79072	10.20928	10.07017	10.27945	4.17374	4.67626	18
43	9.72075	9.92970	9.79100	10.20900	10.07024	10.27925	4.17418	4.67604	17
44	9.72096	9.92968	9.79128	10.20872	10.07032	10.27904	4.17463	4.67581	16
45	9.72116	9.92960	9.79156	10.20844	10.07040	10.27884	4.17507	4.67558	15
46	9.72137	9.92952	9.79185	10.20815	10.07048	10.27863	4.17551	4.67536	14
47	9.72157	9.92944	9.79213	10.20787	10.07056	10.27843	4.17596	4.67513	13
48	9.72177	9.92936	9.79241	10.20759	10.07064	10.27823	4.17640	4.67490	12
49	9.72198	9.92929	9.79269	10.20731	10.07071	10.27802	4.17685	4.67467	11
50	9.72218	9.92921	9.79297	10.20703	10.07079	10.27782	4.17729	4.67445	10
51	9.72238	9.92913	9.79326	10.20674	10.07087	10.27762	4.17773	4.67422	9
52	9.72259	9.92905	9.79354	10.20646	10.07095	10.27741	4.17817	4.67399	8
53	9.72279	9.92897	9.79382	10.20618	10.07103	10.27721	4.17862	4.67377	7
54	9.72299	9.92889	9.79410	10.20590	10.07111	10.27701	4.17906	4.67354	6
55	9.72320	9.92881	9.79438	10.20562	10.07119	10.27680	4.17950	4.67331	5
56	9.72340	9.92874	9.79466	10.20534	10.07126	10.27660	4.17994	4.67308	4
57	9.72360	9.92866	9.79495	10.20505	10.07134	10.27640	4.18038	4.67286	3
58	9.72381	9.92858	9.79523	10.20477	10.07142	10.27619	4.18082	4.67263	2
59	9.72401	9.92850	9.79551	10.20449	10.07150	10.27599	4.18127	4.67240	1
60	9.72421	9.92842	9.79579	10.20421	10.07158	10.27579	4.18171	4.67217	0
	[illegible]	Sine.	Co-tan.	Tangent	Co-sec.			V. Sine	M

(58 Deg.)

M	Sine	Co-sin	Tang.	Co-tan.	Secant	Co-sec.	V. Sine		
0	9.72421	9.92842	9.79579	10.20421	10.07158	10.27579	4.18171	4.67217	60
1	9.72441	9.92834	9.79607	10.20393	10.07166	10.27559	4.18215	4.67194	59
2	9.72461	9.92826	9.79635	10.20365	10.07174	10.27539	4.18259	4.67172	58
3	9.72482	9.92818	9.79663	10.20337	10.07182	10.27518	4.18303	4.67149	57
4	9.72502	9.92810	9.79691	10.20309	10.07190	10.27498	4.18347	4.67126	56
5	9.72522	9.92803	9.79719	10.20281	10.07197	10.27478	4.18391	4.67103	55
6	9.72542	9.92795	9.79747	10.20253	10.07205	10.27458	4.18435	4.67080	54
7	9.72562	9.92787	9.79776	10.20224	10.07213	10.27438	4.08478	4.67058	53
8	9.72582	9.92779	9.79804	10.20196	10.07221	10.27418	4.18522	4.67035	52
9	9.72602	9.92771	9.79832	10.20168	10.07229	10.27398	4.18566	4.67012	51
10	9.72622	9.92763	9.79860	10.20140	10.07237	10.27378	4.18610	4.66989	50
11	9.72643	9.92755	9.79888	10.20112	10.07245	10.27357	4.18654	4.66966	49
12	9.72663	9.92747	9.79916	10.20084	10.07253	10.27337	4.18698	4.66943	48
13	9.72683	9.92739	9.79944	10.20056	10.07261	10.27317	4.18741	4.66920	47
14	9.72703	9.92731	9.79972	10.20028	10.07269	10.27297	4.18785	4.66897	46
15	9.72723	9.92723	9.80000	10.20000	10.07277	10.27277	4.18829	4.66874	45
16	9.72743	9.92715	9.80028	10.19972	10.07285	10.27257	4.18872	4.66852	44
17	9.72763	9.92707	9.80056	10.19944	10.07293	10.27237	4.18916	4.66829	43
18	9.72783	9.92699	9.80084	10.19916	10.07301	10.27217	4.18960	4.66806	42
19	9.72803	9.92691	9.80112	10.19888	10.07309	10.27197	4.19003	4.66783	41
20	9.72823	9.92683	9.80140	10.19860	10.07317	10.27177	4.19047	4.66760	40
21	9.72843	9.92675	9.80168	10.19832	10.07325	10.27157	4.19090	4.66737	39
22	9.72863	9.92667	9.80195	10.19805	10.07333	10.27137	4.19134	4.66714	38
23	9.72883	9.92659	9.80223	10.19777	10.07341	10.27117	4.19178	4.66691	37
24	9.72902	9.92651	9.80251	10.19749	10.07349	10.27098	4.19221	4.66668	36
25	9.72922	9.92643	9.80279	10.19721	10.07357	10.27078	4.19265	4.66645	35
26	9.72942	9.92635	9.80307	10.19693	10.07365	10.27058	4.19308	4.66622	34
27	9.72962	9.92627	9.80335	10.19665	10.07373	10.27038	4.19351	4.66599	33
28	9.72982	9.92619	9.80363	10.19637	10.07381	10.27018	4.19395	4.66576	32
29	9.73002	9.92611	9.80391	10.19609	10.07389	10.26998	4.19438	4.66553	31
30	9.73022	9.92603	9.80419	10.19581	10.07397	10.26978	4.19482	4.66530	30
31	9.73041	9.92595	9.80447	10.19553	10.07405	10.26959	4.19525	4.66507	29
32	9.73061	9.92587	9.80474	10.19526	10.07413	10.26939	4.19568	4.66484	28
33	9.73081	9.92579	9.80502	10.19498	10.07421	10.26919	4.19611	4.66461	27
34	9.73101	9.92571	9.80530	10.19470	10.07429	10.26899	4.19655	4.66438	26
35	9.73121	9.92563	9.80558	10.19442	10.07437	10.26879	4.19698	4.66415	25
36	9.73140	9.92555	9.80586	10.19414	10.07445	10.26860	4.19741	4.66392	24
37	9.73160	9.92546	9.80614	10.19386	10.07454	10.26840	4.19784	4.66369	23
38	9.73180	9.92538	9.80642	10.19358	10.07462	10.26820	4.19828	4.66346	22
39	9.73200	9.92530	9.80669	10.19331	10.07470	10.26800	4.19871	4.66322	21
40	9.73219	9.92522	9.80697	10.19303	10.07478	10.26781	4.19914	4.66299	20
41	9.73239	9.92514	9.80725	10.19275	10.07486	10.26761	4.19957	4.66276	19
42	9.73259	9.92506	9.80753	10.19247	10.07494	10.26741	4.20000	4.66253	18
43	9.73278	9.92498	9.80781	10.19219	10.07502	10.26722	4.20043	4.66230	17
44	9.73298	9.92490	9.80808	10.19192	10.07510	10.26702	4.20086	4.66207	16
45	9.73318	9.92482	9.80836	10.19164	10.07518	10.26682	4.20129	4.66184	15
46	9.73337	9.92473	9.80864	10.19136	10.07527	10.26663	4.20172	4.66161	14
47	9.73357	9.92465	9.80892	10.19108	10.07535	10.26643	4.20215	4.66137	13
48	9.73377	9.92457	9.80919	10.19081	10.07543	10.26623	4.20258	4.66114	12
49	9.73396	9.92449	9.80947	10.19053	10.07551	10.26604	4.20301	4.66091	11
50	9.73416	9.92441	9.80975	10.19025	10.07559	10.26584	4.20344	4.66068	10
51	9.73435	9.92433	9.81003	10.18997	10.07567	10.26565	4.20387	4.66045	9
52	9.73455	9.92425	9.81030	10.18970	10.07575	10.26545	4.20429	4.66021	8
53	9.73474	9.92416	9.81058	10.18942	10.07584	10.26526	4.20472	4.65998	7
54	9.73494	9.92408	9.81086	10.18914	10.07592	10.26506	4.20515	4.65975	6
55	9.73513	9.92400	9.81113	10.18887	10.07600	10.26487	4.20558	4.65952	5
56	9.73533	9.92392	9.81141	10.18859	10.07608	10.26467	4.20601	4.65929	4
57	9.73552	9.62384	9.81169	10.18831	10.07616	10.26448	4.20643	4.65905	3
58	9.73572	9.92376	9.81196	10.18804	10.07624	10.26428	4.20686	4.65882	2
59	9.73591	9.92367	9.81224	10.18776	10.07633	10.26409	4.20729	4.65859	1
60	9.73611	9.92359	9.81252	10.18748	10.07641	10.26389	4.20771	4.65836	0
	Co-sine	Sine	Co-tan.	Tang.	Co-sec.	Secant		V. Sine	M

M	Sine	Co-sine	Tang.	Co-tan.	Secant	Co-sec.	V.Sine		
0	9.73611	9.92359	9.81252	10.18748	10.07641	10.26389	4.20771	4.65836	60
1	9.73630	9.92351	9.81279	10.18721	10.07649	10.26370	4.20814	4.65812	59
2	9.73650	9.92343	9.81307	10.18693	10.07657	10.26350	4.20857	4.65789	58
3	9.73669	9.92334	9.81335	10.18665	10.07666	10.26331	4.20899	4.65766	57
4	9.73689	9.92326	9.81362	10.18638	10.07674	10.26311	4.20942	4.65742	56
5	9.73708	9.92318	9.81390	10.18610	10.07682	10.26292	4.20984	4.65719	55
6	9.73727	9.92310	9.81418	10.18582	10.07690	10.26273	4.21027	4.65696	54
7	9.73747	9.92302	9.81448	10.18555	10.07698	10.26253	4.21069	4.65673	53
8	9.73766	9.92293	9.81473	10.18527	10.07707	10.26234	4.21112	4.65649	52
9	9.73785	9.92285	9.81500	10.18500	10.07715	10.26215	4.21154	4.65626	51
10	9.73805	9.92277	9.81528	10.18472	10.07723	10.26195	4.21197	4.65602	50
11	9.73824	9.92269	9.81556	10.18444	10.07731	10.26176	4.21239	4.65579	49
12	9.73843	9.92260	9.81583	10.18417	10.07740	10.26157	4.21282	4.65556	48
13	9.73863	9.92252	9.81611	10.18389	10.07748	10.26137	4.21324	4.65532	47
14	9.73882	9.92244	9.81638	10.18362	10.07756	10.26118	4.21366	4.65509	46
15	9.73901	9.92235	9.81666	10.18334	10.07765	10.26099	4.21409	4.65486	45
16	9.73921	9.92227	9.81693	10.18307	10.07773	10.26079	4.21451	4.65462	44
17	9.73940	9.92219	9.81721	10.18279	10.07781	10.26060	4.21493	4.65439	43
18	9.73959	9.92211	9.81748	10.18252	10.07789	10.26041	4.21535	4.65415	42
19	9.73978	9.92202	9.81776	10.18224	10.07798	10.26022	4.21578	4.65392	41
20	9.73997	9.92194	9.81803	10.18197	10.07806	10.26003	4.21620	4.65369	40
21	9.74017	9.92186	9.81831	10.18169	10.07814	10.25983	4.21622	4.65345	39
22	9.74036	9.92177	9.81858	10.18142	10.07823	10.25964	4.21704	4.65322	38
23	9.74055	9.92169	9.81886	10.18114	10.07831	10.25945	4.21746	4.65298	37
24	9.74074	9.92161	9.81913	10.18087	10.07839	10.25926	4.21788	4.65275	36
25	9.74093	9.92152	9.81941	10.18059	10.07848	10.25907	4.21831	4.65251	35
26	9.74113	9.92144	9.81968	10.18032	10.07856	10.25887	4.21873	4.65228	34
27	9.74132	9.92136	9.81996	10.18004	10.07864	10.25868	4.21915	4.65204	33
28	9.74151	9.92127	9.82023	10.17977	10.07873	10.25849	4.21957	4.65181	32
29	9.74170	9.92119	9.82051	10.17949	10.07881	10.25830	4.21999	4.65157	31
30	9.74189	9.92111	9.82078	10.17922	10.07889	10.25811	4.22041	4.65134	30
31	9.74208	9.92102	9.82106	10.17894	10.07898	10.25792	4.22083	4.65110	29
32	9.74227	9.92094	9.82133	10.17867	10.07906	10.25773	4.22125	4.65087	28
33	9.74246	9.92086	9.82161	10.17839	10.07914	10.25754	4.22166	4.65063	27
34	9.74265	9.92077	9.82188	10.17812	10.07923	10.25735	4.22208	4.65040	26
35	9.74284	9.92069	9.82215	10.17785	10.07931	10.25716	4.22250	4.65016	25
36	9.74303	9.92060	9.82243	10.17757	10.07940	10.25697	4.22292	4.64993	24
37	9.74322	9.92052	9.82270	10.17730	10.07948	10.25678	4.22334	4.64969	23
38	9.74341	9.92044	9.82298	10.17702	10.07956	10.25659	4.22376	4.64946	22
39	9.74360	9.92035	9.82325	10.17675	10.07965	10.25640	4.22418	4.64922	21
40	9.74379	9.92027	9.82352	10.17648	10.07973	10.25621	4.22459	4.64898	20
41	9.74398	9.92018	9.82380	10.17620	10.07982	10.25602	4.22501	4.64875	19
42	9.74417	9.92010	9.82407	10.17593	10.07990	10.25583	4.22543	4.64851	18
43	9.74436	9.92002	9.82435	10.17565	10.07998	10.25564	4.22584	4.64828	17
44	9.74455	9.91993	9.82462	10.17538	10.08007	10.25545	4.22626	4.64804	16
45	9.74474	9.91985	9.82489	10.17511	10.08015	10.25526	4.22668	4.64780	15
46	9.74493	9.91976	9.82517	10.17483	10.08024	10.25507	4.22709	4.64757	14
47	9.74512	9.91968	9.82544	10.17456	10.08032	10.25488	4.22751	4.64733	13
48	9.74531	9.91959	9.82571	10.17429	10.08041	10.25469	4.22793	4.64709	12
49	9.74549	9.91951	9.82599	10.17401	10.08049	10.25451	4.22834	4.64686	11
50	9.74568	9.91942	9.82626	10.17374	10.08058	10.25432	4.22876	4.64662	10
51	9.74587	9.91934	9.82653	10.17347	10.08066	10.25413	4.22917	4.64638	9
52	9.74606	9.91925	9.82681	10.17319	10.08075	10.25394	4.22959	4.64615	8
53	9.74625	9.91917	9.82708	10.17292	10.08083	10.25375	4.23000	4.64591	7
54	9.74644	9.91908	9.82735	10.17265	10.08092	10.25356	4.23042	4.64567	6
55	9.74662	9.91900	9.82762	10.17238	10.08100	10.25338	4.23083	4.64544	5
56	9.74681	9.91891	9.82790	10.17210	10.08109	10.25319	4.23125	4.64520	4
57	9.74700	9.91883	9.82817	10.17183	10.08117	10.25300	4.23166	4.64496	3
58	9.74719	9.91874	9.82844	10.17156	10.08126	10.25281	4.23207	4.64472	2
59	9.74737	9.91866	9.82871	10.17129	10.08134	10.25263	4.23249	4.64449	1
60	9.74756	9.91857	9.82899	10.17101	10.08143	10.25244	4.23290	4.64425	0
	Co-sine	Sine	Co-tan.	Tangent	Co-sec.	Secant		V. Sine	M

(56 Deg.)

M	Sine	Co-sine	Tang.	Co-tang.	Secant	Co-sec.	V. Sine		
0	9.74756	9.91857	9.82899	10.17101	10.08143	10.25244	4.23390	4.64425	60
1	9.74775	9.91849	9.82926	10.17074	10.08151	10.25225	4.23331	4.64401	59
2	9.74794	9.91840	9.82953	10.17047	10.08160	10.25206	4.23373	4.64377	58
3	9.74812	9.91832	9.82980	10.17020	10.08168	10.25188	4.23414	4.64354	57
4	9.74831	9.91823	9.83008	10.16992	10.08177	10.25169	4.23455	4.64330	56
5	9.74850	9.91815	9.83035	10.16965	10.08185	10.25150	4.23496	4.64306	55
6	9.74868	9.91806	9.83062	10.16938	10.08194	10.25132	4.23538	4.64282	54
7	9.74887	9.91798	9.83089	10.16911	10.08202	10.25113	4.23579	4.64258	53
8	9.74906	9.91789	9.83117	10.16883	10.08211	10.25094	4.23620	4.64235	52
9	9.74924	9.91781	9.83144	10.16856	10.08219	10.25076	4.23661	4.64211	51
10	9.74943	9.91772	9.83171	10.16829	10.08228	10.25057	4.23702	4.64187	50
11	9.74961	9.91763	9.83198	10.16802	10.08237	10.25039	4.23743	4.64163	49
12	9.74980	9.91755	9.83225	10.16775	10.08245	10.25020	4.23784	4.64139	48
13	9.74999	9.91746	9.43252	10.16748	10.08254	10.25001	4.23825	4.64115	47
14	9.75017	9.91738	9.83280	10.16720	10.08262	10.24983	4.23866	4.64091	46
15	9.75036	9.91729	9.83307	10.16693	10.08271	10.24964	4.23907	4.64068	45
16	9.75054	9.91720	9.83334	10.16666	10.08280	10.24946	4.23948	4.64044	44
17	9.75073	9.91712	9.83361	10.16639	10.08288	10.24927	4.23989	4.64020	43
18	9.75091	9.91703	9.83388	10.16612	10.08297	10.24909	4.24030	4.63996	42
19	9.75110	9.91695	9.83415	10.16595	10.08305	10.24890	4.24071	4.63972	41
20	9.75128	9.91686	9.83442	10.16558	10.08314	10.24872	4.24112	4.63948	40
21	9.75147	9.91677	9.83470	10.16530	10.08323	10.24853	4.24153	4.63924	39
22	9.75165	9.91669	9.83407	10.16503	10.08331	10.24835	4.24194	4.63900	38
23	9.75184	9.91660	9.83524	10.16476	10.08340	10.24816	4.24235	4.63876	37
24	9.75202	9.91651	9.83551	10.16449	10.08349	10.24798	4.24276	4.63852	36
25	9.75221	9.91643	9.83578	10.16422	10.08357	10.24779	4.24316	4.63828	35
26	9.75239	9.91634	9.83603	10.16395	10.08366	10.24761	4.24357	4.63804	34
27	9.75258	9.91625	9.83632	10.16368	10.08375	10.24742	4.24398	4.63780	33
28	9.75276	9.91617	9.83659	10.16341	10.08383	10.24724	4.24439	4.63756	32
29	9.75294	9.91608	9.83686	10.16314	10.08392	10.24706	4.24479	4.63732	31
30	9.75313	9.91599	9.83713	10.16287	10.08401	10.24687	4.24520	4.63708	30
31	9.75331	9.91591	9.83740	10.16260	10.08409	10.24669	4.24561	4.63684	29
32	9.75350	9.91582	9.83768	10.16232	10.08418	10.24650	4.24601	4.63660	28
33	9.75368	9.91573	9.83795	10.16205	10.08427	10.24632	4.24642	4.63636	27
34	9.75386	9.91565	9.83822	10.16178	10.08435	10.24614	4.24683	4.63612	26
35	9.75405	9.91556	9.83849	10.16151	10.08444	10.24595	4.24723	4.63588	25
36	9.75423	9.91547	9.83876	10.16124	10.08453	10.24577	4.24764	4.63564	24
37	9.75441	9.91538	9.83903	10.16097	10.08462	10.24559	4.24804	4.63540	23
38	9.75459	9.91530	9.83930	10.16070	10.08470	10.24541	4.24845	4.63516	22
39	9.75478	9.91521	9.83957	10.16043	10.08479	10.24522	4.24885	4.63492	21
40	9.75496	9.91512	9.83984	10.16016	10.08488	10.24504	4.24926	4.63468	20
41	9.75514	9.91504	9.84011	10.15989	10.08496	10.24486	4.24966	4.63444	19
42	9.75533	9.91495	9.84038	10.15962	10.08505	10.24467	4.25007	4.63420	18
43	9.75551	9.91486	9.84065	10.15935	10.08514	10.24449	4.25047	4.63395	17
44	9.75569	9.91477	9.84092	10.15908	10.08523	10.24431	4.25088	4.63371	16
45	9.75587	9.91469	9.84119	10.15881	10.08531	10.24413	4.25128	4.63347	15
46	9.75605	9.91460	9.84146	10.15854	10.08540	10.24395	4.25168	4.63323	14
47	9.75624	9.91451	9.84173	10.15827	10.08549	10.24376	4.25209	4.63299	13
48	9.75642	9.91442	9.84200	10.15800	10.08558	10.24358	4.25249	4.63275	12
49	9.75660	9.91433	9.84227	10.15773	10.08567	10.24340	4.25289	4.63251	11
50	9.75678	9.91425	9.84254	10.15746	10.08575	10.24322	4.25330	4.63226	10
51	9.75696	9.91416	9.84280	10.15720	10.08584	10.24304	4.25370	4.63202	9
52	9.75714	9.91407	9.84307	10.15693	10.08593	10.24286	4.25410	4.63178	8
53	9.75733	9.91398	9.84334	10.15666	10.08602	10.24267	4.25450	4.63154	7
54	9.75751	9.91389	9.84361	10.15639	10.08611	10.24249	4.25491	4.63130	6
55	9.75769	9.91381	9.84388	10.15612	10.08619	10.24231	4.25531	4.63105	5
56	9.75787	9.91372	9.84415	10.15585	10.08628	10.24213	4.25571	4.63081	4
57	9.75805	9.91363	9.84442	10.15558	10.08637	10.24195	4.25611	4.63057	3
58	9.75823	9.91354	9.84469	10.15531	10.08646	10.24177	4.25651	4.63033	2
59	9.75841	9.91345	9.84496	10.15504	10.08655	10.24159	4.25691	4.63008	1
60	9.75859	9.91336	9.84523	10.15477	10.08664	10.24141	4.25731	4.62984	0
	Co-sine	Sine	Co-tan	Tangent	Co-sec.	Secant		V. Sine	M.

M	Sine	Co-sine	Tang.	Co-tang.	Secant	Co-sec.	V Sine		
0	9.75859	9.91336	9.84523	10.15477	10.08664	10.24141	4.25731	4.62984	60
1	9.75877	9.91328	9.84550	10.15450	10.08672	10.24123	4.25771	4.62960	59
2	9.75895	9.91319	9.84576	10.15424	10.08681	10.24105	4.25811	4.62936	58
3	9.75913	9.91310	9.84603	10.15397	10.08690	10.24087	4.25851	4.62911	57
4	9.75931	9.91301	9.84630	10.15370	10.08699	10.24069	4.25891	4.62887	56
5	9.75949	9.91292	9.84657	10.15343	10.08708	10.24051	4.25931	4.62863	55
6	9.75967	9.91283	9.84684	10.15316	10.08717	10.24033	4.25971	4.62838	54
7	9.75985	9.91274	9.84711	10.15289	10.08726	10.24015	4.26011	4.62814	53
8	9.76003	9.91266	9.84738	10.15262	10.08734	10.23997	4.26051	4.62790	52
9	9.76021	9.91257	9.84764	10.15236	10.0874	10.23979	4.26091	4.62765	51
10	9.76039	9.91248	9.84791	10.15209	10.08752	10.23961	4.26131	4.62741	50
11	9.76057	9.91239	9.84818	10.15182	10.08761	10.23943	4.26171	4.62717	49
12	9.76075	9.91230	9.84845	10.15155	10.08770	10.23925	4.26211	4.62692	48
13	9.76093	6.91221	9.84872	10.15128	10.08779	10.23907	4.26251	4.62668	47
14	9.76111	9.91212	9.84899	10.15101	10.08788	10.23889	4.26290	4.62644	46
15	9.76129	9.91203	9.84925	10.15075	10.08797	10.23871	4.26330	4.62619	45
16	9.76146	9.91194	9.84952	10.15048	10.08806	10.23854	4.26370	4.62595	44
17	9.76164	9.91185	9.84979	10.15021	10.08815	10.23836	4.26410	4.62570	43
18	9.76182	9.91176	9.85006	10.14994	10.08824	10.23818	4.26449	4.62546	42
19	9.76200	9.91167	9.85033	10.14967	10.08833	10.23800	4.26489	4.62521	41
20	9.76218	9.91158	9.85059	10.14941	10.08842	10.23782	4.26529	4.62497	40
21	9.76236	9.91149	9.85086	10.14914	10.08851	10.23764	4.26568	4.62473	39
22	9.76253	9.91141	9.85113	10.14887	10.08859	10.23747	4.26608	4.62448	38
23	9.76271	9.91132	9.85140	10.14860	10.08868	10.23729	4.26648	4.62424	37
24	9.76289	9.91123	9.85166	10.14834	10.08877	10.23711	4.26687	4.62399	36
25	9.76307	9.91114	9.85193	10.14807	10.08886	10.23693	4.26727	4.62375	35
26	9.76324	9.91105	9.85220	10.14780	10.08895	10.23676	4.26767	4.62350	34
27	9.76342	9.91096	9.85247	10.14753	10.08904	10.23658	4.26806	4.62326	33
28	9.76360	9.91087	9.85273	10.14727	10.08913	10.23640	4.26845	4.62301	32
29	9.76378	9.91078	9.85300	10.14700	10.08922	10.23622	4.26885	4.62277	31
30	9.76395	9.91069	9.85327	10.14673	10.08931	10.23605	4.26924	4.62252	30
31	9.76413	9.91060	9.85354	10.14646	10.08940	10.23587	4.26964	4.62221	29
32	9.76431	9.91051	9.85380	10.14620	10.08949	10.23569	4.27003	4.62203	28
33	9.76448	9.91042	9.85407	10.14593	10.08958	10.23552	4.27042	4.62179	27
34	9.76466	9.91033	9.85434	10.14566	10.08967	10.23534	4.27081	4.62154	26
35	9.76484	9.91023	9.85460	10.14540	10.08977	10.23516	4.27121	4.62129	25
36	9.76501	9.91014	9.85487	10.14513	10.08986	10.23499	4.27161	4.62105	24
37	9.76519	9.91005	9.85514	10.14486	10.08995	10.23481	4.27200	4.02080	23
38	9.76537	9.90996	9.85540	10.14460	10.09004	10.23463	4.27239	4.62056	22
39	9.76554	9.90987	9.85567	10.14433	10.09013	10.23446	4.27279	4.62031	21
40	9.76572	9.90978	9.85594	10.14406	10.09022	10.23428	4.27317	4.62006	20
41	9.76590	9.90969	9.85620	10.14380	10.09031	10.23410	4.27356	4.61982	19
42	9.76607	9.90960	9.85647	10.14353	10.09040	10.23393	4.27396	4.61957	18
43	9.76625	9.90951	9.85674	10.14326	10.09049	10.23375	4.27435	4.61933	17
44	9.76642	9.90942	9.85700	10.14300	10.09058	10.23358	4.27474	4.61908	16
45	9.76660	9.90933	9.85727	10.14273	10.09067	10.23340	4.27513	4.61883	15
46	9.76677	9.90924	9.85754	10.14246	10.09076	10.23323	4.27552	4.61859	14
47	9.76695	9.90915	9.85780	10.14220	10.09085	10.23305	4.27592	4.61834	13
48	9.76712	9.90905	9.85807	10.14193	10.09095	10.23288	4.27631	4.61809	12
49	9.76730	9.90896	9.85834	10.14166	10.09104	10.23270	4.27670	4.61785	11
50	9.76747	9.90887	9.85860	10.14140	10.09113	10.23253	4.27719	4.61760	10
51	9.76765	9.90878	9.85887	10.14113	10.09122	10.23235	4.27748	4.61735	9
52	9.76782	9.90869	9.85913	10.14087	10.09131	10.23218	4.27796	4.61710	8
53	9.76800	9.90860	9.85940	10.14060	10.09140	10.23200	4.27867	4.61686	7
54	9.79817	9.90851	9.85967	10.14033	10.09149	10.23183	4.27852	4.61661	6
55	9.76835	9.90842	9.85993	10.14007	10.09158	10.23165	4.27904	4.61636	5
56	9.76852	9.90832	9.86020	10.13980	10.09168	10.23148	4.27943	4.61611	4
57	9.76870	9.90823	9.86046	10.13954	10.09177	10.23130	4.27982	4.61587	3
58	9.76887	9.90814	9.86073	10.13927	10.09186	10.23113	4.28021	4.61562	2
59	9.76904	9.90805	9.86100	10.13900	10.09195	10.23096	4.28069	4.61537	1
60	9.76922	9.90796	9.86126	10.13874	10.09204	10.23078	4.28099	4.61512	0
	Co-sine	Sine.	Co-tan.	Tangent	Co-sec.	Secant		V. Sine	M

(54 Deg.)

M	Sine	Co-sine	Tang.	Co-tang.	Secant	Co-sec.	V. Sine		
0	9.76922	9.90796	9.86126	10.13874	10.09204	10.23078	4.28099	4.61512	60
1	9.76939	9.90787	9.86153	10.13847	10.09213	10.23061	4.28138	4.61488	59
2	9.76957	9.90777	9.86179	10.13821	10.09223	10.23043	4.28177	4.61463	58
3	9.76974	9.90768	9.86206	10.13794	10.09232	10.23026	4.28216	4.61438	57
4	9.76991	9.90759	9.86232	10.13768	10.09241	10.23009	4.28255	4.61413	56
5	9.77009	9.90750	9.86259	10.13741	10.09250	10.22991	4.28294	4.61388	55
6	9.77026	9.90741	9.86285	10.13715	10.09259	10.22974	4.28332	4.61363	54
7	9.77043	9.90731	9.86312	10.13688	10.09269	10.22957	4.28371	4.61339	53
8	9.77061	9.90722	9.86338	10.13662	10.09278	10.22939	4.28410	4.61314	52
9	9.77078	9.90713	9.86365	10.13635	10.09287	10.22922	4.28449	4.61289	51
10	9.77095	9.90704	9.86392	10.13608	10.09296	10.22905	4.28487	4.61264	50
11	9.77112	9.90694	9.86418	10.13582	10.09306	10.22888	4.28526	4.61239	49
12	9.77130	9.90685	9.86445	10.13555	10.09315	10.22870	4.28565	4.61214	48
13	9.77147	9.90676	9.86471	10.13529	10.09324	10.22853	4.28603	4.61189	47
14	9.77164	9.90667	9.86498	10.13502	10.09333	10.22836	4.28642	4.61164	46
15	9.77181	9.90657	9.86524	10.13476	10.09343	10.22819	4.28681	4.61139	45
16	9.77199	9.90648	9.86551	10.13449	10.09352	10.22801	4.28719	4.61115	44
17	9.77216	9.90639	9.86577	10.13423	10.09361	10.22784	4.28758	4.61090	43
18	9.77233	9.90630	9.86603	10.13397	10.09370	10.22767	4.28796	4.61065	42
19	9.77250	9.90620	9.86630	10.13370	10.09380	10.22750	4.28835	4.61040	41
20	9.77268	9.90611	9.86656	10.13344	10.09389	10.22732	4.28873	4.61015	40
21	9.77285	9.90602	9.86683	10.13317	10.09398	10.22715	4.28912	4.60990	39
22	9.77302	9.90592	9.86709	10.13291	10.09408	10.22698	4.28950	4.60965	38
23	9.77319	9.90583	9.86736	10.13264	10.09417	10.22681	4.28989	4.60940	37
24	9.77336	9.90574	9.86762	10.13238	10.09426	10.22664	4.29027	4.60915	36
25	9.77353	9.90565	9.86789	10.13211	10.09435	10.22647	4.29066	4.60890	35
26	9.77370	9.90555	9.86815	10.13185	10.09445	10.22630	4.29104	4.60865	34
27	9.77387	9.90546	9.86842	10.13158	10.09454	10.22613	4.29142	4.60840	33
28	9.77405	9.90537	9.86868	10.13132	10.09473	10.22595	4.29181	4.60815	32
29	9.77422	9.90527	9.86894	10.13106	10.09463	10.22578	4.29219	4.60790	31
30	9.77439	9.90518	9.86921	10.13079	10.09482	10.22561	4.29257	4.60764	30
31	9.77456	9.90509	9.86947	10.13053	10.09491	10.22544	4.29296	4.60739	29
32	9.77473	9.90499	9.86974	10.13026	10.09501	10.22527	4.29334	4.60713	28
33	9.77490	9.90490	9.87000	10.13000	10.09510	10.22510	4.29372	4.60689	27
34	9.77507	9.90480	9.87027	10.12973	10.09520	10.22493	4.29410	4.60664	26
35	9.77524	9.90471	9.87053	10.12947	10.09529	10.22476	4.29449	4.60639	25
36	9.77541	9.90462	9.87079	10.12921	10.09538	10.22459	4.29487	4.60614	24
37	9.77558	9.90453	9.87106	10.12894	10.09548	10.22442	4.29525	4.60589	23
38	9.77575	9.90443	9.87132	10.12868	10.09557	10.22425	4.29563	4.60564	22
39	9.77592	9.90434	9.87158	10.12842	10.09566	10.22408	4.29601	4.60539	21
40	9.77609	9.90424	9.87185	10.12815	10.09576	10.22391	4.29639	4.60513	20
41	9.77626	9.90415	9.87211	10.12789	10.09585	10.22374	4.29678	4.60488	19
42	9.77643	9.90405	9.87238	10.12762	10.09595	10.22357	4.29716	4.60463	18
43	9.77660	9.90396	9.87264	10.12736	10.09604	10.22340	4.29754	4.60438	17
44	9.77677	9.90386	9.87290	10.12710	10.09614	10.22323	4.29792	4.60414	16
45	9.77694	9.90377	9.87317	10.12683	10.09623	10.22306	4.29830	4.60388	15
46	9.77711	9.90368	9.87343	10.12657	10.09632	10.22289	4.29868	4.60362	14
47	9.77727	9.90358	9.87368	10.12631	10.09642	10.22272	4.29906	4.60337	13
48	9.77744	9.90349	9.87396	10.12604	10.09651	10.22256	4.29944	4.60312	12
49	9.77761	9.90339	9.87422	10.12578	10.09661	10.22239	4.29982	4.60287	11
50	9.77778	9.90330	9.87448	10.12552	10.09670	10.22222	4.30020	4.60261	10
51	9.77795	9.90320	9.87475	10.12525	10.09680	10.22205	4.30058	4.60236	9
52	9.77812	9.90311	9.87501	10.12499	10.09689	10.22188	4.30096	4.60211	8
53	9.77829	9.90301	9.87527	10.12473	10.09699	10.22171	4.30134	4.60186	7
54	9.77846	9.90292	9.87554	10.12440	10.09708	10.22154	4.30171	4.60160	6
55	9.77862	9.90282	9.87580	10.12420	10.09718	10.22138	4.30209	4.60135	5
56	9.77879	9.90273	9.87606	10.12394	10.09727	10.22121	4.30247	4.60110	4
57	9.77896	9.90263	9.87633	10.12367	10.09737	10.22104	4.30285	4.60084	3
58	9.77913	9.90254	9.87659	10.12341	10.09746	10.22087	4.30323	4.60059	2
59	9.77930	9.90244	9.87685	10.12315	10.09756	10.22070	4.30361	4.60034	1
60	9.77946	9.90235	9.87711	10.12289	10.09765	10.22054	4.30398	4.60008	0
	Co-sine	Sine	Co-tang.	Tangent	Co-sec.	Secant		V. Sine	M

M	Sine	Co-sine	Tang.	Co-tang.	Secant	Co-sec.	V. Sine		
0	9.77946	9.90235	9.87711	10.12289	10.09765	10.22054	4.30398	4.60008	60
1	9.77963	9.90225	9.87738	10.12262	10.09775	10.22037	4.30436	4.59983	59
2	9.77980	9.90216	9.87764	10.12236	10.09784	10.22020	4.30474	4.59958	58
3	9.77997	9.90206	9.87790	10.12210	10.09794	10.22003	4.30511	4.59932	57
4	9.78013	9.90197	9.87817	10.12183	10.09803	10.21987	4.30549	4.59907	56
5	9.78030	9.90187	9.87843	10.12157	10.09813	10.21970	4.30587	4.59882	55
6	9.78047	9.90178	9.87869	10.12131	10.09822	10.21953	4.30624	4.59856	54
7	9.78063	9.90168	9.87895	10.12105	10.09832	10.21937	4.30662	4.59831	53
8	9.78080	9.90159	9.87922	10.12078	10.09841	10.21920	4.30700	4.59805	52
9	9.78097	9.90149	9.87948	10.12052	10.09851	10.21903	4.30737	4.59780	51
10	9.78113	9.90139	9.87974	10.12026	10.09861	10.21887	4.30775	4.59755	50
11	9.78130	9.90130	9.88000	10.12000	10.09870	10.218[illegible]	4.30812	4.59729	49
12	9.78147	9.90120	9.88027	10.11973	10.09880	10.21853	4.30850	4.59704	48
13	9.78163	9.90111	9.88053	10.11947	10.09889	10.21837	4.30888	4.59678	47
14	9.78180	9.90101	9.88079	10.11921	10.09899	10.21820	4.30925	4.59653	46
15	9.78197	9.90091	9.88105	10.11895	10.09909	10.21803	4.30963	4.59627	45
16	9.78213	9.90082	9.88131	10.11869	10.09918	10.21787	4.31000	4.59602	44
17	9.78230	9.90072	9.88158	10.11842	10.09928	10.21770	4.31036	4.59576	43
18	9.78246	9.90063	9.88184	10.11816	10.09937	10.21754	4.31075	4.59551	42
19	9.78263	9.90053	9.88210	10.11790	10.09947	10.21737	4.31112	4.59525	41
20	9.78280	9.90043	9.88236	10.11764	10.09957	10.21720	4.31150	4.59500	40
21	9.78296	9.90034	9.88262	10.11738	10.09966	10.21704	4.31187	4.59474	39
22	9.78313	9.90024	9.88289	10.11711	10.09976	10.21687	4.31225	4.59449	38
23	9.78329	9.90014	9.88315	10.11685	10.09986	10.21671	4.31262	4.59423	37
24	9.78346	9.90005	9.88341	10.11659	10.09995	10.21654	4.31299	4.59398	36
25	9.78362	9.89995	9.88367	10.11633	10.10004	10.21638	4.31337	4.59372	35
26	9.78379	9.89985	9.88393	10.11607	10.10013	10.21621	4.31374	4.59347	34
27	9.78395	9.89976	9.88420	10.11580	10.10024	10.21605	4.31411	4.59321	33
28	9.78412	9.89966	9.88446	10.11554	10.10034	10.21588	4.31448	4.59295	32
29	9.78428	9.89956	9.88472	10.11528	10.10044	10.21572	4.31486	4.59270	31
30	9.78445	9.89947	9.88498	10.11502	10.10053	10.21555	4.31523	4.59244	30
31	9.78461	9.89937	9.88524	10.11476	10.10063	10.21539	4.31560	4.59219	29
32	9.78478	9.89927	9.88550	10.11450	10.10073	10.21522	4.31597	4.59193	28
33	9.78494	9.89918	9.88576	10.11424	10.10082	10.21506	4.31634	4.59167	27
34	9.78510	9.89908	9.88603	10.11397	10.10092	10.21490	4.31672	4.59142	26
35	9.78527	9.89898	9.88629	10.11371	10.10102	10.21473	4.31709	4.59116	25
36	9.78543	9.89888	9.88655	10.11345	10.10112	10.21457	4.31746	4.59090	24
37	9.78560	9.89879	9.88681	10.11319	10.10121	10.21440	4.31783	4.59065	23
38	9.78576	9.89869	9.88707	10.11293	10.10131	10.21424	4.31820	4.59039	22
39	9.78592	9.89859	9.88733	10.11267	10.10142	10.21408	4.31857	4.59013	21
40	9.78609	9.89849	9.88759	10.11241	10.10151	10.21391	4.31894	4.58988	20
41	9.78625	9.89840	9.88786	10.11214	10.10160	10.21375	4.31931	4.58962	19
42	9.78642	9.89830	9.88812	10.11188	10.10170	10.21358	4.31968	4.58936	18
43	9.78658	9.89820	9.88838	10.11162	10.10180	10.21342	4.32005	4.58910	17
44	9.78674	9.89810	9.88864	10.11136	10.10190	10.21326	4.32042	4.58885	16
45	9.78691	9.89801	9.88890	10.11110	10.10199	10.21309	4.32079	4.38859	15
46	9.78707	9.89791	9.88916	10.11084	10.10209	10.21293	4.32116	4.58833	14
47	9.78723	9.89781	9.88942	10.11058	10.10219	10.21277	4.32153	4.58807	13
48	9.78739	9.89771	9.88968	10,11032	10.10229	10.21261	4.32190	4.58782	12
49	9.78756	9.89761	9.88994	10.11006	10.10239	10.21244	4.32227	4.58756	11
50	9.78772	9.89752	9.89020	10.10980	10.10248	10.21228	4.32264	4.58730	10
51	9.78788	9.89742	9.89046	10.10954	10.10258	10.21212	4.32300	4.58704	9
52	9.78805	9.89732	9.89073	10.10927	10.10268	10.21195	4.32338	4.58678	8
53	9.78821	9.89722	9.89099	10.10901	10.10278	10.21179	4.32374	4.58652	7
54	9.78837	9.89712	9.89125	10.10875	10.10288	10.21103	4.32411	4.58627	6
55	9.78853	9.89702	9.89151	10.10849	10.10298	10.21147	4.32448	4.58601	5
56	9.78869	9.89693	9.89177	10.10823	10.10307	10.21131	4.32484	4.58575	4
57	9.78886	9.89683	9.89203	10.10797	10.10317	10.21114	4.32521	4.88549	3
58	9.78902	9.89673	9.89229	10.10771	10.10327	10.21098	4.32558	4.58523	2
59	9.78918	9.89663	9.89255	10.10745	10.10337	10.21082	4.32595	4.58497	1
60	9.78934	9.89653	9.89281	10.10719	10.10347	10.21066	4.32631	4.58471	0
	Co-sine	Sine.	Co-tan.	Tangent	Co-sec.	Secant		V. Sine	M

(52 Deg.)

M	Sine	Co-fine	Tang.	Co-tang.	Secant	Co-fec.	V. Sine		
0	9.78934	9.89653	9.89281	10.10719	10.10347	10.21066	4.32631	4.58471	60
1	9.78950	9.89643	9.89307	10.10693	10.10357	10.21050	4.32668	4.58445	59
2	9.78967	9.89633	9.89333	10.10667	10.10367	10.21033	4.32705	4.58420	58
3	9.78983	9.89624	9.89359	10.10641	10.10376	10.21017	4.32741	4.58394	57
4	9.78999	9.89614	9.89385	10.10615	10.10386	10.21001	4.32778	4.58368	56
5	9.79015	9.89604	9.89411	10.10589	10.10396	10.20985	4.32815	4.58342	55
6	9.79031	9.89594	9.89437	10.10563	10.10406	10.20969	4.32851	4.58316	54
7	9.79047	9.89584	9.89463	10.10537	10.10416	10.20953	4.32888	4.58290	53
8	9.79063	9.89574	9.89489	10.10511	10.10426	10.20937	4.32924	4.58264	52
9	9.79079	9.89564	9.89515	10.10485	10.10436	10.20921	4.32961	4.58238	51
10	9.79095	9.89554	9.89541	10.10459	10.10446	10.20905	4.32997	4.58212	50
11	9.79111	9.89544	9.89567	10.10433	10.10456	10.20889	4.33034	4.58186	49
12	9.79128	9.89534	9.89593	10.10407	10.10466	10.20872	4.33070	4.58160	48
13	9.79144	9.89524	9.89619	10.10381	10.10476	10.20856	4.33107	4.58134	47
14	9.79160	9.89514	9.89645	10.10355	10.10486	10.20840	4.33143	4.58108	46
15	9.79176	9.89504	9.89671	10.10329	10.10496	10.20824	4.33180	4.58082	45
16	9.79192	9.89495	9.89697	10.10303	10.10505	10.20808	4.33216	4.58056	44
17	9.79208	9.89485	9.89723	10.10277	10.10515	10.20792	4.33253	4.58030	43
18	9.79224	9.89475	9.89749	10.10251	10.10525	10.20776	4.33289	4.58004	42
19	9.79240	9.89465	9.89775	10.10225	10.10535	10.20760	4.33325	4.57978	41
20	9.79256	9.89455	9.89801	10.10199	10.10545	10.20744	4.33362	4.57951	40
21	9.79272	9.89445	9.89827	10.10173	10.10555	10.20728	4.33398	4.57925	39
22	9.79288	9.89435	9.89853	10.10147	10.10565	10.20712	4.33434	4.57899	38
23	9.79304	9.89425	9.89879	10.10121	10.10575	10.20696	4.33471	4.57873	37
24	9.79319	9.89415	9.89905	10.10095	10.10585	10.20681	4.33507	4.57847	36
25	9.79335	9.89405	9.89931	10.10069	10.10595	10.20665	4.33543	4.57821	35
26	9.79351	9.89395	9.89957	10.10043	10.10605	10.20649	4.33579	4.57795	34
27	9.79367	9.89385	9.89983	10.10017	10.10615	10.20633	4.33616	4.57769	33
28	9.79383	9.89375	9.90009	10.09991	10.10625	10.20617	4.33652	4.57742	32
29	9.79399	9.89364	9.90035	10.09965	10.10636	10.20601	4.33688	4.57716	31
30	9.79415	9.89354	9.90061	10.09939	10.10646	10.20585	4.33724	4.57690	30
31	9.79431	9.89344	9.90086	10.09914	10.10656	10.20569	4.33760	4.57664	29
32	9.79447	9.89334	9.90112	10.09888	10.10666	10.20553	4.33797	4.57638	28
33	9.79463	9.89324	9.90138	10.09862	10.10676	10.20537	4.33833	4.57611	27
34	9.79478	9.89314	9.90164	10.09836	10.10686	10.20522	4.33869	4.57585	26
35	9.79494	9.89304	9.90190	10.09810	10.10696	10.20506	4.33905	4.57559	25
36	9.79510	9.89294	9.90216	10.09784	10.10706	10.20490	4.33941	4.57533	24
37	9.79526	9.89284	9.90242	10.09758	10.10716	10.20474	4.33977	4.57506	23
38	9.79542	9.89274	9.90268	10.09732	10.10726	10.20458	4.34013	4.57484	22
39	9.79558	9.89264	9.90294	10.09706	10.10736	10.20442	4.34049	4.57454	21
40	9.79573	9.89254	9.90320	10.09680	10.10746	10.20427	4.34085	4.57428	20
41	9.79589	9.89244	9.90346	10.09654	10.10756	10.20411	4.34121	4.57401	19
42	9.79605	9.89233	9.90371	10.09629	10.10767	10.20395	4.34157	4.57375	18
43	9.79621	9.89223	9.90397	10.09603	10.10777	10.20379	4.34193	4.57349	17
44	9.79636	9.89213	9.90423	10.09577	10.10787	10.20364	4.34229	4.57322	16
45	9.79652	9.89203	9.90449	10.09551	10.10797	10.20348	4.34265	4.57296	15
46	9.79668	9.89193	9.90475	10.09525	10.10807	10.20332	4.34301	4.57270	14
47	9.79684	9.89183	9.90501	10.09499	10.10817	10.20316	4.34377	4.57243	13
48	9.79699	9.89173	9.90527	10.09473	10.10827	10.20301	4.34373	4.57217	12
49	9.79715	9.89162	9.90553	10.09447	10.10838	10.20285	4.34409	4.57191	11
50	9.79731	9.89152	9.90578	10.09422	10.10848	10.20269	4.34444	4.57164	10
51	9.79746	9.89142	9.90604	10.09396	10.10858	10.20254	4.34480	4.57138	9
52	9.79762	9.89132	9.90630	10.09370	10.10868	10.20238	4.34516	4.57111	8
53	9.79778	9.89122	9.90656	10.09344	10.10878	10.20222	4.34552	4.57085	7
54	9.79793	9.89112	9.90682	10.09318	10.10888	10.20207	4.34588	4.57059	6
55	9.79809	9.89101	9.90708	10.09292	10.10899	10.20191	4.34623	4.57032	5
56	9.79825	9.89091	9.90734	10.09266	10.10909	10.20175	4.34659	4.57006	4
57	9.79840	9.89081	9.90759	10.09241	10.10919	10.20160	4.34695	4.56979	3
58	9.79856	9.89071	9.90785	10.09215	10.10929	10.20144	4.34731	4.56953	2
59	9.79872	9.89060	9.90811	10.09189	10.10940	10.20128	4.34766	4.56926	1
60	9.79887	9.89050	9.90837	10.09163	10.10950	10.20113	4.34802	4.56900	0
	Co-fine	Sine	Co-tan.	Tang.	Co-fec.	Secant		V. Sine	M

M	Sine	Co-sine	Tan.	Co-tan.	Secant.	Co-sec.	V. Sine		
0	9.79887	9.89050	9.90837	10.09163	10.10950	10.20013	4.34802	4.56900	60
1	9.79903	9.89040	9.90863	10.09137	10.10960	10.20097	4.34838	4.56873	59
2	9.79918	9.89030	9.90889	10.09111	10.10970	10.20082	4.34873	4.56847	58
3	9.79934	9.89020	9.90914	10.09085	10.10980	10.20066	4.34909	4.56820	57
4	9.79950	9.89009	9.90940	10.09060	10.10991	10.20050	4.34945	4.56794	56
5	9.79965	9.88999	9.90966	10.09034	10.11001	10.20035	4.34980	4.56767	55
6	9.79981	9.88989	9.90992	10.09009	10.11011	10.20019	4.35016	4.56741	54
7	9.79996	9.88978	9.91018	10.08982	10.11022	10.20004	4.35051	4.56714	53
8	9.80012	9.88968	9.91043	10.08957	10.11032	10.19988	4.35087	4.56688	52
9	9.80027	9.88958	9.91069	10.08931	10.11042	10.19973	4.35122	4.56661	51
10	9.80043	9.88948	9.91095	10.08905	10.11052	10.19957	4.35158	4.56635	50
11	9.80058	9.88937	9.91121	10.08879	10.11063	10.10942	4.35193	4.56608	49
12	9.80074	9.88927	9.91 47	10.08853	10.11073	10.19926	4.35229	4.56581	48
13	9.80089	9.88917	9.91172	10.08828	10.11083	10.19911	4.35264	4.56555	47
14	9.80105	9.88906	9.91198	10.08802	10.11094	10.19895	4.35300	4.56528	46
15	9.80120	9.88896	9.91224	10.08776	10.11104	10.19880	4.35335	4.56596	45
16	9.80136	9.88886	9.91250	10.08750	10.11114	10.19864	4.35371	4.56475	44
17	9.80151	9.88875	9.91276	10.08724	10.11125	10.19849	4.35406	4.56448	43
18	9.80166	9.88865	9.91301	10.08699	10.11135	10.19834	4.35442	4.56422	42
19	9.80182	9.88855	9.91327	10.08673	10.11145	10.19818	4.35477	4.56395	41
20	9.80197	9.88844	9.91353	10.08647	10.11156	10.19803	4.35512	4.56367	40
21	9.80213	9.88834	9.91379	10.08621	10.11166	10.19787	4.35548	4.56341	39
22	9.80228	9.88824	9.91404	10.08596	10.11176	10.19772	4.35563	4.56315	38
23	9.80244	9.88813	9.91430	10.08570	10.11187	10.19756	4.35618	4.56288	37
24	9.80259	9.88803	9.91456	10.08544	10.11197	10.19741	4.35654	4.56261	36
25	9.80274	9.88793	9.91482	10.08518	10.11207	10.19726	4.35689	4.56235	35
26	9.80290	9.88782	9.91507	10.08493	10.11218	10.19710	4.35724	4.56208	34
27	9.80305	9.88772	9.91533	10.08467	10.11228	10.19695	4.35759	4.56181	33
28	9.80320	9.88761	9.91559	10.08441	10.11239	10.19680	4.35795	4.56154	32
29	9.80336	9.88751	9.91585	10.08415	10.11249	10.19664	4.35830	4.56128	31
30	9.80351	9.88741	9.91610	10.08390	10.11259	10.19649	4.35865	4.56101	30
31	9.80366	9.88730	9.91636	10.08364	10.11270	10.19634	4.35900	4.56074	29
32	9.80382	9.88720	9.91662	10.08338	10.11280	10.19618	4.35935	4.56047	28
33	9.80397	9.88709	9.91688	10.08312	10.11291	10.19603	4.35970	4.56020	27
34	9.80412	9.88699	9.91713	10.08287	10.11301	10.19588	4.36006	4.55964	26
35	9.80428	9.88688	9.91739	10.08261	10.11312	10.19572	4.36041	4.55967	25
36	9.80443	9.88678	9.91765	10.08235	10.11322	10.19557	4.36076	4.55946	24
37	9.80458	9.88668	9.91791	10.08209	10.11332	10.19542	4.36111	4.55913	23
38	9.80473	9.88657	9.91816	10.08184	10.11343	10.19527	4.36146	4.55886	22
39	9.80489	9.88647	9.91842	10.08158	10.11353	10.19511	4.36121	4.55859	21
40	9.80504	9.88636	9.91868	10.08132	10.11364	10.19496	4.36216	4.55832	20
41	9.80519	9.88626	9.91893	10.08107	10.11374	10.19481	4.36251	4.55806	19
42	9.80534	9.88615	9.91919	10.08081	10.11385	10.19466	4.36286	4.55779	18
43	9.80550	9.88605	9.91945	10.08055	10.11395	10.19450	4.36321	4.55752	17
44	9.80565	9.88594	9.91971	10.08029	10.11406	10.19435	4.36356	4.55725	16
45	9.80580	9.88584	9.91996	10.08004	10.11416	10.19420	4.36391	4.55698	15
46	9.80595	9.88573	9.92022	10.07978	10.11427	10.19405	4.36426	4.55671	14
47	9.80610	9.88563	9.92048	10.07952	10.11437	10.19390	4.36461	4.55644	13
48	9.80625	9.88552	9.92073	10.07927	10.11448	10.19375	4.36496	4.55617	12
49	9.80641	9.88542	9.92099	10.07901	10.11458	10.19359	4.36531	4.55590	11
50	9.80656	9.88531	9.92125	10.07875	10.11469	10.19344	4.36565	4.55563	10
51	9.80671	9.88521	9.92150	10.07850	10.11479	10.19329	4.36600	4.55536	9
52	9.80686	9.88510	9.92176	10.07824	10.11490	10.19314	4.36635	4.55509	8
53	9.80701	9.88499	9.92202	10.07798	10.11501	10.19299	4.36670	4.55482	7
54	9.80716	9.88489	9.92227	10.07773	10.11511	10.19284	4.36705	4.55455	6
55	9.80731	9.88478	9.92253	10.07747	10.11522	10.19269	4.36740	4.55428	5
56	9.80746	9.88468	9.92279	10.07721	10.11532	10.19254	4.36774	4.55401	4
57	9.80762	9.88457	9.92304	10.07696	10.11543	10.19238	4.36809	4.55374	3
58	9.80777	9.88447	9.92330	10.07670	10.11553	10.19223	4.36844	4.55347	2
59	9.80792	9.88436	9.92356	10.07644	10.11564	10.19208	4.36879	4.55320	1
60	9.80807	9.88425	9.92381	10.07619	10.11575	10.19193	4.36913	4.55293	0
	Co-sine	Sine	Co-tan.	Tangent	Co-sec.	Secant		V. Sine	M

M	Sine	Co-sine	Tan	Co-tan.	Secant.	Co-sec.	V. Sine		
0	80807	9.88425	9.92381	10.07619	10.11575	10.19193	4.36913	4.55293	60
1	9.80822	9.88415	9.92407	10.07593	10.11585	10.19178	4.36948	4.55266	59
2	9.80837	9.88404	9.92433	10.07567	10.11596	10.19163	4.36983	4.55238	58
3	9.80852	9.88394	9.92458	10.07542	10.11606	10.19148	4.37017	4.55211	57
4	9.80867	9.88383	9.92484	10.07516	10.11617	10.19133	4.37052	4.55184	56
5	9.80882	9.88372	9.92510	10.07490	10.11628	10.19118	4.37087	4.55157	55
6	9.80897	9.88362	9.92535	10.07465	10.11638	10.19103	4.37121	4.55130	54
7	9.80912	9.88351	9.92561	10.07439	10.11649	10.19088	4.37156	4.55103	53
8	9.80927	9.88340	9.92587	10.07413	10.11660	10.19073	4.37191	4.55076	52
9	9.80942	9.88330	9.92612	10.07388	10.11670	10.19058	4.37225	4.55048	51
10	9.80957	9.88319	9.92638	10.07362	10.11681	10.19043	4.37260	4.55021	50
11	9.80972	9.88308	9.92663	10.07337	10.11692	10.19028	4.37294	4.54994	49
12	9.80987	9.88298	9.92689	10.07311	10.11702	10.19013	4.37329	4.54967	48
13	9.81002	9.88287	9.92715	10.07285	10.11713	10.18998	4.37363	4.54940	47
14	9.81017	9.88276	9.92740	10.07260	10.11724	10.18983	4.37398	4.54912	46
15	9.81032	9.88266	9.92766	10.07234	10.11734	10.18968	4.37432	4.54885	45
16	9.81046	9.88255	9.92792	10.07208	10.11745	10.18954	4.37467	4.54858	44
17	9.81061	9.88244	9.92817	10.07183	10.11756	10.18939	4.37501	4.54831	43
18	9.81076	9.88234	9.92843	10.07157	10.11766	10.18924	4.37536	4.54803	42
19	9.81091	9.88223	9.92868	10.07132	10.11777	10.18909	4.37570	4.54776	41
20	9.81106	9.88212	9.92894	10.07106	10.11788	10.18894	4.37604	4.54749	40
21	9.81121	9.88201	9.92920	10.07080	10.11799	10.18879	4.37639	4.54721	39
22	9.81136	9.88191	9.92945	10.07055	10.11809	10.18864	4.37673	4.54694	38
23	9.81151	9.88180	9.92971	10.07029	10.11820	10.18849	4.37708	4.54667	37
24	9.81166	9.88169	9.92996	10.07004	10.11831	10.18834	4.37742	4.54639	36
25	9.81180	9.88158	9.93022	10.06978	10.11842	10.18820	4.37776	4.54612	35
26	9.81195	9.88148	9.93048	10.06952	10.11852	10.18805	4.37810	4.54585	34
27	9.81210	9.88137	9.93073	10.06927	10.11863	10.18790	4.37845	4.54557	33
28	9.81225	9.88126	9.93099	10.06901	10.11874	10.18775	4.37879	4.54530	32
29	9.81240	9.88115	9.93124	10.06876	10.11885	10.18760	4.37913	4.54503	31
30	9.81254	9.88105	9.93150	10.06850	10.11895	10.18746	4.37948	4.54475	30
31	9.81269	9.88094	9.93175	10.06825	10.11906	10.18731	4.37982	4.54448	29
32	9.81284	9.88083	9.93201	10.06799	10.11917	10.18716	4.38016	4.54420	28
33	9.81299	9.88072	9.9322	10.06773	10.11928	10.18701	4.38050	4.54393	27
34	9.81314	9.88061	9.93252	10.06748	10.11939	10.18686	4.38084	4.54366	26
35	9.81328	9.88051	9.93278	10.06722	10.11949	10.18672	4.38119	4.54338	25
36	9.81343	9.88040	9.93303	10.06697	10.11960	10.18657	4.38153	4.54311	24
37	9.81358	9.88029	9.93329	10.06671	10.11971	10.18642	4.38187	4.54283	23
38	9.81372	9.88018	9.93354	10.06646	10.11982	10.18628	4.38221	4.54256	22
39	9.81387	9.88007	9.93380	10.06620	10.11993	10.18613	4.38255	4.54228	21
40	9.81402	9.87996	9.93406	10.06594	10.12004	10.18598	4.38289	4.54201	20
41	9.81417	9.87985	9.93431	10.06569	10.12015	10.18583	4.38323	4.54173	19
42	9.81431	9.87975	9.93457	10.06543	10.12025	10.18569	4.38357	4.54146	18
43	9.81446	9.87964	9.93482	10.06518	10.12036	10.18554	4.38391	4.54118	17
44	9.81461	9.87953	9.93508	10.06492	10.12047	10.18539	4.38426	4.54091	16
45	9.81475	9.87942	9.93533	10.06467	10.12058	10.18525	4.38460	4.54063	15
46	9.81490	9.87931	9.93559	10.06441	10.12059	10.18510	4.38494	4.54035	14
47	9.81505	9.87920	9.93584	10.06416	10.12080	10.18495	4.38528	4.54008	13
48	9.81519	9.87909	9.93610	10.06390	10.12091	10.18481	4.38562	4.53980	12
49	9.81534	9.87898	9.93636	10.06364	10.12102	10.18466	4.38595	4.53953	11
50	9.81549	9.87887	9.93661	10.06339	10.12113	10.18451	4.38629	4.53925	10
51	9.81563	9.87877	9.93687	10.06313	10.12123	10.18437	4.38663	4.53897	9
52	9.81578	9.87866	9.93712	10.06288	10.12134	10.18422	4.38697	4.53870	8
53	9.81592	9.87855	9.93738	10.06262	10.12145	10.18408	4.38731	4.53842	7
54	9.81607	9.87844	9.93763	10.06237	10.12156	10.18393	4.38765	4.53815	6
55	9.81622	9.87833	9.93789	10.06211	10.12167	10.18378	4.38799	4.53787	5
56	9.81636	9.87822	9.93814	10.06186	10.12178	10.18364	4.38833	4.53759	4
57	9.81651	9.87811	9.93840	10.06160	10.12189	10.18349	4.38867	4.53731	3
58	9.81665	9.87800	9.93865	10.06135	10.12200	10.18335	4.38900	4.53704	2
59	9.81680	9.87789	9.93891	10.09109	10.12211	10.18320	4.38934	4.53676	1
60	9.81694	9.87778	9.93916	10.09084	10.12222	10.18306	4.38968	4.53648	0
	Co-sine	Sine	Co-tan.	Tangent	Co-sec.	Secant		V. Sin.	M

M	Sine	Co-sine	Tang.	Co-tan.	Secant	Co-sec.	V. Sine		
0	9.81694	9.87778	9.93916	10.06084	10.12222	10.18306	4.38968	4.53648	60
1	9.81709	9.87767	9.93942	10.06058	10.12233	10.18291	4.39002	4.53621	59
2	9.81723	9.87756	9.93997	10.06033	10.12244	10.18277	4.39036	4.53593	58
3	9.81738	9.87745	9.93993	10.06007	10.12255	10.18262	4.39069	4.53565	57
4	9.81752	9.87734	9.94018	10.05982	10.12266	10.18248	4.39103	4.53537	56
5	9.81767	9.87723	9.94044	10.05956	10.12277	10.18233	4.39137	4.53510	55
6	9.81781	9.87712	9.94069	10.05931	10.12288	10.18219	4.39171	4.53482	54
7	9.81796	9.87701	9.94095	10.05905	10.12299	10.18204	4.39204	4.53454	53
8	9.81810	9.87690	9.94120	10.05880	10.12310	10.18190	4.39238	4.53426	52
9	9.81825	9.87679	9.94146	10.05854	10.12321	10.18175	4.39272	4.53398	51
10	9.81839	9.87668	9.94171	10.05829	10.12332	10.18161	4.39305	4.53371	50
11	9.81854	9.87657	9.94197	10.05803	10.12343	10.18146	4.39339	4.53343	49
12	9.81868	9.87646	9.94222	10.05778	10.12354	10.1813	4.39372	4.53315	48
13	9.81882	9.87635	9.94248	10.05752	10.12365	10.1811	4.39406	4.53287	47
14	9.81897	9.87624	9.94273	10.05727	10.12376	10.18103	4.39440	4.53259	46
15	9.81911	9.87613	9.94299	10.05701	10.12387	10.18089	4.39473	4.53231	45
16	9.81926	9.87601	9.94324	10.05676	10.12399	10.18074	4.39507	4.53203	44
17	9.81940	9.87590	9.94350	10.05650	10.12410	10.18060	4.39540	4.53176	43
18	9.81954	9.87579	9.94375	10.05625	10.12421	10.18046	4.39574	4.53148	42
19	9.81969	9.87568	9.94401	10.05599	10.12432	10.18031	4.39607	4.53120	41
20	9.81983	9.87557	9.94426	10.05574	10.12443	10.18017	4.39641	4.53092	40
21	9.81998	9.87546	9.94452	10.05548	10.12454	10.1800	4.39674	4.53064	39
22	9.82012	9.87535	9.94477	10.05523	10.12465	10.17988	4.39708	4.53036	38
23	9.82026	9.87524	9.94503	10.05497	10.12476	10.17974	4.39741	4.53008	37
24	9.82041	9.87513	9.94528	10.05472	10.12487	10.17959	4.39775	4.52980	36
25	9.82055	9.87501	9.94554	10.05446	10.12499	10.17945	4.39808	4.52952	35
26	9.82069	9.87490	9.94579	10.05421	10.12510	10.17931	4.39842	4.52924	34
27	9.82084	9.87479	9.94604	10.05396	10.12521	10.17916	4.39875	4.52896	33
28	9.82098	9.87468	9.9463	10.05370	10.12532	10.17902	4.39908	4.52868	32
29	9.82112	9.87457	9.94655	10.05345	10.12543	10.17888	4.39942	4.52840	31
30	9.82126	9.87446	9.94681	10.05319	10.12554	10.17874	4.39975	4.52812	30
31	9.82141	9.87434	9.94706	10.05294	10.12566	10.17859	4.40008	4.42784	29
32	9.82155	9.87423	9.94732	10.05268	10.12577	10.17845	4.40042	4.52756	28
33	9.82169	9.87412	9.94757	10.05243	10.12588	10.17831	4.40075	4.52728	27
34	9.82184	9.87401	9.94783	10.05217	10.12599	10.17816	4.40108	4.52700	26
35	9.82198	9.87390	9.94808	10.05192	10.12610	10.17802	4.40147	4.92672	25
36	9.82212	9.87378	9.94834	10.05166	10.12622	10.17788	4.40185	4.52643	24
37	9.82226	9.87367	9.94859	10.05141	10.12633	10.17774	4.40208	4.52615	23
38	9.82240	9.87356	9.94884	10.05116	10.12644	10.17760	4.40241	4.52587	22
39	9.82255	9.87345	9.94910	10.05090	10.12655	10.17745	4.40275	4.52559	21
40	9.82269	9.87334	9.94935	10.05065	10.12666	10.17731	4.40308	4.52531	20
41	9.82283	9.87322	9.94961	10.05039	10.12678	10.17717	4.40341	4.52503	19
42	9.82297	9.87311	9.94986	10.05014	10.12689	10.17703	4.40374	4.52475	18
43	9.82311	9.87300	9.95012	10.04988	10.12700	10.17689	4.40407	4.52446	17
44	9.82326	9.87288	9.95037	10.04963	10.12712	10.17674	4.40440	4.52418	16
45	9.82340	9.87277	9.95062	10.04938	10.12723	10.17660	4.40474	4.52390	15
46	9.82354	9.87266	9.95088	10.04912	10.12734	10.17646	4.40507	4.52362	14
47	9.82368	9.87255	9.95113	10.04887	10.12745	10.17632	4.40540	4.52334	13
48	9.82382	9.87243	9.95139	10.04861	10.12757	10.17618	4.40573	4.52305	12
49	9.82396	9.87232	9.95164	10.04836	10.12768	10.17604	4.40606	4.52277	11
50	9.82410	9.87221	9.95190	10.04810	10.12779	10.17590	4.40639	4.52249	10
51	9.82424	9.87209	9.95215	10.04785	10.12791	10.17576	4.40672	4.52221	9
52	9.82439	9.87198	9.95240	10.04760	10.12802	10.17561	4.40705	4.52192	8
53	9.82453	9.87187	9.95266	10.04734	10.12813	10.17547	4.40738	4.52164	7
54	9.82467	9.87175	9.95291	10.04709	10.12825	10.17533	4.40771	4.52136	6
55	9.82481	9.87164	9.95317	10.04683	10.12836	10.17519	4.40804	4.52107	5
56	9.82495	9.87153	9.95342	10.04658	10.12847	10.17505	4.40837	4.52079	4
57	9.82509	9.87141	9.95368	10.04632	10.12859	10.17491	4.40870	4.52051	3
58	9.82523	9.87130	9.95393	10.04607	10.12870	10.17477	4.40903	4.52022	2
59	9.82537	9.87119	9.95418	10.04582	10.12881	10.17463	4.40936	4.51994	1
60	9.82551	9.87107	9.95444	10.04556	10.12893	10.17449	4.40969	4.51966	0
	Co-sine	Sine	Co-tan.	Tang.	Co-sec.	Secant		V. Sine	M

(48 Deg.)

M	Sine	Co-sine	Tang.	Co-tang.	Secant	Co-sec.	V. Sine		
0	9.82551	9.87107	9.95444	10.04556	10.12893	10.17449	4.40969	4.51966	60
1	9.82565	9.87096	9.95469	10.04531	10.12904	10.17435	4.41002	4.51937	59
2	9.82579	9.87085	9.95495	10.04505	10.12915	10.17421	4.41035	4.51909	58
3	9.82593	9.87073	9.95520	10.04480	10.12927	10.17407	4.41067	4.51880	57
4	9.82607	9.87062	9.95545	10.04455	10.12938	10.17393	4.41100	4.51852	56
5	9.82621	9.87050	9.95571	10.04429	10.12950	10.17379	4.41133	4.51824	55
6	9.82635	9.87039	9.95596	10.04404	10.12961	10.17365	4.41166	4.51795	54
7	9.82649	9.87028	9.95622	10.04378	10.12972	10.17351	4.41199	4.51767	53
8	9.82663	9.87016	9.95647	10.04353	10.12984	10.17337	4.41232	4.51738	52
9	9.82677	9.87005	9.95672	10.04328	10.12995	10.17323	4.41264	4.51710	51
10	9.82691	9.86993	9.95698	10.04302	10.13007	10.17309	4.41297	4.51681	50
11	9.82705	9.86982	9.95723	10.04277	10.13018	10.17295	4.41330	4.51653	49
12	9.82719	9.86970	9.95748	10.04252	10.13030	10.17281	4.41363	4.51624	48
13	9.82735	9.86959	9.95774	10.04226	10.13041	10.17267	4.41395	4.51596	47
14	9.82747	9.86947	9.95799	10.04201	10.13053	10.17253	4.41428	4.51567	46
15	9.82761	9.86936	9.95825	10.04175	10.13064	10.17239	4.41461	4.51539	45
16	9.82775	9.86924	9.95850	10.04150	10.13076	10.17225	4.41494	4.51510	44
17	9.82788	9.86913	9.95875	10.04125	10.13087	10.17212	4.41526	4.51482	43
18	9.82802	9.86902	9.95901	10.04098	10.13098	10.17198	4.41559	4.51453	42
19	9.82816	9.86890	9.95926	10.04074	10.13110	10.17184	4.41592	4.51425	41
20	9.82830	9.86879	9.95952	10.04048	10.13121	10.17170	4.41624	4.51396	40
21	9.82844	9.86867	9.95977	10.04023	10.13133	10.17156	4.41657	4.51367	39
22	9.82858	9.86855	9.96002	10.03998	10.13145	10.17142	4.41689	4.51339	38
23	9.82872	9.86844	9.96028	10.03972	10.13156	10.17128	4.41722	4.51310	37
24	9.82885	9.86832	9.96053	10.03947	10.13168	10.17115	4.41755	4.51281	36
25	9.82899	9.86821	9.96078	10.03922	10.13179	10.17101	4.41787	4.51253	35
26	9.82913	9.86809	9.96104	10.03896	10.13191	10.17087	4.41820	4.51224	34
27	9.82927	9.86798	9.96129	10.03871	10.13202	10.17073	4.41852	4.51195	33
28	9.82941	9.86786	9.96155	10.03845	10.13214	10.17059	4.41885	4.51167	32
29	9.82955	9.86775	9.96180	10.03820	10.13225	10.17045	4.41917	4.51138	31
30	9.82968	9.86763	9.96205	10.03795	10.13237	10.17032	4.41950	4.51109	30
31	9.82982	9.86752	9.96231	10.03769	10.13248	10.17018	4.41982	4.51081	29
32	9.82996	9.86740	9.96256	10.03744	10.13260	10.17004	4.42015	4.51052	28
33	9.83010	9.86728	9.96281	10.03719	10.13272	10.16990	4.42047	4.51023	27
34	9.83023	9.86717	9.96307	10.03693	10.13283	10.16977	4.42080	4.50994	26
35	9.83037	9.86705	9.96332	10.03668	10.13295	10.16963	4.42112	4.50966	25
36	9.83051	9.86694	9.96357	10.03643	10.13306	10.16949	4.42144	4.50937	24
37	9.83065	9.86682	9.96383	10.03617	10.13318	10.16935	4.42177	4.50908	23
38	9.83078	9.86670	9.96408	10.03592	10.13330	10.16922	4.42209	4.50879	22
39	9.83092	9.86659	9.96433	10.03567	10.13341	10.16908	4.42242	4.50851	21
40	9.83106	9.86647	9.96459	10.03541	10.13353	10.16894	4.42274	4.50822	20
41	9.83119	9.86635	9.96484	10.03516	10.13365	10.16881	4.42306	4.50793	19
42	9.83133	9.86624	9.96510	10.03490	10.13376	10.16867	4.42339	4.50764	18
43	9.83147	9.86612	9.96535	10.03465	10.13388	10.16853	4.42371	4.50735	17
44	9.83161	9.86600	9.96560	10.03440	10.13400	10.16839	4.42403	4.50706	16
45	9.83174	9.86589	9.96586	10.03414	10.13411	10.16826	4.42435	4.50677	15
46	9.83188	9.86577	9.96611	10.03389	10.13423	10.16812	4.42468	4.50649	14
47	9.83202	9.86569	9.96636	10.03364	10.13435	10.16798	4.42500	4.50620	13
48	9.83215	9.86554	9.96662	10.03338	10.13446	10.16785	4.42532	4.50591	12
49	9.83229	9.86542	9.96687	10.03313	10.13458	10.16771	4.42504	4.50562	11
50	9.83242	9.86530	9.96712	10.03288	10.13470	10.16758	4.42597	4.50533	10
51	9.83256	9.86518	9.96738	10.03262	10.13482	10.16[illegible]44	4.42629	4.505[illegible]4	9
52	9.83270	9.86507	9.96763	10.03237	10.13493	10.16730	4.42661	4.50475	8
53	9.83283	9.86495	9.96788	10.03212	10.13505	10.16717	4.42693	4.50446	7
54	9.83297	9.86483	9.96814	10.03186	10.13517	10.16703	4.42725	4.50417	6
55	9.83310	9.86472	9.96839	10.03161	10.13528	10.16690	4.42758	4.50388	5
56	9.83324	9.86460	9.96864	10.03136	10.13540	10.16676	4.42790	4.50359	4
57	9.83338	9.86448	9.96890	10.03110	10.13552	10.16662	4.42822	4.50330	3
58	9.83351	9.86436	9.96915	10.03085	10.13564	10.16649	4.42854	4.50301	2
59	9.83365	9.86425	9.96940	10.03060	10.13575	10.16635	4.42886	4.50272	1
60	9.83378	9.86413	9.96966	10.03034	10.13587	10.16622	4.42918	4.50243	0
	Co-sine	Sine	Co tan.	Tangent	Co-sec.	Secant		V. Sine	M

M	Sine	Co-sine	Tang.	Co-tang.	Secant	Co-sec.	V Sine		
0	9.83378	9.86413	9.96966	10.03034	10.13587	10.16622	4.42918	4.50243	60
1	9.83392	9.86401	9.96991	10.03009	10.13599	10.16608	4.42950	4.50214	59
2	9.83405	9.86389	9.97016	10.02984	10.13611	10.16[illegible]95	4.42982	4.50185	58
3	9.83419	9.86377	9.97042	10.02958	10.13623	10.16581	4.43014	4.50156	57
4	9.83432	9.86366	9.97067	10.02933	10.13634	10.16568	4.43046	4.50127	56
5	9.83446	9.86354	9.97092	10.02908	10.13646	10.16554	4.43078	4.50098	55
6	9.83459	9.86342	9.97118	10.02882	10.13658	10.16541	4.43110	4.50068	54
7	9.83473	9.86330	9.97143	10.02857	10.13670	10.16527	4.43142	4.50039	53
8	9.83486	9.86318	9.97168	10.02832	10.13682	10.16514	4.43174	4.50010	52
9	9.83500	9.86306	9.97193	10.02807	10.13694	10.16500	4.43206	4.50981	51
10	9.83513	9.86295	9.97219	10.02781	10.13705	10.16487	4.43238	4.50952	50
11	9.83527	9.86283	9.97244	10.02756	10.13717	10.16473	4.43270	4.49923	49
12	9.83540	9.86271	9.97269	10.02731	10.13729	10.16460	4.43302	4.49893	48
13	9.83554	9.86259	9.97295	10.02705	10.13741	10.16446	4.43334	4.49864	47
14	9.83567	9.86247	9.97320	10.02680	10.13753	10.16433	4.43366	4.49835	46
15	9.83581	9.86235	9.97345	10.02655	10.13765	10.16419	4.43398	4.49806	45
16	9.83594	9.86223	9.97371	10.02629	10.13777	10.16406	4.43429	4.49777	44
17	9.83607	9.86211	9.97396	10.02604	10.13789	10.16393	4.43461	4.49747	43
18	9.83621	9.86200	9.97431	10.02579	10.13800	10.16379	4.43493	4.49718	42
19	9.83634	0.86188	9.97447	10.02553	10.13812	10.16366	4.43525	4.49689	41
20	9.83648	9.86176	9.97472	10.02528	10.13824	10.16352	4.43557	4.49660	40
21	9.83661	9.86164	9.97497	10.02503	10.13836	10.16339	4.43589	4.49630	39
22	9.83674	9.86152	9.97523	10.02477	10.13848	10.16326	4.43620	4.49601	38
23	9.83688	9.86140	9.97548	10.02452	10.13860	10.16312	4.43652	4.49572	37
24	9.83701	9.86128	9.97573	10.02427	10.13872	10.16299	4.43684	4.49542	36
25	9.83715	9.86116	9.97598	10.02402	10.13884	10.16285	4.43716	4.49513	35
26	9.83728	9.86104	9.97624	10.02376	10.13896	10.16272	4.43747	4.49484	34
27	9.83741	9.86092	9.97649	10.02351	10.13908	10.16259	4.43780	4.49454	33
28	9.83755	9.86080	9.97674	10.02326	10.13920	10.16245	4.43811	4.49425	32
29	9.83768	9.86068	9.97700	10.02300	10.13932	10.16232	4.43842	4.49395	31
30	9.83781	9.86056	9.97725	10.02275	10.13944	10.16219	4.438[illegible]4	4.49366	30
31	9.83795	9.86[illegible]44	9.97750	10.02250	10.13956	10.16205	4.43906	4.49337	29
32	9.83808	9.86032	9.97776	10.02224	10.13968	10.16192	4.43937	4.49307	28
33	9.83821	9.86020	9.97801	10.02199	10.13980	10.16179	4.43969	4.49278	27
34	9.83834	9.86008	9.97826	10.02174	10.13992	10.16166	4.44001	4.49248	26
35	9.83848	9.85[illegible]96	9.97851	10.02149	10.14004	10.16152	4.44032	4.49219	25
36	9.83861	9.85984	9.97877	10.0212[illegible]	10.14016	10.16139	4.44064	4.49189	24
37	9.83874	9.85972	9.97902	10.02098	10.14028	10.16126	4.44095	4.49160	23
38	9.83887	9.85960	9.97927	10.02073	10.14040	10.16113	4.44127	4.49130	22
39	9.83901	9.[illegible]5948	9.97953	10.02047	10.14052	10.16099	4.44159	4.49101	21
40	9.83914	9.85936	9.97978	10.0202[illegible]	10.14064	10.16086	4.44190	4.49071	20
41	9.83927	9.85924	9.98003	10.01997	10.14076	10.16073	4.44222	4.49042	19
42	9.83940	9.85912	9.98029	10.01971	10.14088	10.16060	4.44253	4.49012	18
43	9.83954	9.859[illegible]0	9.98054	10.01946	10.14100	10.16046	4.44285	4.48983	17
44	9.83967	9.85888	9.98079	10.01921	10.14112	10.16033	4.44316	4.48953	16
45	9.83980	9.85876	9.98104	10.01896	10.14124	10.16020	4.44348	4.48924	15
46	9.83993	9.85864	9.98130	10.01870	10.14136	10.16907	4.44379	4.48894	14
47	9.84006	9.85851	6.98155	10.01845	10.14149	10.15994	4.44410	4.48864	13
48	9.84020	9.85839	9.98180	10.01820	10.14161	10.15980	4.44442	4.48835	12
49	9.84033	9.85827	9.98206	10.01794	10.14173	10.15967	4.44473	4.48805	11
50	9.84046	9.85815	9.98231	10.01769	10.14185	10.15954	4.44505	4.48776	10
51	9.84059	9.85803	9.98256	10.01744	10.14197	10.15941	4.44536	4.48746	9
52	9.84072	9.85791	9.98281	10.01719	10.14209	10.15928	4.44568	4.48716	8
53	9.84085	9.85779	9.98307	10.01693	10.14221	10.15915	4.44599	4.48687	7
54	9.84098	9.85766	9.98332	10.01668	10.14234	10.15902	4.44630	4.48657	6
55	6.84112	9.85754	9.98357	10.01643	10.14246	10.15888	4.44662	4.48127	5
56	9.84125	9.85742	9.98383	10.01617	10.14258	10.15875	4.44693	4.48598	4
57	9.84138	9.85730	9.98408	10.01592	10.14270	10.15862	4.44724	4.48568	3
58	9.84151	9.85718	9.98433	10.01567	10.14282	10.15849	4.44756	4.48538	2
59	9.84164	9.85706	9.98458	10.01542	10.14294	10.15836	4.44787	4.48508	1
60	9.84177	9.85693	9.98484	10.01516	10.14307	10.15823	4.44818	4.48479	0
	Co-sine	Sine.	Co-tan.	Tangent	Co-sec.	Secant		V. Sine	M

(46 Deg.)

M	Sine	Co-fine	Tang.	Co-tang.	Secant	Co-fec.	V.Sine		
0	9.84177	9.85693	9.98484	10.01516	10.14307	10.15823	4.44818	4.48479	60
1	9.84190	9.85681	9.98509	10.01491	10.14319	10.15810	4.44849	4.48449	59
2	9.84203	9.85669	9 98534	10.01466	10.14331	10.15797	4.44881	4.48419	58
3	9.84216	9.85657	9.98560	10.01440	10.14343	10.15784	4.44912	4.48389	57
4	9.84229	9.85645	9.98585	10.01415	10.14355	10.15771	4.44943	4.48359	56
5	9.84242	9.85632	9.98610	10.01390	10.14368	10.15758	4.44974	4.48330	55
6	9.84255	9.85620	9.98635	10.01365	10.14380	10.15745	4 45005	4.4830[illegible]	54
7	9.84269	9.85608	9.98661	10.01339	10.14392	10.15731	4.45037	4.48270	53
8	9.84282	9.85596	9.98686	10.01314	10.14304	10.15718	4.45068	4.48240	52
9	9.84295	9.85583	9.98811	10.01289	10.14417	10.15705	4.45099	4.48210	51
10	9.84308	9.85571	9.98737	10.01263	10.14429	10.15692	4.45130	4.48180	50
11	9.84321	9.85559	9.98762	10.01238	10.14441	10.15679	4.45161	4.48150	49
12	9.84334	9.85546	9.98787	10 01213	10.14454	10.15666	4.45192	4.48121	48
13	9.84347	9.85534	9.98812	10.01188	10.14466	10.15653	4.45223	4.48091	47
14	9.84360	9.85522	9.98838	10.01162	10.14478	10.15640	4.45255	4.48061	46
15	9.84372	9.85510	9.98863	10.01137	10.14490	10.05628	4.45286	4.48031	45
16	9.84385	9.85497	9 98888	10.01112	10.14503	10.15615	4.45317	4.48001	44
17	9.84398	9.85485	9.98913	10.01087	10 14515	10.15602	4.45348	4.47971	43
18	9.84411	9.85473	9.98939	10.01061	10.14527	10 15589	4.45379	4.47941	42
19	9.84424	9.85460	9.98964	10.01036	10.14540	10.15576	4.45410	4.47911	41
20	9 84437	9.85448	9.98989	10.01011	10.14552	10.15563	4.45441	4.47881	40
21	9.84450	9 85436	9.99015	10 00985	10.14564	10.15550	4.45472	4.47851	39
22	9.84463	9.85423	9 99040	10.00960	10.14577	10.15537	4.45503	4.47821	38
23	9 84476	9.85411	9.99065	10.00935	10.14589	10.15524	4.45534	4.47791	37
24	9 84489	9.85399	9.99090	10.00910	10 14601	10.15511	4.45565	4.47761	36
25	9.84502	9.85386	9.99116	10.00884	10.14614	10.15498	4.45596	4.47731	35
26	9.84515	9.85374	9.99141	10.00859	10 14626	10.15485	4.45627	4.47701	34
27	9.84528	9.85361	9.99166	10.00834	10.14639	10.15472	4.45658	4.47671	33
28	9.84540	9.85349	9.99191	10.00809	10.14651	10.15460	4.45688	4.47641	32
29	9.84553	9.85337	9.99217	10.00783	10.14663	10.05447	4.45719	4.47610	31
30	9.84566	9.85324	9.99242	10.00758	10.14676	10 15434	4.45750	4.47580	30
31	9.84[illegible]9	9.85312	9 99267	10.00733	10.14688	10.15421	4.45781	4.47550	29
32	9.84592	9.85299	9 99293	10.00707	10.14701	10.15408	4.45812	4.47520	28
33	9.84605	9.85287	9.99318	10.00682	10.14713	10 15395	4.45843	4.47490	27
34	9.84618	9.85274	9.99343	10.00657	10 14726	10.15382	4.45874	4.47460	26
35	9.84830	9.85262	9.99368	10.00632	10.14738	10.15370	4.45905	4.47430	25
36	9.84643	9.85250	9.99394	10.00606	10.14750	10.15357	4.45935	4.47399	24
37	9.84656	9.85237	9.99419	10.00581	10.14763	10.15344	4.45966	4.47369	23
38	9.84669	9.85225	9.99444	10.00556	10.14775	10.15331	4.45997	4.47339	22
39	9.8468[illegible]	9 85212	9.99469	10.00531	10.14788	10.15318	4.46028	4.47309	21
40	9.84694	9.85200	9.99495	10.00505	10.14800	10.15306	4.46058	4.47278	20
41	9.84707	9.85187	9.99520	10.00480	10.14813	10.15293	4.46089	4.47248	19
42	9.84720	9 85175	9.99545	10.00455	10.14825	10.15280	5.46120	4.47218	18
43	9.84733	9.85162	9.99570	10.00430	10.14838	10.15267	4.46151	4.47188	17
44	9.84745	9.85150	9.99596	10.00404	10.14850	10.15255	4.46181	4.47157	16
45	9.84758	9 85137	9.99621	10.00379	10.14863	10.15242	4.46212	4.47127	15
46	9.84771	9.85125	9.99646	10.00354	10.14875	10.15229	4.46243	4.47097	14
47	9.84784	9.85112	9.99672	10.00328	10 14888	10.15216	4.46273	4.47066	13
48	9.84796	9.85100	9.99697	10 00303	10.14900	10.15204	4.46304	4.47036	12
49	9.84809	9.85087	9.99722	10.00278	10.14913	10.15191	4.46335	4.47006	11
50	9.84822	9.85074	9.99747	10.00253	10.14926	10.15178	4.46365	4.46975	10
51	9.84834	9.85062	9.99773	10.00227	10.14938	10.15166	4.46396	4.46945	9
52	9.84847	9.85049	9.99798	10.00202	10.14951	10.15153	4.46427	4.46915	8
53	9 84860	9.85037	9.99823	10.00177	10.14963	10.15140	4.46457	4.46884	7
54	9.84873	9.85024	9.99848	10.00152	10.14976	10.15127	4.46488	4.46854	6
55	9.84885	9.85012	9.99874	10.00226	10.14988	10.15115	4.46518	4.46823	5
56	9 84898	9.84999	9.99899	10.00101	10.15001	10.15102	4.46549	4.46793	4
57	9.84911	9.84986	9 99924	10.00076	10.15014	10.15089	4.46579	4.46762	3
58	9.84923	9.84974	9.99949	10.00051	10.15026	10.15077	4.46610	4.46732	2
59	9.84936	9.84961	9.99975	10.00025	10.15039	10.15074	4.46640	4.46701	1
60	9.84948	9.84948	10.00000	10.00000	10 15052	10.15052	4.46671	4.46671	0
	Co-fine	Sine.	Co-tan.	Tngent	Co-fec.	Secant		V. Sine	M

A

TABLE

OF

LOGARITHMIC SINES, TANGENTS AND SECANTS

TO EVERY

Point, Half Point, and Quarter Point of the COMPASS.

Prts.	Sines	Co-ſines	Tangents	Co-tang.	Secants	Co-ſec.	
0	0.00000	10.00000	0.00000	Infinite	10.00000	11.10901	8
0¼	8.69079	9.99948	8.69131	11.00869	10.00052	11.30921	7¾
0½	8.99130	9.99790	8.99340	11.00660	10.00210	11.00870	7½
0¾	9.16652	9.99527	9.17125	10.82875	10.00473	10.83348	7¼
1	9.29024	9.91157	9.29866	10.70134	10.00843	10.70976	7
1¼	9.38557	9.98679	9.39878	10.60122	10.01321	10.61443	6¾
1½	9.46282	9.98088	9.48194	10.51806	10.01912	10.53718	6½
1¾	9.52749	9.97384	9.55365	10.44635	10.02616	10.47251	6¼
2	9.58284	9.96562	9.61722	10.38278	10.03438	10.41716	6
2¼	9.63099	9.95616	9.67482	10.32518	10.04184	10.36901	5¾
2½	9.67339	9.94543	9.72796	10.27204	10.05457	10.32661	5½
2¾	9.71105	9.93335	9.77770	10.22234	10.06665	10.28895	5¼
3	9.74474	9.91985	9.82489	10.17511	10.08015	10.25526	5
3¼	9.77503	9.80483	9.87020	10.12980	10.09517	10.22497	4¾
3½	9.80236	9.88819	9.91417	10.08583	10.11181	10.19764	4½
3¾	9.82708	9.86979	9.95729	10.04271	10.13021	10.17292	4¼
4	9.84948	9.84948	10.00000	10 00000	10,15052	10.15052	4
	Co-ſine	Sines	Co-tang.	Tangents	Co-ſec.	Secants	Prts.

MENSURATION OF SURFACES.

Lineal Measures.

12 inches	-	1 foot.
3 feet	- -	1 yard.
6 feet	- -	1 fathom.
$16\frac{1}{2}$ feet or $5\frac{1}{2}$ yards	-	1 pole, rood, or perch.
40 poles	-	1 furlong.
8 furlongs	-	1 mile.

Square Measures.

144 ſquare inches	-	1 foot.
9 feet	- - -	1 yard.
36 feet	- -	1 fathom.
$272\frac{1}{4}$ feet or $30\frac{1}{4}$ yards	- -	1 pole or rood.
1600 poles	- -	1 furlong.
64 furlongs	- -	1 mile.

The Engliſh chain is divided into 100 links, each link being 7·92 inches. And 80 chain-lengths is 1 Engliſh mile.

Lineal Scots Measures.

37 inches	- - -	1 ell.
6 ells, or $18\frac{1}{2}$ feet	-	1 fall.
4 falls, or 74 feet	- -	1 chain.
10 chains or 40 falls	-	1 furlong.
8 furlongs or 80 chains	-	1 mile.

SQUARE MEASURES.

1369 square inches	- -	1 ell.
36 ells, or $342\frac{1}{4}$ feet	-	1 fall.
16 falls, or 547 feet	-	1 chain.
100 chains, or 1600 falls	-	1 furlong.
64 furlongs, or 6400 chains	-	1 mile.

The Scots chain is divided in 100 links, each link being 8.88 inches, and 80 chains is one Scots mile.

To measure surfaces and solids by duodecimals, or cross multiplication.

RULE.

Multiply each denomination of the length by the feet of breadth, beginning at the lower place, and setting each product under that denomination of the multiplicand from which it arises, carrying by 12, when necessary, to the higher place. Then multiply by the inches of breadth (if any) setting each product one place to the right hand, carrying by 12 as above. *Lastly*, Multiply by the parts, setting down each product, another place to the right, and so on.

EXAMPLE.

A pavement 16 feet 4 inches long, 7 feet 6 inches broad, How many square feet ?

	F.	in.	
Multiply the length	16	4	
By the breadth -	7	6	
	114	4	
	8	2	0
	122	6	0

First,

First, say, 7 times 4 is 28, which is 2 to carry, and 4 over. Write down 4, the excess in the column of inches; then say, 7 times 16 is 112, and the 2 you carry is 114, which set down in the column of feet. Again, 6 times 4 is 24, which is 2 to carry and 0 over; then say, 6 times 16 is 96, and 2 is 98, which is 8 feet 2 inches; which, when you'll have placed in their proper columns, add, and the sum will be 122 feet 6 inches.

Note, The superficial content of masons, joiners, plasterers, painters, &c. work, is frequently cast up by the preceding rule; for the better understanding of which observe, that

The superficies of any rectangle is found by multiplying the length by the breadth; and the content of any triangle is found by multiplying half the base by the perpendicular altitude, as shall be explained afterwards.

Any lineal measure, multiplied by the same lineal measure, produces squares of the same. Thus, lineal feet, multiplied by lineal feet, produce square feet; lineal inches into lineal inches produce square inches, and so on.

Lineal feet into lineal inches produce rectangles 1 foot long, and 1 inch broad, which, divided by 12 quots feet; and the remainder multiplied by 12, produces square inches.

Lineal feet into lineal lines produce rectangles 1 foot long and 1 line broad; which, divided by 144 quots square feet, and each unit in the remainder are rectangles, equal to square inches.

Examples for practice.

	Length.			Breadth.			Answers.				
	F.	i.	pts.	F.	i.	pts.	F.	i.	pts.		
1.	18	4	6	2	3	2	41	7	2	″3	‴0
2.	27	3	0	1	6	0	40	10	6		
3.	16	4	9	1	9	3	29	0	4	11	3
4.	14	8	0	2	0	6	29	11	4		
5.	20	10	0	0	4	6	7	9	9		
6.	18	9	1	3	0	0	56	3	3		
7.	32	6	7	0	0	8	1	9	8	4	8
8.	14	0	0	2	6	0	35	0	0		
9.	19	0	6	1	4	3	25	9	5	1	6
10.	25	3	2	1	9	9	45	9	5	10	6
11.	36	1	0	1	6	0	54	1	6		
12.	162	3	0	32	5	0	5259	7	3		
13.	103	2	6	10	0	0	1032	1	0		
14.	13	5	0	4	3	2	57	2	5	10	0
15.	19	3	2	1	3	0	24	0	11	6	0
16.	14	6	0	2	4	0	33	10	0		
17.	20	8	0	0	6	0	10	4	0		
18.	23	7	0	1	3	0	29	5	9		
19.	37	11	6	2	6	0	94	10	9		
20.	37	7	5	4	8	6	177	1	5	0	6
21.	311	4	7	36	7	5	11402	2	4	11	11
22.	87	5	0	35	8	0	3117	10	4		
23.	24	6	0	9	6	0	232	9	0		
24.	4	7	9			1 5″ 3‴	6	8	1	8	3
25.	14	0	0	1	6	0	21	0	0		
26.	18	4	6	2	0	0	36	0	9		
27.	10	0	0	0	10	0	8	4	0		

PRO-

PROBLEM I. *Plate 5. fig. 70.*

To find the area of a square.

RULE.

Multiply the side by itself, and the product will be the area.

EXAMPLE I.

How many square yards are in a square, whose side is $15\frac{1}{2}$ feet?

Duodecimally.		*Decimally.*
F.	*in.*	*F.*
15	6	15.5
15	6	15.5
232	6	775
7	9	775
		155
9)240	3	9)240.25
26 yds 6 feet 3 inches.		Yds. 26 6 3

By reduction.

F. *in.* *in.*

15 6 = 186

186

1116

1488

186

9

144)34596(240 feet.

288

26 yards 6 feet.

579

576

12)36

3 inch.

Ex. 2.

Ex. 2. Required the area of a ſquare, whoſe ſide is 12 feet?
Anſ. 144 *ſquare feet.*

Ex. 3. How many ſquare feet are in a ſquare, whoſe ſide is 6 feet 3 inches? *Anſ.* 39 *feet* 0 *in.* 9 *pts.*

Ex. 4. How many ſquare yards are in a ſquare court, whoſe ſide is $80\frac{1}{4}$ feet? *Anſ.* 715 *yds.* 5 *feet* 0 *in.* 9 *pts.*

Ex. 5. How many ſquare chains are in a field, whoſe ſide is 1 mile? *Anſ.* 6400 *ſq. chains.*

Ex. 6. Required the area of a ſquare, whoſe ſide is 3 chains?
Anſ. 9 *ſq. ch.*

PROBLEM II. *Plate* 5. *fig.* 71.

To find the area of a rectangle.

RULE.

Multiply the length by the breadth, and the product is the area.

EXAMPLE I.

Required the area of a rectangle, whoſe height is 3000 links, and breadth 1670 links of the Engliſh chain.

1670
3000

50,10000 ſquare links.
4

.40000
40

16,00000

Anſ. 50 *acres,* 0 *roods* 16 *poles.*

Here,

Here, becaufe the chain is divided into 100 links, and that 100×100 is 10000 (the number of fquare links in one fquare chain) and $10000 \times 10 = 100000$ (the number of fquare links in one acre) divide the product by 100000, the quot gives acres and decimals of an acre; and this decimal is reduced to value by multiplying by 4, by 40, by $30\frac{1}{4}$. Or, inftead of dividing the fquare links by 100000, cut off five decimal places towards the right hand, the integral part gives acres, and thofe cut off are decimals of an acre, and are reduced to value accordingly.

Ex. 2. Required the area of a rectangular field, whofe fides are 5.5 and 2.54 Scots chains.

Anf. 1 *acre* 1 *rood* 23 *falls* 18.72 *ells.*

Ex. 3. Required the area of a rectangle, whofe length is $15\frac{1}{2}$ feet, and breadth 12 feet. *Anf.* 186 *fquare feet.*

Ex 4. Required the area of a rectangle, whofe length is 10 inches, and breadth 6 inches. *Anf.* 60 *inches.*

Ex. 5. Required the area of a rectangle, whofe fides are 56 feet, and 18 feet 6 inches? *Anf.* 1036 *fquare feet.*

Ex. 6. Required the area of a rectangle, whofe length is $16\frac{1}{2}$, and breadth $10\frac{1}{5}$ yards *Anf.* 168.3 *yds.*

PROBLEM III. *Plate* 5. *fig.* 72

To find the area of a rhombus or rhomboid.

RULE.

Multiply the length by the perpendicular breadth, and the product is the area.

EXAMPLE I.

Required the area of a rhombus, whofe fide is 750 links, and one of its acute angles 60°.

To

To find DE the perpendicular.			To find the area.
As rad. -	90 = 10.00000		AB - = 750
is to AD 750 -	2.87506		DE - = 649.5
So is sine ang A 60°	9.93753		324750
To DE the per. 649.5 =	2.81259		454654
			4,87125,0
			4
			3,485000
			40
			19,400000
			36
			14,400000

Ans. 4 acres 3 rods 19 falls 14 ells.

When one of the angles of a rhombus or rhomboid are given, the area may be found by the following proportion.

As rad : sine included ang. : : the prod. containing sides : area.

As rad. 90° - -	10.00000		4,87100
is to sine 60° -	9.93753		4
So is prod. 562500 -	5.75012		3,48400
To the area = 487100 -	5.68765		40
			19,36000
			36
			216000
			108000
			12,96000

Ans. 4 acres 3 roods 19 falls 13 ells nearly.

Ex. 2. Required the area of a rhombus, whose length is 15 feet, and perpendicular breadth 12 feet.

Anſ. 180 *ſquare feet.*

Ex. 3. Required the area of a rhomboid, its length being 24, and perpendicular 14 Scots chains.

Anſ. 33 *acres* 2 *roods* 16 *falls.*

Ex. 4. What is the area of a rhombus, when the ſide is 1260 links, and its acute angles 54° 30′?

Anſ. 12 *acres* 3 *roods* 27 *falls* 7 *ells.*

Ex. 5. Required the area of a rhomboid whoſe ſides are 3200, 2400 links of the Engliſh chain, and acute angle 30.′

Anſ 38 *a.* 1 *r.* 24 *p.*

Ex. 6. Required the area of a rhomboid, when the length is 50 feet 6 inches, and perpendicular breadth 6 feet 6 inches.

Anſ. 328¼ *feet.*

Ex. 7. How many ſquare yards are in a rhombus, whoſe ſide is 15½ feet, and perpendicular height 3¼ feet?

Anſ. 5 *yards* 5 *feet* 4 *inches* 6 *parts.*

PROBLEM IV.

To find the ſide, or the perpendicular breadth of any parallelogram, the other ſide and the area being given.

RULE.

Divide the area by the given ſide, and the quotient will be the other ſide.

EXAMPLE I.

The area of a rectangle being 100000 ſquare links, and one of its ſides 1000 links, required the other ſide.

1000)100000(100 links, or 1 chain length,
1000

R EXAM-

Ex. 2. A ſquare field contains 120 acres, required the length of its ſide. - - 120 acres = 12000000 ſquare links.

And $\sqrt{12000000} = 3464$ links.

Ex. 3. The area of a rhomboid is $63\frac{1}{2}$ ſquare feet, and the length 10 feet. Required the perpendicular breadth.

Anſ. 6 *feet* 4.2 *inches.*

Ex. 4. The perpendicular breadth of a rhombus is $4\frac{1}{4}$ feet, and area 30 ſq. feet. Required the ſide. *Anſ.* 7.0588 *feet.*

Ex. 5. The area of a ſquare is 6740 ſquare yards. Required its ſide. *Anſ.* 82.097 *yards.*

Examples in this problem and in the three preceding problems prove each other.

PROBLEM V. *Plate 6. fig.* 74.

To find the area of a triangle, its perpendicular height and baſe being given.

RULE.

Multiply the baſe by half the perpendicular height, and the product will be the area.

EXAMPLE.

Required the area of a triangle, whoſe baſe is 64, and perpendicular 80 feet.

$$\begin{array}{r} 64 \text{ baſe.} \\ 80 \text{ perp.} \\ \hline 2)5120 \\ \hline 2560 \text{ area.} \end{array}$$

Ex. 2 How many ſquare yards are in a triangle, whoſe baſe is 99 feet, and perpendicular 85 feet 6 inches.

De-

Decimally.

```
      85.5
        99
   -------
     7695
    7695
   -------
  2)8464.5
   -------
  9)4232.25 area in feet.
   -------
Yds. 470  2  3
```

Duodecimally.

```
      F.   in.
      85    6
      99    0
   ----------
      49    6
     765    0
    765     0
   ----------
  2)8464    6
   ----------
  9)4232    3
   ----------
Yds 470   2 feet 3 in.
```

By reduction.

```
 F.   in.      in.
 85    6   =  1026
 99    0   =  1188
             ------
              8208
             8208
            1026
           1026
          ---------
         2)121888.8
          ---------  9
       144)609444(4232)
           576
           ---   Ys. 470  2  3
            334
            288
            ---
             464
             432
             ---
              324
              288
              ---
          12)  36
              ---
              3 inches.
```

Ex. 3. Required the area of a triangle, whoſe baſe is 12 feet 3 inches, and perpendicular 8 feet 9 inches.

Anſ. 53 *feet* 7 *in.* 1 *pt.* 6″.

Ex. 4. How many ſquare yards are contained in a triangular garden, the length of one of its ſides being 80 yards, and the perpendicular diſtance between that ſide and the oppoſite angle 70 yards? *Anſ.* 2800 *ſquare yards.*

Ex. 5. What is the expence of paving a triangular court, at 4s. 6d. *per* ſquare yard, one of its ſides being 48 feet 6 inches, and perpendicular $30\frac{1}{2}$ feet? *Anſ.* 18l. 9s. $9\frac{1}{4}$d.

PROBLEM VI. *Plate 6. fig.* 74.

One of the angles of a triangle and the containing ſides being given, to ſind the area.

RULE I.

As radius, is to the ſine of the included angle, ſo is half the product of the containing ſides, to the area.

RULE 2. Find the perpendicular by trigonometry, and proceed as in the preceding problem.

EXAMPLE I.

Required the area of a triangle, whoſe included angle is 63° 30′, and the containing ſides 806 and 700 links of the Engliſh chain.

By RULE I.

As radius 90	-	=10.00000
To ſine 63° 30′	-	9.95179
So is AB × AC = 282100	=	5.45040
To area 2.52500	=	5.40219

Anſ. 2 *acres* 2 *roods* 4 *perches.*

By

By Rule II.

To find BD the perpen.

As rad. 90 - 10.00000
is to AB 700 = 2.84510
So is sine $63°$ 30′ - 9.95179

To BD 626.5 - 2.79689
And 626.5 × 806 = 504959
2)504959

2.52479 = 2 acres 2 roods, 4 per. nearly.

Ex. 2. How many square yards are in a triangle, whose sides are 100.98, feet, and included angle 45° ?

Ans. 384 *sq. yds* $8\frac{4}{5}$ *or* 8.8 *feet.*

Ex. 3. Required the area of a triangle, when the containing sides are 409 and 220 yards, and the included angle 30°.

Ans. 22495 *sq. yds.*

Ex. 4. Required the area of a triangular field ABC, AB= 6000, AC 8000 links of the Scots chain, and angle A 39° 36′.

Ans. 153 *acres.*

Ex. 5. Required the area of a triangle, the containing sides being $21\frac{1}{4}$ and 25 yards, and the contained angle 50°.

Ans. $203\frac{1}{2}$ *yards.*

PROBLEM VII. *Plate 6. fig. 75.*

The three sides of any triangle being given, to find the area.

RULE.

Add the three given sides, and from half their sum subtract the sides severally: Multiply the half sum and the three remainders

mainders continually, and the ſquare root of the laſt product will be the area.

EXAMPLE I.

Required the area of a triangle, its three ſides being 20, 30, 40 Scots chains.

```
       30     45  45  45
       20     30  20  40
       40     --  --  --
     ----     15  25   5
    . 2)90
     ----
Half ſum 45
       15
     ----
      225                    Sq. Chains.
      45                   10)290,4738
     ----                  -----------
      675                     29,04738
      25                             4
     ----                  -----------
     3375                        .18952
    1350                             40
     ----                  -----------
    16875                       7.58080
        5                            36
     ----                  -----------
    84375 (290,4738              348480
    4                           174240
     ----                  -----------
    443                        20,90880
    441
    ------
 5804)27500
      23216
    --------
 58087)428400
       406609
    ---------
 580942179100, &c
```

So the area is 29 ac. 0 ro. 7 falls 21 ells nearly.

METHOD

METHOD II. *By Logarithms.*

RULE.

Add the logarithms of the three remainders and half ſum together, and half their ſum will be the logarithm of the area.

40	45	45	45
30	40	30	20
20	—	—	—
——	5	15	25
2)90			

Half ſum 45 = 1.65321

Rem. $\begin{cases} 5 = 0.69897 \\ 15 = 1.17609 \\ 25 = 1.39794 \end{cases}$

2)4,92621 ——— 290,4738 = 2.463105

10)290,4738 ——— 29 ac. 0 ro. 7 f. 21 ells.

METHOD III.

AC : AB + BC : : AB—BC : AD—DC.

That is, 40 : 50 : : 10 : 12.5 diff. ſeg. baſe.

To half baſe	20	
Add half diff.	6.25	
The greater ſeg.	26.25	AD
From half baſe	20	
Subtr. half diff.	6.25	
The leſſ. ſeg.	13.75	DC

Now to find BD the perpen.

$AB^2 - AD^2 = BD^2$, or

$BC^2 - DC^2 = BD^2$

$AB^2 = 900$

$AD^2 = 689.0625$

$BD = 210.9375$

$BD = \sqrt{210.9375} = 14.52369$ chains.

20 half baſe.

290.47380

Anſ. 29 *ac.* 0 *r.* 7 *falls* 21 *ells.*

METHOD

METHOD IV.

To find ang. A.			To find the area.		
As AD =	26,25	1,41913	As radius -	90	10,00000
is to rad.	90°	10,00000	is to sine 28° 57′		9,68489
So is AB -	30	1,47712	So is AB × AC	600	2,77815
To sec.ang. A 28° 57′		10,05799	To area 2) 290,4738 =		2.46304

Which is 29 ac. 0 r. 7 falls, 21 ells, as before.

From these four different varieties, it appears, that the logarithmic operation is the easiest. It were to be wished, that all land surveyors would take the trouble of computing their measurements by logarithms; then would they agree in their calculations, and depend less upon the accuracy of their scales.

Ex. 2. Required the area of a triangle, whose three sides are 500, 300, and 400 links. *Ans.* 2 *roods*, 16 *falls.*

Ex. 3. Required the area of a triangle, whose sides are 80, 60, 100, feet. *Ans.* 2400 *feet.*

Ex 4. How many square yards are in a triangular court, whose three sides are 36, 24, and 30 feet? *Ans.* 39 *yards*, 6.17 *feet.*

Ex. 5. How many square yards are in a triangle, whose three sides are 63, 123,5 and 148 yards? *Ans.* $4168\frac{1}{4}$ *yards.*

Ex. 6. How many square yards are in a triangle, whose sides are 39, 42, and 45 feet? *Ans.* 84 *yards.*

Ex 7. Required the area of a triangle, whose sides are 90, 84 and 78 yards. *Ans.* 3024 *yards.*

PRO-

PROBLEM VII. *Plate 6. fig. 75.*

Two ſides of a right-angled triangle being given, to find the other ſide.

RULE.

To find the hypothenuſe, add the ſquare of both the legs, and the ſquare root of the ſum is the hypothenuſe.

To find one of the legs, ſubtract the ſquare of the given leg from the ſquare of the hypothenuſe, and the ſquare root of the remainder is the leg required.

EXAMPLE I.

The hypothenuſe is 60, and the baſe AC 45; required the perpendicular.

45	60
45	60
225	3600 = AB^2
180	2025 AC^2
2025	1575(39.7 BC
	9
	69)675
	621
	787)5400
	5409

Ex. 2. Required the length of a ladder, to reach the top of a tower 56 feet high, the foot of the ladder being 48 feet from the wall. *Anſ. 73 feet 9.672 inches.*

Ex. 3. The hypothenuſe is 600, and one of the legs 360: Required the other leg. *Anſ.* 480.

Ex. 4. The legs of a right-angled triangle are 64, and 48: Required the hypothenuſe. *Anſ.* 80.

Ex. 5. The hypothenuſe of a right-angled triangle is 100, and one of the legs 80: Required the other leg. *Anſ.* 60.

PROBLEM VIII. *Plate 6. fig. 76.*

To find the area of a trapezoid.

RULE.

Multiply one half of the ſum of the parallel ſides by the perpendicular diſtance between them, and the product will be the area.

EXAMPLE I.

Required the area of a trapezoid, whoſe parallel ſides are 15, $19\frac{1}{2}$ chains, and their perpendicular diſtance 14 chains.

$$
\begin{array}{r}
\text{BC } 15 \\
\text{AD } 19.5 \\
\hline
2)34.5 \\
\hline
17.25 \\
14 \\
\hline
6900 \\
1725 \\
\hline
10)241.50 \\
\hline
24.15
\end{array}
$$

Anſ. 24 *ac.* 0 *r.* 24 *falls.*

Ex

Ex. 2. Required the area of trapezoid, whofe fides are 12, $18\frac{1}{3}$ feet, and the perpendicular diftance between 7 feet.

Anf. 106 *fquare feet* 2 *inches.*

Ex. 3. Required the area of a trapezoid, the parallel fides being 180 and 200 yards, and their perpendicular diftance 100 yards. *Anf.* 19000 *fquare yards.*

Ex. 4. How many fquare yards are in a trapezoid, whofe parallel fides are 90 and 100 feet, and breadth 50 feet?

Anf. 527 *yards* 7 *feet.*

Ex. 5. Required the area of a trapezoid, whofe parallel fides are 3, 4 feet, and perpendicular breadth 3 feet.

Anf. $10\frac{1}{2}$ *feet.*

Ex. 6. How many fquare feet are in a plank, 13 inches broad at one end, and 15 at the other, the length being 16 feet 5 inches? *Anf.* 19 *feet* 1 *inch* 10 *parts.*

Ex. 2. Required the expence of caufewaying a bridge 150 feet long and 30 broad, at 1s. 6d. *per* fquare yard.

Anf. 37*l.* 10*s.*

PROBLEM IX. *Plate 6. fig. 77.*

To find the area of a trapezium.

RULE.

Refolve the trapezium into triangles; compute the area of each of the triangles feparately, and the fum will be the area of the trapezium.

EXAMPLE I.

Required the area of a trapezium ABCD, the diagonal AC 60, BF 50, and DE 40 feet.

50
40
———
90
60
———
2)5400
———
2700 *Ans.*

Ex. 2. In the trapezium ABCD, AB is 64, BC 46, CD 60, DA 66, and the diagonal AC 72, English chains.

To find ABC.				To find ACD.			
64	91	91	91	72	99	99	99
46	64	46	72	60	72	60	66
72	—	—	—	66	—	—	—
———	27	45	19	———	27	39	33
2)182				2)198			

To find ABC.	To find ACD.
91 = 1.95904	99 = 1.99563
27 = 1.43136	27 = 1.43136
45 = 1.65321	39 = 1.59106
19 = 1.27875	33 = 1.51851
2)6.32236	2)6.53656
1449.5 = 3.16118	1855 = 3.26828

BAC 1449.5
ACD= 1855.
———
10)3304.5
———
330.45
4
———
1.80
40
———
32.00

Ans. 330 *ac.* 1 *rood* 32 *perches*

E x. 3

Ex. 3. Required the area of the trapezium ABCD, whose diagonal AC is 20, and perpendiculars BE and DE, 8 and 10 Scots chains. *Ans.* 18 *acres.*

Ex. 4. In the trapezium ABCD, the sides AB is 45, BC 39, CD 42, DA 36, and the diagonal AC 48: Required the area. *Ans.* 1552, 7223.

Ex. 5. Required the area of the trapezium ABCD, whereof the side AB is 10.25, BC 35, CD 50, DA 30, and the diagonal AC 40 chains. *Ans.* 76 *acres* 2 *roods* 19 *falls* 6 *ells.*

Ex. 6. How many square yards paving are in a trapezium, whose diagonal is 20, and perpendiculars $10\frac{4}{5}$ and $6\frac{1}{2}$ feet? *Ans.* 19 *yards* 2 *feet.*

Ex. 7. How many acres are in a field ABCD, of which the side AB is 8000, AD 6000, and AC the diagonal 9560 links of the Scots chain: Also the angles * BAC, CAD are each of them 30°? *Ans.* 334 *acres* 2 *roods* 16 *falls.*

PROBLEM X. *Plate 6. fig. 77.*

To find the area of a trapezium, its two diagonals and the included angle being given.

RULE.

As radius,
Is to the sine of the included angle:
So is ½ product of the diagonals,
to the area.

EXAMPLE.

---

* It will be worth the learner's while to observe, that when one of the angles of a right angled triangle is 30°, the leg opposite to it will be exactly one half of the hypothenuse. Hence the perpendiculars BF and DE are 4000, 3000 the halves of the sides AB, AD.

EXAMPLE I.

Required the area of a trapezium, whoſe diagonals are 100, 80 feet, and the included angle 60°.

As radius 90°	-	10.00000
is to ſine 60°	-	9.93753
So is $\frac{1}{2}$ prod. 4000	-	3.60206
To the area 3464 =	-	3.53959

Ex. 2. Required the area of a trapezium, whoſe diagonals are 120, and 140 yards, and the included angle 30°.

Anſ. 4200 *ſquare yards.*

Ex. 3. What is the area of a trapezium, of which the diagonals are 80 and 60 Scots chains, and the included angle 60°?

Anſ. 207 *acres* 3 *roods* 8 *falls.*

If the trapezium be inſcribed in a circle, its area may be found by the following rule.

Add all the four ſides together; from half their ſum ſubtract the ſides ſeverally; then multiply the remainders continually into each other, and the ſquare root of the laſt product will be the area.

EXAMPLE.

Required the area of a trapezium, whoſe ſides are 12, 13, 14, 15.

12	27	27	27	27
13	12	13	14	15
14	—	—	—	—
15	15	14	13	12
2)54				
27				

$15 \times 14 \times 13 \times 12 = 32760$ and $\sqrt{32760} = 180,997$ *Anſ.*

PROBLEM

PROBLEM XI.

To find the area of an irregular polygon.

RULE.

Resolve the polygon into triangles by diagonals; find the area of each triangle separately, and their sum will be the area of the whole polygon.

EXAMPLE I.

Required the area of the following figure, ABCDEF, whose perpendiculars and diagonals are given. *Fig.* 78. *plate* 6.

$$\left.\begin{array}{l} AC = 1050 \\ Bb = 320 \\ Ff = 420 \\ FD = 980 \\ Cc = 600 \\ Ee = 200 \end{array}\right\} \text{links.}$$

To find the area of ABC.

```
  1050 AC
   320 Bb
 ------
  21000
 3150
 ------
2)336000
 ------
168000 in square links.
```

To find ACE.

```
  1050 AC
   420 Ff
 ------
  21000
 4200
 ------
2)441000
 ------
220500 square links.
```

To find CED.

```
  980 FD
  600 Cc
 ------
2)588000
 ------
294000 sq. links.
```

To find the area of FDE.

```
  980 FD
  200 Ee
 ------
2)196000
 ------
98000 sq. links
```

The

$$\text{The area of}\left\{\begin{array}{l}ABC = 168000\\ ACF = 220500\\ CED = 294000\\ FDE = 98000\end{array}\right.$$

$$\begin{array}{r}\overline{7.80500}\\ 4\\ \hline 3.22000\\ 40\\ \hline 8,80000\\ 30\frac{1}{4}\\ \hline 24.00000\\ 20000\\ \hline 24.20000\end{array}$$

Anſ. 7 *acres* 3 *roods* 8 *perches* $24\frac{1}{5}$ *yards.*

The above example may be rendered more ſimple, by reducing the figure to trapezias.

$$\begin{array}{r}Bb = 320\\ Ff = 420\\ \hline 2)740\\ \hline 370\end{array} \quad\Bigg|\quad \begin{array}{r}1050\\ 370\\ \hline 73500\\ 3150\\ \hline 388500\end{array}\ \text{the trap. ABCF.}$$

$$\begin{array}{r}Cc\ 600\\ Ee\ 200\\ \hline 2)800\\ \hline 400\end{array} \quad\Bigg|\quad \begin{array}{r}980\\ 400\\ \hline 392000\end{array} = \text{FCDE.}$$

$$\begin{array}{r}388500\\ 392000\\ \hline 7.80500\end{array}$$

Anſ. 7 *ac.* 3 *ro.* 8 *per.* $24\frac{1}{5}$ *yds.* 7.80500 as before.

Required

Required the area of the irregular figure ABCDEF, of which the ſide AB is 40, AC 50, AD 55, AE 69, AF 36 Scots chains; and the angles are as follow. *Plate* 6. *fig*. 80.

BAC 40°.
CAD 43°.
DAE 40° 30′.
EAF 48° 20′.

METHOD I.

To find the area of BAC.

As rad. 90	-	10.00000
is to ſine 40°	-	9.80807
So is $\frac{BA \times AC}{2}$ 1000		3.00000
To area 642.8 =	-	2.80807

To find CAD.

As rad. 90	-	10.00000
is to ſine 43°	-	9,83378
So is $\frac{AC \times AD}{2}$ 1375=		3,13830
To area 937,7 =	-	2,97208

To find DAC.

As rad. 90	-	10.00000
is to ſine 40° 30″		9,81254
So is $\frac{DA \times AE}{2}$ 1898		3.27830
To area 1232	- -	3.09084

To find EAF.

As rad. 90	- -	10.00000
is to ſine 48° 20′	-	9,87334
So is $\frac{EA \times AF}{2}$ 1242		3,09412
To area 927,8	-	2,96746

The area of ABC	-	642.8
of CAD	- -	937.7
of DAE	-	1232.0
of EAF	- -	927.8
		10)3740.3
		374.03

Anſ. 374.03 *acres.*

METHOD II. *By finding the perpendiculars.*

To find Bb.		To find Cc.	
As rad. 90° -	10.00000	As rad. = 90 = -	10.00000
is to AB 40 -	1,60206	is to AC = 50 =	1,69897
So is sine 40° -	9,80807	So is sine 43° -	9,83378
To Bb 25.71 - -	1,41013	To Cc 34.1 = -	1.53275

To find Dd.		To find Ff.	
As rad. 90 = -	10,00000	As rad. 90° -	10.00000
is to AD = 55 =	1,74036	is to AF = 36 =	1.55630
So is sine 40° 30′ -	9,81254	So is sine 48° 20′ -	9,87334
To Dd 35,72 -	1,55290	To Ff 26.89 = -	1,42964

Now, to find the area by bases and perpendiculars.

25.71 × 50 = 1285.5
34,1 × 55 = 1875.5
35.72 × 69 = 2464.68
26.89 × 69 = 1855.41

2)7481,09 twice the area.

10)3740.545

374.0545

Ans. 374.0545 *acres.*

Ex. 3. Required the area of the following polygon, whereof the sides are as follow, viz. AF 31.5, ,FE 33.5 ED 25.5, DC 38.5, CB 43.5, BA 34.5, AE 60.5, AD 81.7, BD 74.3 English chains. *Ans.* 277 *acres* 3 *ro.* 12 *perches.*

PRO-

PROBLEM XII.

To find the angles of any regular polygon.

By cor. 1ſt, I. 32. *Euclid.* All the anterior angles of any rectilineal figure, together with four right angles, are equal to twice as many right angles as the figure has ſides. Hence the following rule.

RULE.

From double the number of ſides ſubtract 4, and the remainder is the number of right angles contained by all the ſides of the polygon. Multiply the remainder by 90, and divide the product by the number of ſides, the quot gives the degrees in any of the angles.

EXAMPLE I.

Required the angle of a pentagon.

$$\begin{array}{rl} 5 & \text{No. sides.} \\ 2 & \\ \hline 10 & \\ 4 & \\ \hline 6 & \text{rem.} \\ 90 & \\ \hline 5)540 & \\ \hline 108 & \text{degrees in each angle.} \end{array}$$

Ex. 2. Required the angle of a heptagon. *Anſ.* $128° \, 34\frac{2}{7}'$
Ex. 3. ———————— of a hexagon. *Anſ.* 120.
Ex. 4. ———————— of a decagon. *Anſ.* 144.
Ex. 5. ———————— of an octagon. *Anſ.* 135.

PROBLEM XIII.

To find the area of a regular polygon.

RULE.

Find the area of a triangle, conſtructed on one of the ſides of the polygon, and whoſe vertex is in the centre; then multiply this area by the number of ſides, and the product will be the area of the polygon. Or,

Multiply the perimeter by the radius of the inſcribed circle, and half the product is the area of the polygon.

EXAMPLE I.

Required the area of a pentagon, whoſe ſide is 10.

1ſt, To find the angle.

	5
	2
	10
	4
	6
	90
	5)540
Angle	108

To find the rad. of the inſcribed.

As rad. 90 =	-	10,00000
is to EG 5	-	0,69897
So is tang. 54 =	-	10,13874
To FG 6,882 =	-	0,83771

The perpen.	6,882
$\frac{1}{2}$ the baſe -	5
	34.410
No. ſides -	5
Area -	172.050

Ex. 2. Required the area of a hexagon, whoſe ſide is 30.

Anſ. 2338.2

Ex. 3.

Ex. 3. Required the area of a heptagon, whose side is 1.

Ans. 3.633912.

Ex. 4. Required the radius of the inscribed circle, area, internal angles, and angles at the centre, for the following polygons: *viz.* The trigon, tetragon, pentagon, hexagon, heptagon, octagon, enneagon, decagon, undecagon, and dodecagon, the side of each being 1,

Names.	No of Sides.	Rad. inf. circ.	Areas.	Int. Ang.		Ang. t cent.	
Trigon	3	0.288674	0.433013	60°	0′	120°	0′
Tetragon	4	0.5	1.	90	0	90	0
Pentagon	5	0.688190	1.720475	108	0	72	0
Hexagon	6	0.866024	2.598072	120	0	60	0
Heptagon	7	1.038260	3.633912	128	$34\frac{2}{7}$	51	$25\frac{5}{7}$
Octagon	8	1.207106	4.828427	135	0	45	0
Enneagon	9	1.373738	6.181824	140	0	40	0
Decagon	10	1.538841	7.694205	144	0	36	0
Undecagon	11	1.702840	9.365620	147	$16\frac{4}{11}$	32	$43\frac{7}{11}$
Dodecagon	12	1.866024	11.196144	150	0	30	0

Regular polygons of the like number of sides are similar, and similar surfaces are to one another in the duplicate ratio of their homologous sides; but the sides of the polygons in the foregoing table are each of them 1; therefore, as the square of 1 is to the tabular area, so is the square of the side of any given polygon to the area required: Hence the following

RULE.

Multiply the square of the side of any given polygon into the tabular area of the like polygon, and the product will be the area of the polygon.

Ex. 5. Required the area of a pentagon whose side is 20 feet.

20	Pentagonal tablular area	1.720475
20	Sq. of the given Pol. side	400
400		688.190000 *sq. feet.*

Ex. 6. What is the area of a hexagon, whoſe ſide is 50 yards? *Anſ.* 6495.18 *ſquare yards.*

Ex. 7. Required the area of a hexagon, whoſe ſide is 20 feet. *Anſ.* 1039.2288

Ex. 8. Required the area of a pentagon, whoſe ſide is 4. *Anſ.* 27.5276.

Ex. 9. How many ſquare yards are in a decagon, whoſe ſide is 12 feet? *Anſ.* 123 *yards* 11 *inches* $6\frac{1}{2}$ *pts.*

PROBLEM XIV.

The area of a polygon being given to find the ſide.

RULE.

Divide the area of the given polygon by the tabular area of the like polygon, and the ſquare root of the quotient will be the ſide of the given polygon.

EXAMPLE I.

Required the ſide of a pentagon, whoſe area is 61.9371.

1.720475)61.937100(36
51 61425
————
10322850
10322850

$\sqrt{36} = 6$ the ſide of poly.

Required the ſide of a decagon, whoſe area is 3077.682 ſquare yards. *Anſ.* 20 *yards.*

Ex. 3 What is the ſide of a trigon, whoſe area is 173.2025 ſquare yards? *Anſ.* 20.

Ex. 4. Required the ſide of a pentagon, whoſe area is 27.5276 ſquare yards. *Anſ.* 4 *yards.*

OF

OF THE CIRCLE.

A CIRCLE may be considered as a regular polygon of an infinite number of sides.

The area of a circle is equal to the area of a triangle, whose base is equal to the circumference, and height the radius.

The proportion of the diameter to the circumference may be found thus: Describe a polygon of a great number of sides, about a circle of a known diameter, and inscribe another of the like number of sides; find the perimeter of each, and the square root of their product will be the circumference. Hence the circumference of a circle whose diameter is 1, is 3.141592653 58979323846264338327950288, of which number 3.1416 may be used, it being sufficiently accurate for most practical purposes.

PROBLEM XIV. *Plate 6. fig.* 82.

The diameter being given to find the circumference

RULE.

Multiply 3.1416 by the given diameter; the product will be the circumference.

EXAMPLE I.

Required the circumference of a circle, whose diameter is 14 feet.

3.1416

$$\begin{array}{r} 3.1416 \\ 14 \\ \hline 125664 \\ 31416 \\ \hline 43.9824 \\ 12 \\ \hline 11.7888 \\ 12 \\ \hline 9.4656 \end{array}$$

Anſ. 43 *feet* 11 *inches* $9\frac{1}{2}$ *parts nearly.*

Ex. 2. The diſtance between the cogs of a mill-wheel, and the centre of the axle, is 5 feet; how many cogs, at 4 inches pitch, will the wheel admit of? *Anſ.* 94.248 *cogs.*

Ex. 3. The ſame thing being given, required the true pitch for 61 cogs. *Anſ.* 6 *inches* 2″ 2‴ *nearly.*

Ex. 4. Suppoſing the earth to be an exact ſphere, required its circumference, the diameter being 7958 miles.

Anſ. 25000.8528 *miles.*

Ex. 5. Required the circumference of a circle, whoſe radius is 15 feet. *Anſ.* $94\frac{248}{1000}$ *feet.*

Ex 6. What is the circumference of a circle, whoſe diameter is $48\frac{1}{2}$ inches? *Anſ.* 12 *feet* 8 *inches* $4\frac{1}{2}$ *parts.*

PROBLEM XIII.

The circumference of a circle being given, to find the diameter.

RULE.

Divide the circumference by 3.1416, and the quotient will be the diameter.

EXAMPLE I.

Required the diameter of a circle whoſe circumference is 9448.

3.1416

3.1416)94.2480(30 feet
94248

Ex. 2. What is the diameter of a circle whose circumference is 80 feet? *Ans.* 25.464 *feet.*

Ex. 3. Required the diameter of a circle, whose circumference is 1000. *Ans.* 318309.

Ex. 4. Required the diameter of a circle, whose circumference is 64 feet. *Ans.* 20.368.

Ex. 5. Required the diameter of a milstone, whose circumference is 22 feet. *Ans.* 7.0028.

PROBLEM XIV.

The diameter and circumference of a circle being given, to find the area.

RULE.

Multiply half the radius into the circumference, and the product is the area.

EXAMPLE I.

Required the area of a circle whose diameter is 1.

3.1416 circum.
.25 half the rad.

157080
62832

.785400

CIRCLES are ſimilar figures; and ſimilar ſurfaces are as the ſquares of their correſponding ſides; therefore, as 1^2 is to .7854 : (the area of a circle whoſe diameter is 1) ſo is the ſquare of any given diameter to the area required. Hence,

RULE II.

Multiply the ſquare of the diameter by the common number .7854, and the product is the area.

EXAMPLE II.

Required the area of a circle, whoſe diameter is 12 feet.
Anſ. 113.0976 *ſquare feet.*

Ex. 3. How many ſquare feet are in a table, whoſe diameter is 6 feet? *Anſ.* 28.2744 *feet.*

Ex. 4. What is the area of a circular court, whoſe diameter is 24 yards? *Anſ.* 452.3904 *ſquare yards.*

Ex. 5. How many ſquare miles are in a great circle of the earth, its diameter being $7957\frac{3}{4}$ miles?
Anſ. 49736071.58880750.

Ex. 6. What is the area of a circle, whoſe diameter is $3\frac{3}{4}$ feet? *Anſ.* 11.0446875 *ſquare feet.*

Ex. 7. Required the area of a circular garden, whoſe radius is 160 links of the Engliſh chain. *Anſ.* 3 *roods* 8 *poles* 20 *yds.*

Ex. 8. What is the area of a circle, whoſe diameter is 2 feet?
Anſ. 3.1416.

Ex. 9. Required the area of the ring between the circumferences of two concentric circles, their diameters being 20 and 15 inches. *Anſ.* 137.445 *ſq. inches.*

PROBLEM

PROBLEM XV.

The circumference of a circle being given, to find the area.

RULE.

Multiply the square of the circumference by .0795775, and the product will be the area.

EXAMPLE I.

Required the area of a circle whose circumference is 1.

$$\begin{array}{r} 1^2=1 \\ .0795775 \\ 1 \\ \hline .0795775 \end{array}$$

Ex. 2. What is the area of a circle, whose circumference is 5 feet? *Ans.* 1.9894375.

Ex. 3. Required the area of a circle whose circumference is 100 yards. *Ans.* 795.775.

Ex. 4. The expence of inclosing a circular court at 8s. *per* yard, amounted to 320l; required the expence of paving it, at 6d. *per* square yard. *Ans.* 1273l. 5s. 7d.

Ex. 5. How many square feet are in a circular table, whose circumference is 25.1328 feet? *Ans.* 50.2656 *sq. feet.*

Ex. 6. Required the area of a circle, whose circumference is 31.416. *Ans.* 78.54.

PROBLEM XVI.

The area of a circle being given, to find the circumference.

RULE I.

Divide the area by .0795775, and the square root of the quotient will give the circumference.

RULE

RULE 2. Divide the area by .7854, the square root of the quotient will give the diameter; then find the circumference by prob. 12.

EXAMPLE I.

The area of a circle being 5026.56 square feet; required the circumference.

By RULE I.

```
.0795775)5026.5600000(63165.3931 sq. circum.]
         4774650
         -------
          2519100
          2387325
          -------
           1317750
            795775
           -------
            5219750
            4774650
            -------
             4451000
             3978875
             -------
             4721250, &c.
```

and $\sqrt{63165.3931} = 251.327$ feet. *Ans.*

By RULE II.

```
.7854)5026.5600(64000 sq. dia.
      47124
      -----
       31416
       31416
       -----
          00
```

and $\sqrt{6400} = 80$ diameter.

```
    3.1416
        80
  --------
  251.3280 feet.  Ans.
```

Ex. 2.

Ex. 2. Required the circumference of a circle, whose area is 795·775. *Ans.* 100.

Ex. 3. What is the circumference, when the area is 452.3904 square yards? *Ans.* 75·3984 *yards*.

PROBLEM XVII. *plate* 6. *fig.* 81.

To find the chord of any arch of a circle, the diameter and versed sine, or height of the arch being given.

Because CD and AB cut each other within a circle, the rectangle contained by the segments of the one is equal to the rectangle contained by the segments of the other. *Euclid* III. 35. That is, $BE \times AE = CE \times ED$; but CE is equal to ED *Euclid* 3. 3. therefore,

$BE \times EA = CE^2$ and $\sqrt{BE \times EA} = CE$, and $2CE = CD$ the chord.

Or, CD may be found thus.

Since BA and AE are given, CG and GE are also given; wherefore $\sqrt{CG^2 - GE^2} = CE$ and twice $CE = CD$. Hence the following rules:

RULE I.

From the diameter subtract the versed sine; then multiply the remainder by the versed sine, and twice the square root of the product will be the chord of the arch.

RULE II.

From the square of the radius subtract the square of the difference between it and the versed sine, and twice the square root of the remainder will give the chord of the arch.

EXAMPLE I.

Required the length of the chord of an arch, whose height is 8 and diameter 40 feet.

By

By Rule 1.

From diam.	40
ſubtr. ver. ſine	8
Rem.	32
V. Sine	8

256(16 half the ch.
1 2

26)156 32 the chord.
156

By Rule 2.

Rad.	20
V. Sine	8
diff.	12

$20^2 = 400$
$12^2 = 144$

256

$\sqrt{256} = 16$ half the chord.
2

32 the chord as before

Ex. 2. What is the chord, when the diameter is 60, and verſed ſine 16 inches? *Anſ.* 53.06 *inches.*

Ex. 3. Required the chord of an arch, when the diameter is 50 and verſed ſine 14 feet. *Anſ.* 44.8998 *feet.*

Ex. 4. What is the chord, when the diameter is 40, and height of the arch 4? *Anſ.* 24.

Ex. 5. When the radius is 68, and the verſed ſine 8, required the chord. *Anſ.* 64.

Ex. 6. What is the chord, when the verſed ſine is 14, and the diameter 70 inches? *Anſ.* 56 *inches.*

PROBLEM XVIII.

The chord and verſed ſine of an arch being given, to find the diameter of the circle of which the arch is a part.

RULE.

Divide the ſquare of half the chord of the arch by the verſed ſine, to the quotient add the verſed ſine, and the ſum will be the diameter.

EXAMPLE

EXAMPLE I.

What is the diameter, when the chord is 48, and the versed sine 8 inches?

$$\begin{array}{ll} 2)48 & 24^2 = 576 \text{ sq. } \tfrac{1}{2} \text{ the chord.} \\ \text{Half chord } 24 & 8)576 \\ & \overline{72} \text{ quot.} \\ & 8 \text{ versed sine.} \\ \text{the diameter is} & \overline{80} \text{ inches.} \end{array}$$

Ex. 2. Required the diameter, when the chord is 36, and height 3. *Ans.* 111 *inches.*

Ex. 3. What is the radius, when the versed sine is 8, and the chord 60? *Ans.* $60\frac{1}{4}$.

Ex. 4. Required the diameter, when the chord is 30, and the versed sine 4 feet. *Ans.* 60.25 *feet.*

Ex. 5. When the height of the arch is 2 feet, and the chord 6 feet, required the diameter. *Ans.* 6.5 feet.

PROBLEM XIX. *plate* 6. *fig.* 81.

To find the chord of half the arch, any two of the following terms being given; namely, the versed sine, chord, and diameter.

When the versed sine and chord are given.

Because $\frac{AC^2}{4} + DB^2 = AB^2$ therefore $\sqrt{\frac{AC^2}{4} + DB^2} = AB$ hence

RULE I.

To one fourth of the square of the chord of the arch add the square of the versed sine, and the square root of their sum will be the chord of half the arch.

When

When the verſed ſine and diameter are given.

Becauſe $AD^2 = BD \times DE$, and $AD^2 + BD^2 = AB^2$, therefore, $BD \times DE + BD^2 = AB^2$ and $\sqrt{BD \times DE + BD^2} = AB$ the chord.

RULE II.

From the diameter ſubtract the verſed ſine, multiply the remainder by the verſed ſine, and to their product add the ſquare of the verſed ſine, and the ſquare root of the ſum will be the chord of half the arch.

When the chord and diameter are given.

$\sqrt{AG^2 - AD^2} = DG$ and $GB - DG = DB$ the verſed ſine.

Then proceed as in RULE I.

EXAMPLE I.

Required the chord of half the arch, when the diameter is 68, and the verſed ſine 4 feet.

```
       68 diameter
        4 verſed ſine
       --
       64
        4 ver. ſine
      ---
      256
       16 ſq. verſed ſine
      ---
      272(16.49
      1..
      ---
   26)172
      156
      ----
  324)1620
      1296
      -----
 3289)30400
      29601
      -----
        799
```

Ex. 2. Required the chord of half the arch, when the chord of the arch is 32, and verſed ſine 4 feet? *Anſ.* 16.49.

Ex. 3. Required the chord of half the arch, of which the diameter is 20, and verſed ſine 8 inches. *Anſ.* 12.65 *inches.*

Ex. 4. Required the chord of half the arch, the chord of the arch being 60, and height 8. *Anſ.* 31.048.

Ex. 5. What is the chord of half the arch, when the diameter is 26, and the verſed ſine 6? *Anſ.* 12.49.

Ex. 6. What is the chord of half the arch, when the verſed ſine is 140 and chord 360 links? *Anſ.* 228.03 *links.*

Ex. 7. When the chord is 31, and verſed ſine $11\frac{1}{2}$; Required the chord of half the arch. *Anſ.* 19.3.

Ex. 8. Required the chord of half the arch, when the diameter is 80, and chord 60 feet. *Anſ.* 32.912.

PROBLEM XX. *Plate 6. fig.* 81.

The chord of half the arch, and the chord of the whole arch being given, to find the height of the arch, alſo the diameter of the circle of which the arch is a part.

$\sqrt{AB^2 - AD^2} = BD$, or verſed ſine.

$\frac{AD^2}{BD} = DE$ and $BD + DE = BE$ the diameter.

RULE.

From the ſquare of the chord of half the arch, ſubtract the ſquare of half the chord of the whole arch, and the ſquare root of the remainder will give the verſed ſine. Then divide the ſquare of half the chord of the arch by the verſed ſine; and to the quotient add the verſed ſine, the ſum will be the diameter.

EXAMPLE I.

Required the diameter when the chord of half the arch is 10, and the chord of the arch 14 inches.

From $10^2 = 100$
Take $7^2 = 49$

51(7.141 versed sine
49

141) 200
141

1424)5900
5696

14281)20400
14281

6119 rem.

7,141)49 sq. ½ chord.

6.861 seg. diamet.
7.141 versed.

14.002 diameter.

Note, When the versed sine is greater than the other segment of the diameter, the arch is greater than a semicircle.

Ex. 2. Required the diameter, when the chord of the arch is 40, and the chord of half the arch 30 feet. *Ans.* 40.71.

Ex. 3. What is the diameter, when the chord is 100, and the chord of half the arch 54? *Ans.* 141.13.

PROBLEM XXI. *Plate 6. fig.* 83.

To find the length of any arch of a circle, any two of these being given,

given, viz. *the diameter or radius, versed sine, chord, or chord of half the arch.*

RULE I.

Find any two sides of the right-angled triangle ADG; then, by trigonometry, find the angle AGB, and twice that angle will measure the arch ABC.

Then say, as 360° is to the number of degrees in the arch ABC, so is the whole circumference of the given circle to the length of the arch.

Rule 2. From 8 times the chord of half the arch, subtract the chord of the whole arch, and $\frac{1}{3}$ the remainder will give the length of the arch nearly.

EXAMPLE I.

Required the length of the arch ABC, its height is 8 feet, and chord 40.

Sq. of $\frac{1}{2}$ chord $= 20^2 = 400$ and $\dfrac{400}{\text{Vers. s. } 8} = 50 = \text{DE}$

$$\begin{array}{r} 50 = \text{DE.} \\ 8 = \text{BD} \\ \hline 2)58 = \text{BE} \\ \hline 29 = \text{AG} \\ 20 = \text{AD.} \end{array}$$

By Rule I.

To find the angle AGD.

As AG = 29 =	-	1.46240
is to rad. 90. =	-	10.00000
So is AD = 20 =	-	1.30103
To sine AGD	43° 36′	9.83863
	2	
Ang. AGC = -	87 12	

3.1416 × 58 = 182.2128 circum.

As 360° : 87.2 : : 182.2128 :

As 45 : 10.9 : : 182.2128 : 44.135989. *Ans.*

By RULE II.

$20^2 = 400 = AD^2$
$8^2 = 64 = BD^2$

464(21.54065 the chord of $\frac{1}{2}$ arch.
4

41)64
41

425)2300
2125

4304)17500
17216

430806)2840000
2584836

4308125)25516400
21540625

3975775

21.54065
8

172.32520
40

3)132.32520

44.1084 *Anſ.*

Ex. 2. Required the height, alſo the diameter of either frigid zone, the diameter of the earth being 7958 miles, and the polar circles 23° 28′ diſtant from their poles.

PS = 7958 diam.
AC = 3979 ſemidiam.
Ang. ACD = 23° 28′

To find AN, the diameter of the zone's baſe.

As rad. 90	-	10.00000
is to AC 3979	-	3.59977
So is ſine ang. C 23° 28′		9.60012
To AD 1574.5	-	3.19989
2		

AN = 3149.0 = FK.

To find PD the height of the zone.

As rad. 90	- -	10.00000
is to AC 3979	-	3.59977
So is co-sine 23° 28′ =	-	9.96251
To DC = 3650 =	-	3.56228
CP—DC=PD, that is		3979
		3650
The height of zone		329

N. B. 1. Here it may be useful to observe, that, because the triangles ADC and TFC are equal and similar, CA : DA : : TC : FC, but AC is equal to TC; therefore, AD=FC and 2AD=2FC, hence it is evident, that the height of the torrid zone is equal to the diameter of either frigid zone.

N. 2. In like manner it might be demonstrated, that the height of both temperate zones, together with the height of the torrid zone, are equal to the diameter of the greater base of either temperate zone, or to the diameter of the base of the torrid zone at the tropics.

PROBLEM XXII. *plate* 6. *fig.* 83.

To find the area of the sector of a circle.

RULE.

Multiply the length of the arch by half the radius, and the product is the area.

EXAMPLE I.

Required the area of the sector ABCG, AC the chord of the arch being 60, and BD the versed sine 8.

To find the radius.

$$\frac{AD^2}{BD} = \frac{900}{8} = 112.5 \text{ and } 112.5+8=120.5 \text{ the diameter.}$$

$60.25 = AG$ rad.

$52.25 = DG$

To

To find the number of degrees in the arch ABC.

As DG	$= 52.25$	$=$	1.71809
is to rad.	90°	-	10.00000
So is AD	$= 30$	- -	1.47712

to tang. AGD $29^\circ\ 52' = 9.75903$
2

$59\ 44 =$ arch ABC.

$360^\circ : 59^\circ : 44' :: 378.5862 : 62.8133.$

62.8133 length of the arch ABC.
60.25

3140665
1256266
37687980

2)3784.501325

1892.2506625 *Ans.*

METHOD II.

Find the length of the arch by prob. 21st.

$30^2 = 900 = AD^2$
$8^2 = 64 = BD^2$

964

$\sqrt{964} = 31.048$ chord of $\frac{1}{2}$ the arch.
8

248.384
60. chord of the arch.

188.384

62.794 length of the arch.

Then

By the RULE.

```
     62.794
      60.25
   --------
     313970
    125588
  3767640
  ---------
2)3783.33850
  ---------
  1891.66925
```

Ex. 2. What is the area of a sector, when the versed sine is 5, and chord of half the arch 10? *Ans.* 104.720 §.

Ex. 3. Required the area of the sector, when the chord o half the arch is 10, and the chord of the whole arch 16 feet.

Ans. 88.88 *feet.*

Ex. 4. Required the area of a sector of a circle, when the diameter of the circle is 60, and the length of the arch 60 yards. *Ans.* 900 *sq. yards.*

Ex. 5. Required the area of a sector, when the length of the arch is 156.28 feet, and the diameter 140 feet.

Ans. 5469.8 *sq. feet.*

Ex. 6. When the length of the arch is 54, and the radius of the circle 60, required the area. *Ans.* 1620.

PROBLEM XXIII. *Plate* 6. *fig.* 83.

To find the area of a segment of a circle.

RULE I.

Find the area of a sector, whose arch is the same with the segment, by the preceding problem.

Then

§ When the chord of half the arch is double the versed sine, four times the versed sine is equal to the diameter, and the arch 120°

Then find the area of a triangle, whoſe two ſides are the radii of the ſector, and baſe the chord of the arch. Subtract or add the area of the triangle, according as the ſegment is greater or leſs than a ſemicircle.

Rule 2. Multiply the chord of half the arch by $1\frac{1}{3}$, to the product add the chord of the whole arch, multiply this ſum by the verſed ſine, and $\frac{4}{10}$ of the product will be the area of the ſegment.

EXAMPLE I.

What is the area of the ſegment ABC, its chord being 60 and radius 50?

By Rule I.

$2500 = AE^2$
$900 = AD^2$

$1600 = DE^2$
$40 = DE$

To find the length of the arch ABC.

As AE=50	1.69897
to Rad.=90	10.00000
So is AD=30	1.47712
To ſine ang. AED 36° 52′	9.77815
2	
73.44	

360° : 73° 44′ : : 314.16 : 64.344 the length of the arch.

64.344 × 25 = 1608.6 area of the ſec. ABCE.

DE=20 × 60 = 1200 = area of tri. ACE

408.6 = area of ſeg. required.

By Rule II.

From 50=BE
Take 40=ED

Rem. 10=BD versed sine.

To 100=BD^2
Add 900=AD^2

1000=AB^2

And $\sqrt{1000}$=31.6228=AB

31.6228
$1\frac{1}{3}$

31.6228
10.54093

42.16373
60.00000 chord of $\frac{1}{2}$ arch.

102.16373
10 versed.

1021.6373
4

10)4086.5492

408.65492 *Ans.* as before.

Or thus:

Divide the versed sine by the diameter, find the quotient in the column of versed sines, and multiply the corresponding area by the square of the diameter for the area of the segment.

The example being the same as before, we have the versed sine equal 10, and diameter 100.

100)10.0(.1
100

In the column of versed sines find .1

And the corresponding area is	.040875
Sq. diameter, - -	10000
Area as before,	408.750000

Ex. 2. Required the area of the segment, when the arch is 90° and diameter 36 feet. *Ans.* 92.4696 *sq. feet.*

Ex. 3. What is the area of the segment of a circle, when the diameter is 25 and versed sine 9? *Ans.* 159.09.

Ex. 4. Required the area of a segment, whose chord is 32, the radius being 20. *Ans.* 178.9168.

Ex. 5. Required the area of a segment, its versed sine being $3\frac{1}{4}$, and diameter 50 yards. *Ans.* 54.1475 *sq. yards.*

PROBLEM XXIV.

To find the area of the cycloid.

Definitions.

1. If the circle ABGE roll on the straight line CD, so that all the points of the circumference be applied to it successively, the point x, that touches the line CD in c, by a motion thus compounded of a circular and rectilineal motion, will describe the curve line CBD, which is called the Cycloid.

2. The straight line CD is called the base.

3. The straight line AB, perpendicular to CD, and bisecting it, is called the axis, and is equal to the diameter of the generating circle.

4. The generating circle is that by whose revolution the curve line is described.

5. The point B is called the vertex.

Note. The base CD is equal to the circumference of the generating circle, and the cycloid CBD is quadruple of the diameter. *Vide* Sir Isaac Newton's Philosophical Discoveries.

RULE

RULE.

Multiply the area of the generating circle by 3, and the product is the area of the cycloid.

EXAMPLE I.

Required the area of the cycloid, when the diameter of the generating circle is 4 feet.

```
       4
       4
      --
      16
   .7854
   -----
   47124
   7854
  ------
 12.5664 area of the generating circle.
       3
 -------
 37.6992 sq. feet, area of the cycloid.
```

Ex. 2. Required the area of the cycloid, whose base is 25.1328. *Ans.* 150.79.

Ex. 3. Required the area of the cycloid, whose length is 400 feet. *Ans.* 23562 *sq. feet.*

PROBLEM XXV.

To find the sine and cosine of a very small arch, such as 1′.

A small arch such as 1′, may be considered nearly equal to its sine. Suppose, then, the radius of a circle to be 100000, in which case the circumference will be 628318.52; therefore $\frac{628318.52}{60 \times 360}$ will quote 29.08, the natural sine of 1′. Since the square of the hypothenuse of a right angled triangle is equal to the sum of the squares of the legs, therefore from the square of the

the radius subtract the square of the sine of any arch, and the square root of the remainder will be the cosine of that arch.

Thus $\sqrt{10000000000 - 845.64} = 99999.9$ the cosine of 1′

The versed sine x B may be found by subtracting the cosine from the radius.

PROBLEM XXVI.

The sine and cosine of any arch being given, to find the sine and cosine of its double.

RULE.

As the radius is to the cosine of any arch, so is twice the sine of that arch to the sine of its double.

EXAMPLE

Required the sine and cosine of two degrees, the sine of 1° being 1745, and cosine 99985.

Rad. Co-sine 1°

100000 : 99985 : : 3490 : 3489.47 natural sine of 2°

$3489.47^2 =$

And $\sqrt{}$

If three arches differ equally, the radius is to the cosine of the middle arch as twice the sine of the difference is to the difference of the sines of the greatest and least arches.

Ex. 2. Required the sine and cosine of 3°, the sine and cosine of 1° and 2° being given.

$$
\begin{array}{l}
100000 : 99939 :: 3490 \\
3490 \\
\hline
8994510 \\
399756 \\
299817 \\
\hline
3487.87110 = AL \text{ the diff. of the extreme arches.} \\
1745 \qquad\qquad \text{Sine of } 1^\circ \\
\hline
5232.87110 \quad \text{the sine of } 3^\circ = AF
\end{array}
$$

The cosine of which is $\sqrt{EA^2 - AF^2} = 99863$ the cosine of 3°

Ex. 3. The sine and cosine 2° and 3° being given, required the sine and cosine of 4°. *Ans.* {Sine 6976, Co-sine 99756

Ex. 4. Required the sine and cosine of 5°, the sine and cosine of 3° and 4° being given. *Ans* {Sine 8715 4844, Co-sine 99619

Ex. 5. The sine and cosine of 4° and 5° being given, required the sine and cosine of 6° *Ans.* {Sine 10452, Co-sine 99455

In like manner, the sine and cosine of every minute and degree of the quadrant may be found; but when the calculations are carried on the length of 60°, the sines of the remaining arches may be found by the following rule:

Take the sine of an arch as much below 60° as the arch whose sine is required is above 60°, to which add the sine of the number of degrees that the proposed arch exceeds 60°; the sum will be the sine required.

Ex. 6. What is the sine of 80°?

$$
\begin{array}{lr}
\text{The sine of } 60^\circ - 20^\circ = 40^\circ \text{ is} & 64279 \\
\text{The sine of } 80^\circ - 60^\circ = 20^\circ \text{ is} & 34202 \\
\hline
\text{The sine of } 80^\circ \quad - & 98481
\end{array}
$$

Ex.

			Answers.
Ex 7.	Required the fine of	70° 15′	94118
8.	of	67° 14′	92209
9.	of	79° 30′	98325
10.	of	89° 45′	99999

The verfed fine is found by fubtracting the cofine from the radius.

PROBLEM XXVII.

To find the tangent and cotangent of every minute and degree of the quadrant, the fines and cofines being given.

The tangent and cotangent of any arch may be found by either of the following proportions:

Becaufe the triangles CED and CBA are fimilar, CD : DE :: CA : AB, therefore $\frac{DE \times CA}{CD} = AB$, the tangent of the arch EA; that is, the rectangle contained by the fine and radius of any arch, is equal to the rectangle contained by the cofine and tangent of that arch. Hence,

RULE I.

To find the tangent, multiply the fine of any arch by the radius, and divide the product by the cofine, the quotient will be the tangent of that arch.

The cotangent of any arch may be found upon the fame principles: Thus, CL : LE : : CF : FK; therefore, $\frac{LE \times CF}{CL} = FK$; hence,

RULE 2. Divide the product of the cofine and radius of any arch by its fine, and the quotient will be the cotangent: or (which is the fame thing) fay, As the fine of any arch is to its co-fine: fo is the radius to the cotangent of that arch.

It

It is alſo obvious, that AB : AC : : AC : FK, therefore $\frac{AC^2}{AB}$=FK, that is to ſay, the radius is a mean proportional between the tangent and co-tangent of any arch. Hence, the co-tangent may be found by the following rule—

RULE 3. Divide the ſquare of the radius by the tangent of any arch, and the quotient will give the co-tangent of that arch.

EXAMPLE I.

Required the tangent and co-tangent of 60°, the co-ſine being 50000, and ſine 86603.

	Co-ſine.	Sine.	Rad.	Tang. of 60°.	
By Rule 1.	50000 :	86603 : .	100000 :	173206	*Anſ.*

By Rule 2. to find the cotangent.

Sine. Co-ſine.
86603 : 50000 : . 100000 : 57734 co-tan. of 60°, or tan. of 30°.

By RULE 3. $\frac{\text{Rad. ſquared, } 10000000000}{\text{tan. } 60° \quad 173206}$=57734, as by Rule 2.

Ex. 2. Required the tangent and co-tangent of 40° 30′.

Anſ. Tang. 85407; co-tang. 117083

Ex. 3. Required the tangent and co-tangent of 15° 32′

Anſ. Tang. 27795; co-tang. 359772

Ex. 4. Required the tangent and co-tangent of 20° 45′

Anſ. Tang. 37886; co-tang. 263949

Ex. 5. Required the tangent and co-tangent of 80° 0′

Anſ. Tang. 567123; co-tang. 17632

PROBLEM

PROBLEM XXVIII.

To find the secant and co-secant of any arch, the sine and co-sine being given.

The figure as in last Prob.

CD : CE : : CA : CB; or rather CD : CA : : CA : CB; therefore the rectangle contained by the co-sine and secant of any arch is equal to the square of the radius : Or the radius is a mean proportional between the co-sine and secant of any arch. Hence the secant is found by

RULE I.

Divide the square of the radius by the co-sine of any arch, and the quotient will give the secant of that arch.

RULE 2. Divide the square of the radius by the sine of any arch, the quotient will be the co-secant of that arch.

EXAMPLE I.

Required the secant and co-secant of 60°

By RULE I.

$$\frac{\text{Sq. rad. } 10000000000}{\text{Co-sine of } 60^\circ \quad 50000} = 200000 \text{ secant of } 60^\circ$$

By RULE II.

$$\frac{\text{Rad. sq. } 10000000000}{\text{Sine of } 60^\circ \quad 86603} = 115469 \text{ co-secant of } 60^\circ$$

Secants may also be calculated by 47. I. Euclid, if the radius and tangents are given. Thus, add the squares of the tangent and radius together, and the square root of their sum will be the secant.

Ex.

Ex. 2. Required the ſecant and co-ſecant of 24° 13′

Anſ. { Sec. 109649 / co-ſec. 243789 }

Ex. 3. Required the ſecant and co-ſecant of 20° 35′

Anſ. { Sec. 106819 / co-ſec. 284438 }

Ex. 4. Required the ſecant and co-ſecant of 10° 0′

Anſ. { Sec. 1015424 / co-ſec. 575871 }

Ex. 5. Required the ſecant and co-ſecant of 35° 40′

Anſ. { Sec. 123089 / co-ſec. 171505 }

PROBLEM. *Fig.* 84.

To find the areas of lunes, or the ſpace included between the interſecting arches of two circles.

RULE.

Find the areas of the two ſegments which form the lune, and their difference will be the area of the lune.

EXAMPLE I.

The length of the chord AB is 80, the height DC 20, and DE 8, required the area of the lune, AEBCA.

$$AD = 40$$
$$AD^2 = 1600$$
$$DE^2 = 64$$

Sq. chord ½ arch, AEB 1664(40.792

```
                 1664(40.792
                 16
                 ------
                 807)6400
                     5649
                 ---------
                 8149)75100
                      73341
                 -----------
                 81582)175900
                       163164
                 ------------
                        12716 &c.
```


40.792

$$
\begin{array}{r}
40.792 \\
1\frac{1}{3} \\
\hline
40.792 \\
13.597 \\
\hline
54.389 \\
80 \\
\hline
134.389 \\
8 \text{ verf. fine.} \\
\hline
1075.112 \\
4 \\
\hline
\text{Seg. AEB } 430.0448
\end{array}
$$

$$
\begin{array}{l}
AD^2 = 1600 \\
DC^2 = 400 \\
\hline
AC^2 = 2000(44.721 \\
16 \\
\hline
84)\ 400 \\
336 \\
\hline
887)6400 \\
6209 \\
\hline
8942)19100 \\
17884 \\
\hline
89441)121600 \\
89441 \\
\hline
32159 \text{ \&c.}
\end{array}
$$

44.721

$$\begin{array}{r l}
44.721 & \\
1\frac{1}{3} & \\
\hline
44.721 & \\
14.907 & \\
\hline
59.628 & \\
80 & \\
\hline
139.628 & \\
20 & \text{vers sine.} \\
\hline
2792.560 & \\
.4 & \\
\hline
1117.0240 & \text{area of ACB.} \\
430.0448 & \text{of AEB.} \\
\hline
686.9792 & \text{area of Lune.}
\end{array}$$

Ex. 2. The chord is 20, and versed sines 10 and 2. Required the area of the lune. *Ans.* 128.522.

Ex. 3. The length of the chord is 48, and the heights of the segments 18 and 7. What is the area? *Ans.* 405.8676

Note. If semicircles be described on the three sides of a right-angled triangle, as diameters, then will the triangle be equal to the two lunes on the legs, taken together.

MENSURATION OF SOLIDS.

Definitions.

1. Solids are figures that have length, breadth, and thickneſs.

2. The boundaries of ſolids are ſuperficies.

3. A ſolid angle is that which is made by the meeting of more than two plane angles in the ſame point, and which are not in the ſame plane.

4. Similar ſolids are ſuch as have their angles ſimilar, and which are contained by the ſame number of ſimilar planes.

5. A cube is a ſolid contained by ſix equal ſquares. *Fig.* 85.

6. A parallelopipedon is a ſolid having ſix rectangular ſides, every oppoſite pair of which are equal and parallel each to each. *Fig.* 86.

7. A priſm is a ſolid whoſe ſides are parallelograms, and is either triangular, ſquare, pentagonal, &c. according to the figure of its end. *Fig.* 87.

8. A cylinder is a round ſolid, whoſe baſes are equal circles. *Fig.* 88.

9. A pyramid is a ſolid, whoſe baſe is a plane figure, and its ſides triangles, whoſe vertices meet in a point, called the vertex of the pyramid, and is either triangular, ſquare, pentagonal, hexagonal, &c. according to the ſigure of its baſe. *Fig.* 89.

10. A cone is a pyramid, having a circular baſe, and is deſcribed by the revolution of a right-angled triangle about one of its legs. It is either right-angled, acute-angled, or obtuſe-angled, according as the revolving leg is equal to, greater, or leſs than the other. *Fig.* 90.

11. The

11. The fixed leg is called the axis of the cone.

12. A ſphere, or globe, is deſcribed by the revolution of a ſemicircle about its diameter; the centre and diameter of the ſphere are the ſame as thoſe of the revolving ſemicircle. *Fig.* 91.

13. A ſegment of any ſolid is a part cut off the top by a plane parallel to the baſe. The fruſtum of a ſolid is that part which remains after the ſegment is cut off. *Fig.* 92.

14. The priſmoid is a ſolid reſembling the fruſtum of a pyramid, having parallel baſes, and theſe baſes both rectangles, but diſproportional. *Fig.* 93.

15. A zone is that part of a ſphere between two parallel planes. *Fig.* 94.

PROBLEM I. *Fig.* 85.

To find the ſuperficies of a cube

RULE.

Multiply the area of one of its ſides by 6, and the product will be the area of the cube.

EXAMPLE I.

Required the ſuperficies of a cube, whoſe ſide is 14 inches.

```
  14
  14
 ---
  56
 14
 ---
 196 area of one of the ſides.
   6
----
1176 Anſ.
```

Ex. 2. How many ſquare yards are in the ſuperficies of a cube, whoſe ſide is 5½ feet? *Anſ.* 20 *ſq. yds.* 1½ *feeo.*

Ex.

Ex. 3. How many ſquare feet are in the ſuperficies of a cube, whoſe ſide is 18 inches? *Anſ.* $13\frac{1}{2}$ *ſq. feet.*

PROBLEM II.

To find the ſolidity of a cube.

RULE.

Multiply the length, breadth, and thickneſs continually, and the product is the ſolidity.

EXAMPLE I.

What is the ſolidity of a cube, whoſe ſide is 8 feet?

$$\begin{array}{r} 8 \\ 8 \\ \hline 64 \\ 8 \\ \hline \end{array}$$

Anſ. 512 ſolid feet.

Ex. 2. Required the ſolidity of a cube, the ſide being 15 feet *Anſ.* 3375 *feet.*

Ex. 3. Required the ſolidity of a cube, whoſe ſide is $3\frac{1}{4}$ yards. *Anſ.* 34·328125 *cub. yds.*

Ex. 4. How many yards digging are in a cubical cellar 12 feet deep? *Anſ.* 64 *cub. yds.*

Ex. 5. How many ſolid yards are in a cubical cellar, whoſe ſide is 10 feet? *Anſ.* $37\frac{1}{27}$ *cub. yds.*

PROBLEM III.

To find the superficies of a parallelopipedon, or prism, and of the cylinder.

RULE.

Multiply the perimeter of the end by the length; to the product add twice the area of the end, and the sum will be the superficies.

EXAMPLE I.

Required the superficies of a parallelopipedon, whose length is 72 feet, breadth 3 feet, and thickness 2 feet.

2+2=4	3
3+3=6	2
10 perimeter.	6 area of one end.
72	2
720	12 area of both ends.
12	
732 feet.	

Ex. 2. Required the surface of a parallelopipedon, whose length is 72 feet, breadth 5, and depth 4 feet.

Ans. 1336 *sq. feet.*

Ex. 3. What is the superficies of a parallelopipedon, whose length is 15, breadth 6, and thickness 4 inches?

Ans. 2 *feet* 5 *inches.*

Ex. 4. Required the surface of a triangular prism, whose length is 10 feet, and sides 3, 4, 5 feet. *Ans.* 132 *feet.*

Ex. 5. Required the superficies of a prism, when the length is $32\frac{1}{2}$ feet, and the end a pentagon, whose side is $6\frac{1}{4}$ feet.

Ans. 1150.037

Ex.

Ex. 6. What is the ſuperficies of a hexagonal priſm, the ſide being 10 inches, and the length 20 feet?

Anſ. 103.6084375 *ſq. feet.*

Ex. 7. Required the convex * ſurface of a cylinder, whoſe diameter is 10 inches, and length $14\frac{1}{2}$ feet.

Anſ. 37.961 *ſq. feet.*

Ex. 8. Required the ſuperficies of a cylinder, whoſe length is $20\frac{1}{3}$ feet, and diameter of its end $5\frac{1}{4}$ feet.

Anſ. 378.660975 *ſq. feet.*

PROBLEM IV.

To find the ſolidity of a parallelopipedon, a priſm, or of a cylinder.

RULE.

Multiply the area of the end by the length, and the product will be the ſolidity.

EXAMPLE I.

Required the ſolidity of a parallelopipedon, whoſe length is 20 feet, breadth 18 inches, and thickneſs 8 inches.

F.	*I.*		*Decimally.*	
20	0	length.	.6	thickneſs.
1	6	breadth.	1.5	
20	0		33	
10	0		66	
30	0		1.00	area of the end.
	8	thickneſs.	20	
20	0	*Anſ.*	20.00	*Anſ.*

20=

* When the convex ſurface is required, the area of both ends is omitted.

F. *By Reduction.*

$$\begin{array}{r} 20 = 240 \\ 8 \\ \hline 1920 \\ 18 \\ \hline 15360 \\ 1920 \\ \hline 1728)34560(20 \textit{ feet Anf.} \\ 3456 \\ \hline 0 \end{array}$$

Ex. 2. Required the ſolidity of a parallelopipedon, whoſe length is 45 feet, breadth 10 feet, and depth $5\frac{1}{2}$ feet.

Anſ. 2475 *cubic feet.*

Ex. 3. Required the ſolidity of a parallelopipedon, whoſe three dimenſions are $30\frac{1}{2}$, $4\frac{3}{4}$, and 2 feet.

Anſ. $289\frac{3}{4}$ *ſolid feet.*

Ex. 4. What is the ſolid content of a parallelopipedon, whoſe length is 25, breadth 3, and thickneſs 2 feet? *Anſ.* 150.

Ex. 5. Required the ſolidity of a triangular priſm, whoſe length is $10\frac{1}{2}$ feet, one ſide of its triangular baſe being 14 inches, and the perpendicular falling upon it from the oppoſite angle, 10 inches. *Anſ.* 5 *feet,* 1 *inch* 3 *parts.*

Ex. 6. Required the ſolid content of a pentagonal priſm, whoſe length is 20 feet, and ſide 10 feet. *Anſ.* 3440.95 *feet.*

Ex. 7. The ſame dimenſions being given, required the ſolidity of an octagonal priſm. *Anſ.* 9656.854 *cubic feet.*

Ex. 8. On the ſame ſuppoſition, required the ſolidity of a decagonal priſm. *Anſ.* 15388.41 *ſolid feet.*

Note, From the foregoing examples it is evident, that the nearer the figure of the baſe approaches to a circle, the greater will the ſolidity be.

Ex. 9. Required the ſolidity of a cylinder, the diameter of its baſe being 15 inches, and length 14 feet.

Anſ. 17.180625 *cubic feet.*

Ex. 10. What is the ſolidity of a pillar 60 inches diameter, and 56 feet high? *Anſ.* 1099.56 *cubic feet.*

PROBLEM V.

To find the ſuperficies of any pyramid or cone.

RULE.

Multiply the primeter of the baſe by one half of the ſlant altitude, to the product add the area of the baſe, the ſum will be the ſuperficies.

The reaſon of this rule is obvious: For if the baſe of the pyramid be any rectilineal figure, each of the ſides will be triangles, whoſe altitude is the ſame with the ſlant altitude of the pyramid.

It is alſo plain, that the convex ſurface of a cone is the ſector of a circle, whoſe radius is the ſlant altitude, and arch the circumference of the cone's baſe.

EXAMPLE I.

Required the ſuperficies of a right cone, whoſe diameter of its baſe is 10 feet, and ſlant altitude 36 feet.

3.1416	.7854
10	100
31.416 circumference.	78.54 area baſe.
18 half the ſlant altitude.	
251328	
31416	
565 488	
78.54	
644.028 ſuperficies.	

Ex.

Ex. 2. Required the surface of a square pyramid, the side of the base being 30 inches, and slant altitude 6 feet.

Ans. $36\frac{1}{4}$ *sq. feet.*

Ex. 3. If the side of the pentagonal base be 10 inches, and the slant altitude 5 feet, required the surface of the pyramid.

Ans. 11.6114 *sq feet.*

Ex. 4. What is the superficies of a hexagonal pyramid, whose side is 15 inches, and slant altitude 4 feet?

Ans. 19.0594875 *sq. feet.*

PROBLEM VI.

To find the solidity of a cone, or any pyramid.

RULE.

Multiply the area of the base by $\frac{1}{3}$, the perpendicular altitude, and the product will be the solidity.

Note. Any pyramid is the third part of a prism of the same base and altitude: Also a cone is equal to one-third the circumscribing cylinder.

EXAMPLE I.

Required the solidity of a pentagonal pyramid, whose perpendicular altitude is 60, and side 8 feet.

```
  1.720475 tabular area of a pentagon.
        64 sq. of the side.
  ---------
   6881900
  10322850
  ---------
110.110400 area of the base.
        20 one third the perp. alt.
  ---------
2202.208000 solidity of the pyramid.
```

Ex. 2. What is the ſolidity of a cone, whoſe ſlant altitude is 96 inches, and diameter of its baſe 20 inches?

Anſ. 9998.45616 *cubic inches.*

Ex. 3. Required the ſolidity of a cone, whoſe perpendicular height is 5 feet, and diameter of its baſe 16 inches.

Anſ. 2.3271 *cubic feet.*

Ex. 4. Required the ſolidity of a triangular pyramid, its height being $14\frac{1}{2}$ feet, and the three ſides of its baſe 12, 14, 10 feet. *Anſ.* 284.13716

PROBLEM VII.

To find the ſuperficies of the fruſtum of a cone, or any pyramid.

RULE.

Add together the primeter of both ends, and multiply one half the ſum by the ſlant altitude, to the product add the area of both ends, and the ſum will be the ſuperficies.

EXAMPLE I.

Required the ſurface of the fruſtum of a ſquare pyramid, the ſides of the leſſer and greater ends being 14, and 24 inches, and ſlant altitude 2 feet 3 inches.

$14 \times 4 = 56$ the perimeter of the lesser end.
$24 \times 4 = 96$ the perimeter of the greater end.

2)152
76 half the sum of the perimeters.
27 slant altitude.

532
152

2052 product.
772 area of both ends.

144)2824(19.61
144

1384
1296

880
864

160
144

16

$14^2 = 196$
$24^2 = 576$

772

	F.	I.	Pts.
Ans.	19	7	4

Ex. 2. Required the surface of the frustum of a cone, the diameter at the greater end being 10, at the lesser 6 feet, and slant altitude $15\frac{1}{2}$ feet. *Ans.* 496.3728 *sq. feet.*

Ex. 3. What is the surface of the frustum of a pentagonal pyramid, its slant altitude being 140 inches, and the sides of the ends 20, and 30 inches? *Ans.* 137.0598 *sq. feet.*

PROBLEM VIII.

To find the solidity of the frustum of a cone, or any pyramid.

RULE

RULE I. *

Add into one ſum the area of both ends, and the mean proportional between them; multiply the ſum by $\frac{1}{3}$ the perpendicular height, and the product will be the ſolidity.—This rule ſerves whether the fruſtum be of a cone or pyramid. The three following applies to the fruſtum of a cone.

RULE 2. To the product of the two diameters add $\frac{1}{3}$ the ſquare of their difference, multiply the ſum by the height, and this product again by .7854 for the ſolidity.

RULE 3. To three times the ſquare of half the ſum of the two diameters, add the ſquare of half their difference; multiply the ſum by $\frac{1}{3}$ the height, and this product again by .7854, the laſt product will be the ſolidity.

RULE 4. Add the ſquares of the two diameters to their product; this ſum, multiplied by .7854, and again by $\frac{1}{3}$ the height, will give the ſolidity.

EXAMPLE I.

Required the ſolidity of the fruſtum of a cone, whoſe height is 20 inches, the greater diameter 32, and leſſer 24 inches.

By RULE I.

```
      32
      32
    ----
    1024
   .7854
  ------
    4096
   5120
  8192
 7168
 -------
 804.2496 area of the greater baſe.
```


* The fruſtum of any pyramid or cone is equal to three complete pyramids

```
      24
      24
     ---
      96
     48
     ---
     576
   .7854
   -----
    2304
   2880
  4608
 4032
 --------
 452.3904 area of the lesser base.
```

$804.2496 \times 452.3904 = 363834.79824384$ And $\sqrt{363834.79824384} = 603.1872$ mean proportional.

```
   804.2496
   603.1872
   452.3904
  ---------
  1859.8272 sum.
         20 height.
  ---------
3)37196.5440
  ----------
 12398.8480 solidity.
```

By

or cones of the same altitude with the frustum, whereof the greatest of the three has its base equal to the greater base of the frustum; the least has its base equal to the less base of the frustum, and the other a mean proportional between them.

By Rule II.

32
24
———
128
64
———
768
21.3
———
789.3
20
———
15786.6

32
24
———
8 diff.
8
———
3)64
21.3

15786.6
.7854
———
631466
7893333
126293333
1105066666
———
12398.84800 *Anſ.* as before.

By Rule III.

32
24
———
2)56
———
28 half the ſum.

32
24
———
2)8
———
4 half the diff.

```
      28
      28
     ---
     224
     56
     ---
     784 sq. half the sum.
       3
     ---
    2352
      16 sq. half diff.
     ---
    2368
      20
    -----
  3)47360
    -----
   15786.6
    .7854
   -------
    631466
   7893333
 126293333
1105066666
----------
12398.84800 Ans. as before.
```

By Rule IV.

```
   24²=  576 sq. less diam.
   32²=1024 sq. greater diam.
24×32=  768 their product.
       ----
       2368
```

```
      2368
      .7854
   ---------
      9472
     11840
    18944
   16576
   ---------
   18598272
        6.6 = 1/3 the height.
   ---------
  111589632
    6199424
    6199424
  ----------
  12398.8480 Ans. as above.
```

Ex. 2. What is the solidity of the frustum of a cone, its height being 50 feet, the diameter at the greater end 20, and at the less end 10 feet? *Ans.* 9163 *solid feet.*

Ex. 3. Required the solid content of the frustum of a cone, the altitude being 9, the greater diameter 4, and the lesser 2. *Ans.* 65.9736.

Ex. 4. Required the solidity of the frustum of a cone, whose height is 38 inches, the diameters being 16, and 9 inches. *Ans.* 4785.1804.

Ex. 5. What is the solidity of a log of wood, whose bases are squares, their sides being 10 and 15 inches, and length 18 feet? *Ans.* 19.7916

Ex. 6. What is the solidity of the frustum of a hexagonal pyramid, the height being 12 feet, the side of the greater end 3 feet, and the lesser 2? *Ans.* 197.453472 *feet.*

Ex. 7. Required the content of the frustum of an octagonal pyramid, its height being 20 feet, the sides of its bases 10 and 6 feet respectively. *Ans.* 6309.14461 *cubic feet.*

Ex. 8. Required the solidity of a mast, $1\frac{1}{2}$ feet diameter at the

the greater base, and 1 foot at the lesser, its length being 72 feet. *Ans.* 89.5356 *cubic feet.*

PROBLEM IX.

To find the solidity of the prismoid.

RULE I.

To the areas of the two ends add four times the area of the middle section. Multiply the sum by the height, and $\frac{1}{6}$ the product will be the solidity.

RULE 2. To the longest side of the lesser base add half the longest side of the greater base, and multiply the sum by the breadth of the lesser base; reserve this product.

Again: To the longest of the greater base add half the long- a Side
est side of the lesser base, and multiply the sum by the breadth of the greater base; and to the product add the product formerly reserved; multiply this sum by the height, and $\frac{1}{3}$ the product will give the solidity.

EXAMPLE I.

Required the solidity of a quadrilateral prismoid, of which the shortest and longest sides of the greater base are 20 and 16 feet, and the corresponding sides of the lesser base 12 and 10 feet, the height being 40 feet

By RULE I.

$$\begin{array}{rl} 20 & \\ 16 & \\ \hline 320 & \text{greater base.} \\ 10 & \\ 12 & \\ \hline 120 & \text{lesser base.} \end{array}$$

$20+12=32$ and $\frac{32}{2}$ is 16

$10+16=26$ and $\frac{26}{2}$ is 13

```
  48
16
----
 208 area of mid. sect.
   4
----
 832
 320
 120
----
1272
  40
------
6)50880
------
 8480 solidity in cubic feet.
```

By Rule II.

```
12 the longest side of the lesser base.
10 half the longest side of the greater base.
---
22
10 the breadth of the lesser base.
---
220 reserved number.
```

Again,

```
20 the longest side of the greater base,
 6 half the longest side of the lesser base.
---
26
 16 breadth of the greater base.
---
416
```

$$\begin{array}{r} 416 \\ 220 \\ \hline 636 \\ 40 \\ \hline 3)25440 \\ \hline 8480 \end{array}$$

8480 cubic feet as above.

Ex. 2. Required the solid content of a trough, in the form of a prismoid, whose greater base is 24 inches by 30, and lesser base 20 inches by 24, the depth being 18 inches. *Ans.* 10728.

Ex. 3. What is the content of the hopper of a mill, 4 feet by 5 at the greater base, and 12 inches by 10 at the lesser, its depth being 4 feet? *Ans.* 57408 solid inches, or 33.2 feet.

PROBLEM X.

To find the solidity of a wedge.

RULE.

Multiply the sum of twice the length of the base, and the length of the edge by the product of the height of the wedge into the breadth of the base, and $\frac{1}{6}$ of the last product will be the solidity.

Note. When the length of the base is equal to that of the edge, the wedge is equal to one half a prism of the same base and altitude.

EXAMPLE I.

How many solid feet are in a wedge whose base is 2 feet 8 inches long and $4\frac{1}{2}$ inches broad, its perpendicular height being 14 inches, and the length of the edge 1 foot 9 inches?

F.	I.	
2	8	14 height.
12		$4\frac{1}{2}$ breadth.
32		56
2		7
64 twice the length of the base.		63
21 the length of the edge.		
85		
63		
255		
510		
6)5355		
$892\frac{1}{2}$		

PROBLEM XI.

To find the superficies of a sphere.

RULE I.

Multiply the circumference by the diameter, and the product will be the surface: Or,

Multiply the square of the diameter by 3.1416 for the surface.

RULE 2. Multiply the square of the axis by .7854, and four times the product will be the superficies.

EXAMPLE I.

How many square inches will cover a globe of 6 inches diameter?

Note. 4 times the area of a great circle of a sphere is equal to its surface.

By

By Rule I.

$$\begin{array}{r} 3.1416 \\ 8 \\ \hline 18.8496 \text{ the circumference.} \\ 6 \text{ the diameter.} \\ \hline \textit{Anf.}\ 113.0976 \text{ sq. inches.} \end{array}$$

By Rule II.

$$\begin{array}{r} .7854 \\ 36 \text{ sq. diam.} \\ \hline 47124 \\ 23562 \\ \hline 28.2744 \text{ area of a great circle.} \\ 4 \\ \hline \textit{Anf.}\ 113.0976 \text{ as above.} \end{array}$$

Ex. 2. Required the ſurface of a ſphere, whoſe diameter is 5 feet 6 inches. *Anſ.* 95.0334 *ſq. feet.*

Ex. 3. What is the ſurface of a ball, whoſe diameter is 1 inch? *Anſ.* 3.1416 *inches.*

Ex. 4. How many inches will cover a globe of 12 inches diameter? *Anſ.* 452.3904.

Ex. 5. Required the ſurface of a globe of 18 inches diameter. *Anſ.* 7.0686 *ſq. feet.*

Ex. 6. Required the ſuperficies of the terraqueous globe, its diameter being 7958 miles. And if only one fourth part of its ſurface be dry land, and two acres ſufficient to produce food for one perſon, how many perſons can live on the earth at one time?

Anſ. { 198956786.5824 *ſq. miles.* 49739196.6456 *dry land.* 15916542927 *perſons.* }

Note. A ſquare mile contains 640 acres.

PROBLEM

PROBLEM XII.

To find the solidity of a sphere.

RULE I.

Multiply the cube of the diameter by .5236, and the product will be the solidity.

RULE 2. A globe may be considered as composed of an infinite number of cones, whose bases are in the surface of the sphere, and common vertex in the centre; therefore the solidity of the globe may be found thus:—Multiply its surface by $\frac{1}{6}$ the diameter, and the product will give the solidity.

RULE 3. Find the solidity of a cylinder, of equal diameter and altitude with the globe, and $\frac{2}{3}$ the result will give the solidity of the globe.

EXAMPLE I.

Required the solidity of a globe, whose diameter is 50 inches.

By RULE I.

$$\begin{array}{r l}
50 & \\
50 & \\
\hline
2500 & \\
50 & \\
\hline
125000 & \text{cube of the axis.} \\
.5236 & \\
\hline
750000 & \\
375000 & \\
250000 & \\
625000 & \\
\hline
65450.0000 &
\end{array}$$

By

By Rule II.

```
        50
        50
      ----
      2500
     3.1416
   --------
  15708000
  62832
  ---------
  7854.0000 surface.
        50
  ---------
 6)392700
   ------
   65450 Ans.
```

By Rule III.

```
        50
        50
      ----
      2500
     .7854
   -------
   3927000
  15708
  ---------
  1963.5000
         50
  ----------
  98175.0000 solid cylin.
         2
  --------
3)196350
  ------
  65450 Ans. as above.
```

Ex. 2. Required the solidity of a sphere of 10 inches diameter. *Ans.* 523.6

Ex. 3. Required the content of a sphere, whose diameter is 25 feet. *Ans.* 8181$\frac{1}{4}$ *cubic feet.*

Ex. 4. What is the solidity of a sphere, whose diameter is 3 feet 1 inch? *Ans.* 15.3483 *cubic feet.*

Ex. 5. Required the solidity of a globe, its diameter being 8 feet 4 inches. *Ans.* 303.0092.

Ex. 6. How many solid miles are in the terraqueous globe, its diameter being 7958 miles? *Ans.* 263883017937.1232.

PROBLEM XIII.

To find the surface of any zone, or segment of a sphere.

RULE.

Multiply the circumference of a great circle of the sphere by the segment's height, and the product will be the superficies.

EXAMPLE I.

Required the superficies of a zone, whose height is 3 inches, the diameter of the sphere being 12 inches.

$$\begin{array}{rl} 3.1416 & \\ 12 & \\ \hline 37.6992 & \text{circumference.} \\ 3 & \text{the zone's height.} \\ \hline 113.0976 & \textit{Ans.} \text{ in square inches.} \end{array}$$

Ex. 2. Required the surface of a segment of a sphere, whose height is 1 foot 9 inches, the diameter being 5 feet. *Ans.* 27.489 *sq. feet.*

Ex. 3. How many square inches will cover a segment, whose height is 1 inch, the diameter of the sphere being 3 inches? *Ans.* 9.4248 *sq. inches.*

PROBLEM

PROBLEM XIV.

To find the solidity of a spherical segment.

RULE I.

From the treple product of the diameter of the sphere, multiplied by the square of the segment's height, subtract twice the cube of the height, and the remainder, multiplied by .5236, will give the solidity.

RULE 2. To thrice the square of the radius of the segment's base, add the square of its height; then multiply the sum by its height, and the product again by .5236, the last product, is the solidity.

EXAMPLE.

Required the solidity of a spherical segment, whose height is 8 inches, and the radius of its base 16 inches.

By RULE I.

```
   16  AE
   16
  ----
8)256
  ----
   32=EF
    8=CE
  ----
   40=CF
    3
  ----
  120 treple prod. of diameter.
   64 sq. of the frust. height.
  ----
  480
 720
 ----
 7680
```

 7680

By Rule II.

7680	16
$1024=2\times8^3$	16
6656	96
.5236	16
39936	256
19968	3
13312	$768=3AE^2$
33280	$64=CE^2$
3485.0816 solid inches.	832
	$8=CE$
	6656
	.5236
	39936
	19968
	13312
	33280
	3485.0816 solid inches as before.

Ex. 2. Required the solidity of a segment, whose base diameter is 100, and its height 13.5 inches.

Ans. 54302.75235 *cubic inches.*

Ex. 3. How many solid miles are in either frigid zone, the height being 329 miles, and diameter of its base 3168 miles?

Ans. 1315766512 *solid miles.*

PROBLEM XV.

To find the solidity of the middle zone of a sphere.

RULE I.

When the ends are unequal, add into one sum the squares of the radii of both ends, and $\frac{1}{3}$ the square of the zone's height; multiply

multiply the sum by the height, and the product again by 1.5708 for the solidity.

Rule 2. From the solidity of the whole sphere, subtract the solidity of the segments ABC and DEF; the remainder is the solidity of the zone.

Rule 3. Add into one sum twice the square of the sphere's diameter, and the square of the diameter of the zone's base; divide this sum by 3.8197, and multiply the quotient by the zone's height; the product is the solidity.

EXAMPLE I.

Required the solidity of the middle zone of a sphere, whose diameter is 80 inches; the diameter of the zone's base being 48, and height 64 inches.

By Rule I.

24
24
———
96
48
———
576
2
———
1152
1365.3
———
2517.3
64
———
100693
151040
———
161109.3
1.5708
———
12888746
1127765333
8055466666
1611093333
———
253070.54080 *Ans.*

By

By Rule II.

$80^3 = 512000$
5236

3072000
1536000
1024000
2560000

268083.2000 solidity of sphere.
15012.6592

253070.5408 *Ans.*

80 diameter.
64 zone's height.

2)16

8 seg. height.

80
3

240
$64 = 8^2$

960
1440

15360
$1024 = 2 \times 8^3$

14336
.5236

86016
43008
28672
71680

7506.3296 solidity of one seg.
2

15012.6592 solidity of both seg.

By

By Rule III.

```
        80        48
        80        48
      ----      ----
      6400       384
         2      192
      ----      ----
     12800      2304 sq. diam. zone's base.
      2304
      ----
3.8197)15104
      ----
   3954.24
        64
   -------
   1581696
  2372544
  --------
  253071.36 Ans.
```

Ex. 2. Required the solidity of a zone, whose greater diameter is 2 feet, the less 1 foot 4 inches, and the height 1 foot 8 inches. *Ans.* 10723.328 *inches.*

Ex. 3. What is the solid content of a zone, whose height is 30, and end diameters 60 and 40 inches?

Ans. 75398·4 *cubic inches.*

Ex. 4. What is the solidity of a zone, whose height is 8 inches, and diameter of the ends 12 inches?

Ans. 1172 864 *cubic inches.*

PROBLEM XVI. *Fig.* 96.

To find the area of a circular spindle.

RULE.

Multiply the length of the spindle by the radius of the revolving arch; again multiply the distance between the centre of the revolving arch and the centre of the spindle by the length

of the revolving arch; subtract this last product from the former, and multiply the remainder by 6.2832 for the superficies.

EXAMPLE.

Required the area of a circular spindle, whose length is 40 and thickness 30 inches.

$\sqrt{AD^2+BD^2} = AB$ the chord of $\frac{1}{2}$ the arch ABC; that is, $\sqrt{400+225} = 25$

$\frac{AD^2}{BD} = DH$ and $\frac{DH+BD}{2} = FB$ rad. also $FB-BD=DF$ cent. dist.

$\frac{400}{15} = 26.6$ and $\frac{26.6+16}{2} = 20.83$ rad. also $20.83-15=5.83=DF$

Now, to find the length of the arch:

As AF = 20.83	=	1.31869
Is to rad. 90		10.00000
So is AD=20		1.30103
To Sine $\frac{1}{2}$ arch 73° 41′	=	9.98234
		2
147° 22′ arch.		

Then say, As 360° : 147° 2 : : 3.1416×41.6 : 53.58 leng. of arch.

Or thus:

$$\begin{array}{r} 25 \\ 8 \\ \hline 200 \\ 40 \\ \hline 3)160 \\ \hline 53.3 \end{array}$$
the arch nearly.

28.83

28.83	53.58
40	5.83
83333	42864
312.55	26790
	1786
520.783	312.550
6.2830	
1041566	
15623500	
416626666	
1041566666	
31247000000	
3272.1858400 square inches.	

Ex. 2. Required the number of square inches which will cover a circular spindle, whose length is 80 and thickness 16 inches?

Ans. 2747.3166336.

Ex. 3. Required the area of a circular spindle, whose length is 12, and thickness 9 inches. *Ans.* 294.3621 *sq. inches.*

PROBLEM XVII.

To find the solidity of a circular spindle.

Multiply the area of the revolving segment by $\frac{1}{2}$ the distance between the centres of the arch and spindle, subtract the product from $\frac{1}{3}$ the cube of half the length of the spindle, then multiply the remainder by 4, and this product again by 3.1416 for the solidity. *See the last figure.*

EXAMPLE I.

Required the area of a circular spindle, whose length is 60 and diameter 45.

$$\frac{AD^2}{BD} = \frac{900}{225} = 40$$

22.5

62.5 diam.
31.25 rad.
22.5

8.75 central dist.

$$\sqrt{BD^2 + AD^2} = AB = \sqrt{506.25 + 900} = 37.5$$

37.5 chord half arch.
8

300.0
60 chord whole arch.

3)240

80 the length of the arch.

31.25 rad.
80 length of the arch.

2)2500.00

1250 the area of the sector AFCB.
8.75 × 30 = 262.5 the area of the triangle ACF.

987.5 the area of the rev. segm. ACB.
4.375 the half the central dist. DF.

49375
69125
29625
29500

4320.3125

```
   30 half the ſpindle.
   30
   ——
  900
   30
  ———
3)27000
  —————
   9000 one-third cube ½ ſpindle.
 4320.3125
 —————————
 4679.6875
         4
 —————————
18718.7500
    3.1416
 —————————
 1123125000
 187187500
748750000
187187500
561572500
———————————————
58806.82500000 ſolidity.
```

Ex. 2. Required the ſolidity of a circular ſpindle, whoſe length is 30, and thickneſs $22\frac{1}{2}$ inches. *Anſ.* 7350.853125.

Ex. 3. Required the ſolidity of a circular ſpindle, whoſe middle diameter is 36, and length 40 inches.

Anſ. $29919\frac{1}{5}$ *cubic inches.*

PROBLEM XVIII.

To find the ſolidity of the middle zone of a circular ſpindle.

RULE.

From the fourth part of the ſquare of the length of the whole ſpindle, ſubtract $\frac{1}{3}$ the ſquare of half the length of the middle fruſtum, and multiply the remainder by $\frac{1}{2}$ the length of

of the fruſtum: Multiply the central diſtance by the revolving area which generates the fruſtum; then ſubtract this latter product from the former, and multiply the remainder by 3.1416, and twice the product will be the ſolidity.

EXAMPLE I.

Required the ſolidity of the fruſtum of a circular ſpindle, whoſe length is 40, greateſt diameter 36, and leaſt 16 inches.

Draw EG parallel to mn, then EF ſhall be equal $\frac{1}{2}$ mn, $=20$
and $EF^2+FB^2=EB^2=500$ chord.

$$\frac{EB^2}{FB} = \frac{500}{10} = 50$$ diameter of the generating circle.

Hence rad. $BD = 25$
and $25-18 = 7$ the central diſt.

$$AL^2=AD^2-LD^2=625-49=576$$

$$\frac{EF}{3} = \frac{400}{3} = \quad . \quad . \quad 133.3$$

$$\begin{array}{r} 133.3 \\ 442.6 \\ 20 \\ \hline 8853.3 \end{array}$$ firſt product.

BE

$$\frac{BE}{2DB} = \frac{10}{50} = \frac{1}{5} = .2$$

Its tabular seg.	.111823
and 50^2 =	2500
	55911500
	223646
Area of seg. EGB =	279.557500
mE × EG = 8 × 40 =	320
Gener. area E m n G =	599.557500
	7
	4196.902500 second product.
	8853.33333 first product.
	4656.43083
	3.1416
	2793858500
	465643083
	186257233333
	465643083333
	1396929250
	14628.633310600
	2
	29257.266621200 solidity.

Ex. 2. Required the solidity of a circular spindle, whose length is 40, its greatest diameter 32, and least 24 inches.

Ans. 27287$\frac{1}{2}$ *cubic inches.*

PROBLEM

PROBLEM XIX.

To find the ſuperficies and ſolidity of the five regular or Platonic bodies.

RULE.

Multiply the ſquare of the given ſide into the correſponding tabular area for the ſuperficies. And

Multiply the cube of the given ſide by the proper tabular ſolidity, for the ſolidity of the given body.

Names.	*Containing ſides.*	*Area.*	*Solidity.*
Tetraedron	4 equilateral trian.	1.732051	0.117851
Hexaedron	6 equal ſquares	6.	1.
Octaedron	8 equal equi. lat. tri.	3.464102	0.471405
Dodecaedron	12 equal pentagons	20.645729	7.663119
Icoſaedron	20 equal equilat. tria.	8.660254	2.181695

This table exhibits the area and ſolidity of any of the above bodies, the ſide being unity.

The areas of the above figures are ſo related to thoſe of regular polygons, and their ſolidities to problems already treated of, that we ſhall leave the conſtruction of the table for the exerciſe of the learner.

EXAMPLE I. *Fig.* 97.

Required the area and ſolidity of a tetraedron, whoſe ſide is 30.

30

30	1.732052 tabular area.
30	900
900	1558.846800 surface.
	30
	30
	900
	30
	27000
	1178511 tab. solidity.
	31809797000 solidity.

Ex. 2. Required the superficial and solid content of a hexaedron, whose side is 6. *Fig.* 98. *Ans.* { Superficies 216, Solidity 216 }

Ex. 3. Required the area and solidity of an octraedron, whose side is 3. *Fig.* 99. *Ans.* { Superficies 31.176918, Solidity 12.7279215 }

Ex. 4. Required the superficies and solidity of the icosaedron, whose side is 2. *Fig.* 100. *Ans.* { Superficies 34.641, Solidity 17.4535 }

Ex. 5. Required the superficies and solidity of a dodecaedron, the side being 4. *Fig.* 101. *Ans.* { Surface 33.03312, Solidity 139.62848 }

PROBLEM XX. *Fig.* 102.

To find the surface and solidity of a cylindric ring.

RULE

Multiply the circumference of the ring by its length for the superficies.

Multiply the area of a section of the ring by the curve, for the solidity.

EXAMPLE I.

Required the surface and solidity of a cylindric ring, whose curve is 12, and the diameter of the ring 3 inches.

To find the surface.

3.1416	.7854
3	9
9.4248 cir. ring.	7.0686 area section.
12 length.	12
113.0976 superficies.	84.8232 solidity.

CONIC SECTIONS.

A Cone may be cut various ways; and, according to the different positions of the cutting plane, the five plane figures following will arise, viz. the *circle*, the *ellipse*, the *parabola*, the *hyperbola*, and the *triangle*.

Definitions.

1. The section is a circle, when the cone is cut parallel to the base.

2. If the section is obliquely to the base, it will form an ellipse. *Fig.* 102.

3. If the plane cut parallel to one of the sides, the section will be a parabola. *Fig.* 103.

4. The

4. The fectioni s an hyperbola, when the cutting plane meets the oppofite cone, and makes another fection fimilar to the former.

5. The fection forms a triangle, when the plane paffes through the vertex and meets the bafe.

6. The vertex of any fection is the point in which the plane meets the oppofite fide of the cone.

7. The tranfverfe axis is a line drawn between two vertices.

8. The centre of an ellipfe is the middle point of the tranfverfe.

9. The conjugate axis is drawn through the centre perpendicular to the tranfverfe.

10. The ordinate is a line perpendicular to the axis.

11. The abfciffa is that part of the axis intercepted between the ordinate and the vertex.

12. The axis of a parabola is a right line drawn from the vertex, fo as to divide the figure into two equal parts.

13. The tranfverfe diameter of an hyperbola is that part of the axis, intercepted between the vertices of the oppofite fections.

PROBLEM I.

To defcribe an ellipfe.

It is a known property of the ellipfe, that any two lines drawn from the foci, meeting in any point of the curve, are together equal to the tranfverfe diameter. Hence the following method of defcribing an ellipfe.

Find the points x y in the tranfverfe, which you are to confider as your foci; there fix two pins, and take a ftring equal to the tranfverfe, and faften its ends each to a pin, then ftretch the ftring with a pencil, and move it round within the thread, fo fhall its path defcribe an ellipfe.

 When

When the transverse and conjugate diameters are given, the foci may be found thus. Draw the transverse AB, and conjugate CD so as they may bisect each other at right angles in the point E, and with the distance AE or EB, and centre C or D, describe arches, cutting the transverse in the points x y, so shall x and y be the foci.

PROBLEM II.

To find the length of the elliptic curve.

RULE.

Multiply the sum of the transverse and conjugate diameters by 3.1416, and half the product will be the circumference nearly.

EXAMPLE I.

Required the length of an elliptic curve, whose conjugate is 40 and transverse 60 feet.

$$\begin{array}{r} 40 \\ 60 \\ \hline 100 \end{array} \qquad\qquad \begin{array}{r} 3.1416 \\ 100 \\ \hline 2)3141.1600 \\ \hline 157.08 \end{array} \quad \textit{Ans.}$$

Ex. 2. What is the length of the circumference, when the diameters are 30, 40 feet? *Ans.* 109.956 *feet.*

Ex. 3. Required the circumference of an ellipse, whose transverse diameter is 20, and conjugate 10 yards.

Ans. 282.744 *feet.*

Ex. 4. What is the periphery of an ellipse, whose axis are 36 feet and 24 feet? *Ans.* 94.248.

PROBLEM

PROBLEM III.

To find the area of an ellipse.

RULE

Multiply the transverse by the conjugate, and this product again by .7854 for the area.

EXAMPLE. I.

Required the area of an ellipse, whose two axes are 30 and 40 feet.

$$
\begin{array}{r}
30 \\
40 \\
\hline
1200 \\
.7854 \\
\hline
942.4800
\end{array}
$$

Ex. 2. Required the area of an ellipse, whose transverse and conjugate are 20 and 10 feet. *Ans.* 157.08.

Ex. 3. Required the area of an ellipse, whose diameters are 48 and 36 yards. *Ans.* 1357.1712.

Ex. 4. Required the area of an ellipse, whose two axes are 14 and 12 feet. *Ans.* 131.9472.

PROBLEM IV.

The transverse, conjugate, and ordinate being given, to find the abscissa.

RULE.

As the conjugate
Is to the tranſverſe,
So is the ſquare root of the difference of the ſquares of the ordinate and ſemi-conjugate
To the diſtance between the ordinate and centre.

Note. This diſt ce is to be added to or ſubtracted from the ſemi-tranſverſe, according as the abſciſſa is greater or leſs than the ſemi-tranſverſe.

EXAMPLE I.

The tranſverſe AB is 60, the conjugate CD 20, and the ordinate Fx 8. It is required to find the abſciſſa.

10	8	100
10	8	64
00 ſq. of the ſemi-conj.	64 ſq. of the ordinate.	36(6 root.
		36

As 20 : 60 : : 6
6

20)360

18 diſtance between the ordinate and centre.
30 ſemi-tranſverſe.

48 the abſciſſa x B.
12 the abſciſſa A x.

Ex. 2. The tranſverſe 90, the conjugate 30, and the ordinate 12, required the abſciſſas. *Anſ.* 72 and 18.

Ex. 3. The tranſverſe 105, the conjugate 35, and the ordinate 14, required the abſciſſas. *Anſ.* 84 and 21.

Ex.

Ex. 4. The tranſverſe diameter is 3, the conjugate 1, and the ordinate $\frac{4}{10}$, required the abſciſſas. *Anſ.* $\frac{6}{10}$ and $2\frac{4}{10}$.

PROBLEM V.

The conjugate, ordinate, and abſciſſa being given, to find the tranſverſe.

RULE.

Find the ſquare root of the difference of the ſquares of the ſemi-conjugate and ordinate, and, according as the greater or leſs obſciſſa is propoſed, add this root to or ſubtract it from the ſemi-conjugate. Then uſe the following proportion—

As the ſquare of the ordinate
Is to the product of the conjugate and abſciſſa,
So is the ſum or difference, found as above,
To the tranſverſe.

EXAMPLE. I.

The conjugate CD is 20, the ordinate Fx is 8, and the abſciſſa Ax 12, required the tranſverſe AB.

$10^2 = 100$
$8^2 = \underline{64}$

36(6 root of the differences of the ſquare of the ſe-
$\underline{36}$ 10 ſemi-conjugate. [mi conj. and ord.

16 ſum.

As 64 : 12 × 20 : : 16
64 : 240 : : 16
$\underline{16}$

64)3840(60 the tranſverſe.
384

Ex.

Ex. 2. The conjugate 30, the ordinate 12, and the abſciſſa 18, required the tranſverſe. *Anſ.* 90.

Ex. 3. The conjugate 35, the ordinate 14, and the abſciſſa 84, required the tranſverſe. *Anſ.* 105.

Ex. 4. The conjugate 1, the abſciſſa $2\frac{4}{10}$, and the ordinate $\frac{4}{10}$, required the tranſverſe. *Anſ.* 3.

PROBLEM VI.

The tranſverſe, ordinate, and abſciſſa being given, to find the conjugate.

RULE.

As the ſquare root of the products of the two abſciſſas
Is to the ordinate,
So is the tranſverſe
To the conjugate.

EXAMPLE I.

The tranſverſe AB is 60, the ordinate Fx 8, and the abſciſſa 12, required the conjugate.

The tranſverſe	60		
One of the abſciſſas	12		
The other abſciſſa	48	As 24 : 8 : :	60
	12		8
	576(24		24)480(20 the conjugate.
	4		40
	44)176		0
	176		

Ex.

Ex. 2. The tranſverſe 90, the ordinate 12, and the abſciſſa 18, required the conjugate diameter. *Anſ.* 30.

Ex. 3. The tranſverſe 105, the ordinate 14, and the abſciſſa 84, it is required to find the conjugate. *Anſ.* 35.

Ev. 4. The tranſverſe 3, the ordinate $\frac{4}{10}$, and the abſciſſa $2\frac{4}{10}$, required the conjugate. *Anſ.* 1.

PROBLEM VII.

The abſciſſa, tranſverſe, and conjugate being given, to find the ordinate.

RULE.

As the tranſverſe
Is to the conjugate,
So is the ſquare root of the product of the two abſciſſas
To the ordinate.

EXAMPLE I.

The tranſverſe AB 60, the abſciſſas 12, 48, and the conjugate 20, required the ordinate.

```
     12               60 : 20 : : 24
     48                           20
    ----                         ----
    576(24                     60)480
    4                            ----
   ----                   Anſ.  8 the ordinate.
 44)176
    176
   ----
```

Ex. 2. The tranſverſe 90, the abſciſſa 18, and the conjugate 30, required the ordinate. *Anſ.* 12.

Ex.

Ex. 3. The transverse 105, the abscissa 84, and the conjugate 35, required the ordinate. *Ans.* 14.

Ex. 4. The transverse 36, the abscissa $28\frac{4}{5}$, and the conjugate 12, required the ordinate. *Ans.* 4.8.

PROBLEM VIII.

To find the area of an elliptic segment, whose base is parallel to either of the axis.

RULE.

Divide the height of the segment by that axe of the ellipse of which it is a part, and find, in the table of circular segments, an area, whose versed sine shall be equal to this quotient. Then multiply the area so found, and the two axes continually, and the last product will give the area of the segment required.

EXAMPLE I.

Required the area of the elliptic segment ECF, whose height is GC 20, and the axes CD and AB 70 and 50.

70)20.0(.2857$\frac{1}{7}$ tabular versed sine.

140	
600	Seg. is .185153
560	70
400	12.960710
350	50
500	648.035500 area.
490	
10	

Ex.

Ex. 2. Required the area of an elliptic ſegment, cut off parallel to the conjugate, at the diſtance of 18 from the centre, the axis being 60 and 20. *Anſ.* 134.1876.

Ex. 3. Required the area of an elliptical ſegment, cut off parallel to the tranſverſe, whoſe height is 6, the diameters being 30 and 20. *Anſ.* 118 9008.

Ex. 4. Required the area of an elliptical ſegment, cut off parallel to the tranſverſe, whoſe height is 10, the diameters being 70 and 50. *Anſ.* 391.3829.

PROBLEM IX.

To deſcribe a parabola, the abſciſſa and ordinate to the axle being given.

RULE.

Biſect the given ordinate BA in G, join VG, and draw GD at right angles to VG, meeting the axis in D, and make VO, OF, each equal to BD, and F will be the focus of the parabola.

Take any number of points, x, x, &c. in the axis, and through theſe points draw double ordinates of an indefinite length.

Then with the radii VF, Vx, &c. and centre F, deſcribe the arches c, c, &c. and through all the points of interſection the curve may be drawn.

Note. The line cFc is called the parameter.

For other methods of conſtruction, See *Gunnery.*

PROBLEM X.

Any three of the four following particulars being given, viz. *any two ordinates and their two abſciſſas, to find the fourth.*

 RULE.

RULE.

As any abſciſſa
Is to the ſquare of its ordinate,
So is any other abſciſſa
To the ſquare of its ordinate.

EXAMPLE I.

Let the abſciſſa VC be 6, and its ordinate AC 5, required the ordinate DF, whoſe abſciſſa VF is 12.

$$6 : 25 :: 12$$
$$\begin{array}{r} 12 \\ \hline 6)300 \\ \hline 50 = DF^2 \end{array} \qquad \text{and } \sqrt{50} = 7.071 \textit{ Anſ.}$$

Ex. 2. The ordinates are 6 and 8, and the leſs abſciſſa 9, required the greater. *Anſ.* 16.

Ex. 3. The ordinate is 18, and its abſciſſa 27, the other abſciſſa is 48, required its correſponding ordinate. *Anſ.* 24.

PROBLEM XI.

To find the length of an arch of a parabolic curve, cut off by a double ordinate.

RULE.

To the ſquare of the ordinate add $\frac{4}{3}$ of the ſquare of the abſciſſa, multiply this ſum by 4, and the ſquare root of the product will be the length of the curve required.

EXAMPLE

EXAMPLE I.

Let the abſciſſa VF be 4, and its ordinate DF 12, required the length of the arch DAVBE.

```
    12                                   4
    12                                   4
   ---                                  ---
   144 ſq. of the ordinate.             16 ſq. of the abſciſſa.
    21.33                                4
   ------                               ---
   165.33                             3)64
        4                               ----
   ------                               21.33
   661.33(25.7162 the length of the arch.
   4
   -----
 45)261
    225
    ---
 507)3633
     3549
     ----
  5141)8333
       5141
      -----
 51426)319233
       308556
       -------
 514322)1067633
        1028644
        -------
          38978 &c.
```

Ex. 2. Required the length of the curve, when the abſciſſa is 8, and the ordinate 16. *Anſ.* 36.951.

Ex. 3. Required the length of the curve, when the abſciſſa is 15, and ordinate 12. *Anſ.* 21.071.

PROBLEM XII.

To find the area of a parabola, the base and height being given.

RULE.

Multiply the base by the height, and $\frac{2}{3}$ the product will be the area required.

Note. Every parabola is equal to $\frac{2}{3}$ of the circumscribing parallelogram.

EXAMPLE I.

Required the area of a parabola, whose base is 16, and height 20.

$$\begin{array}{r} 16 \\ 20 \\ \hline 320 \\ 2 \\ \hline 3)640 \\ \hline 213\frac{1}{3} \end{array}$$

Ex. 2. Required the area of a parabola, whose base is 30, and height 20. *Ans.* 400.

Ex. 3. Required the area of a parabola, whose base is 9, and height 14. *Ans.* 84.

Ex. 4. Required the area of a parabola, whose base is 12, and height 12. *Ans.* 96.

Ex. 5. Required the area of a parabola, whose base and altitude are 15 and 22. *Ans.* 220.

Ex. 6. Required the area, when the base and altitude are 3 and 4. *Ans.* 8.

PROBLEM

PROBLEM XIII.

To find the area of the frustum of a parabola.

RULE.

Divide the difference of the cubes of the two ends of the frustum by the difference of their squares, multiply this quotient by the altitude, and $\frac{2}{3}$ the product will be the area required.

EXAMPLE I.

In the parabolic frustum DABE, the two parallel ends DE, AB, are 12 and 2c, and the altitude FC 6, required the area.

12	20	$AB^2=400$	$AB^3=8000$
12	20	$DE^2=144$	$DE^3=1728$
144 sq.	400	256	256)6272(24.5 quot.
12	20		512
1728 cube.	8000 cube.		1152
			1024
			1280
			1280

And 24.5
6
1470
2
3)2940
980 area required.

Ex. 2. The greater end of a frustum is 20, the less 10, and their distance 12, required the area. *Ans.* $186\frac{2}{3}$.

Ex.

Ex 3. The greater end of a fruſtum is 30, the leſs 20, and their diſtance 15, required the area. *Anſ.* 380.

Ex. 4. The greater end of a fruſtum is 9, the leſs 6, and their diſtance 4, required the area. *Anſ.* $11\frac{2}{5}$.

PROBLEM XIV.

To deſcribe an hyperbola, the tranſverſe and conjugate diameters being given.

RULE.

Draw AB the tranſverſe diameter, and BC the conjugate at right angles to it; biſect AB in c, and and with the centre c, and radius cE, deſcribe the circle EFDf, cutting AB produced in the points F, f, and theſe points will be the foci.

In AB produced take any convenient number of points x, x, &c. and from F and f as centres, and radii Bx, Ax, deſcribe arches interſecting in the points m, m, &c. Join theſe points, and it will form the hyperbolic curve required.

Note. If through the points E and D ſtraight lines be drawn from c, they will be the aſymptotes of the hyperbola.

Any three of the four following particulars being given, to find a fourth, viz. the tranſverſe, conjugate, ordinate, and its abſciſſa.

PROBLEM XV.

The tranſverſe, conjugate, and abſciſſa being given, to find the ordinate.

RULE.

RULE.

As the tranſverſe
Is to the conjugate,
So is the ſquare root of the product of the two abſciſſas
To the ordinate.

EXAMPLE I.

In the hyperbola GBH, the tranſverſe is 60, the conjugate 36, and the abſciſſa AB 20, required the ordinate.

$$2BC : DE :: \sqrt{2BC + BA \times BA} : GA$$

$$\begin{array}{rl} 2BC = & 60 \\ BA = & 20 \\ \hline & 80 \\ BA & 20 \\ \hline & 1600(40 \\ & 16 \\ \hline & 00 \end{array} \qquad \begin{array}{l} \text{As } 60 : 36 :: 40 \\ \qquad\qquad 40 \\ \hline 60)1440 \\ \hline \textit{Anſ.}\ 24 \text{ the ordinate.} \end{array}$$

Ex. 2. The tranſverſe is 50, the conjugate 30, and the abſciſſa $16\frac{2}{3}$, required the ordinate. *Anſ.* 20.

Ex. 3. The tranſverſe is 45, conjugate $22\frac{1}{2}$, and the abſciſſa 15, required the ordinate. *Anſ.* 15.

Ex. 4. The tranſverſe diameter is 24, the conjugate 21, and the leſs abſciſſa 8, required the ordinate. *Anſ.* 14.

PROBLEM XVI.

The tranſverſe, conjugate diameters, and an ordinate, being given, to find the abſciſſas.

RULE.

RULE.

As the conjugate diameter
Is to the tranſverſe,
So is the ſquare root of the ſum of the ſquares of the ordinate and ſemi-conjugate
To the diſtance between the ordinate and centre.

Add to, or ſubtract from, the ſemi-tranſverſe, this fourth proportional, according as the greater or leſs abſciſſa is required.

EXAMPLE I.

The tranſverſe diameter is 60, the conjugate 36, and the ordinate 24, required the two abſciſſas.

$18^2=324$ ſquare of the ſemi-conjugate.
$24^2=576$ ſquare of the ordinate.

$$\begin{array}{r} 900(30 \\ 9 \\ \hline 00 \end{array}$$

$$36 : 60 :: 30$$

$$\begin{array}{rl} 60 & \\ \hline 36)1800(50 & \text{diſt betw. the ordinate and centre.} \\ 180 \quad 30 & \text{ſemi-tranſverſe.} \\ \hline 0 \quad 80 & \text{greater abſciſſa.} \\ 20 & \text{the leſs abſciſſa.} \end{array}$$

The tranſverſe diameter is 50, the conjugate 30, and the ordinate 20, required the abſciſſas, *Anſ.* $66\frac{2}{3}$ and $16\frac{2}{3}$.

Ex. 3. The tranſverſe diameter is 24, the conjugate 21, and the ordinate 14, required the abſciſſas. *Anſ.* 32 and 8.

Ex.

Ex. 3. The transverse diameter is 24, the conjugate 21, and the ordinate 14, required the abscissas. *Ans.* 32 and 8.

Ex. 4. The transverse diameter is 30, the conjugate $22\frac{1}{2}$, and the ordinate 15, required the abscissas. *Ans.* $33\frac{3}{4}$ and $3\frac{3}{4}$.

PROBLEM XVII.

To find the length of an arch of an hyperbolic curve, beginning at the vertex.

RULE.

To 19 times the transverse add 21 times the parameter * of the axis; and, to 9 times the transverse, add 21 times the parameter, then multiply each of these sums by the quotient of the abscissa divided by the transverse.

To each of the products so found add 15 times the parameter, and divide the former by the latter, and multiply this quotient by the ordinate, the product will be the length of the arch nearly.

EXAMPLE I.

In the hyperbola GBH, the transverse is 160, the conjugate 120, the ordinate 20, and abscissa $4\frac{1}{2}$, required the length of the curve GB.

G g First,

* From a well-known property of the hyperbola, the rectangle contained by the transverse and the parameter is equal to the square of the conjugate; that is, the conjugate is a mean proportional between the transverse and the parameter. Hence the following proportion to find the parameter:—

As the transverse, is to the conjugate,
So is the conjugate, to the parameter.

First, To find the parameter:

As 160 : 120 : : 120 : 90 the parameter.

160	160
9	19
1440	1440
1890	160
3330	3040
.028125	1890
16650	4930
6660	.028125
3330	
26640	24650
6660	9860
	4930
93.656250	43440
1350	9860
1443.65625	142.656250
	1350

1443.65625)1492.65625 (1.03047
1443.65625   20

49.0000000   20.60940 *Anf.*
433096875

690312500
577462500

112850000

```
160)4.50(.028125          90
     320                  21
    ----                ----
    1300                1890
    1280
    ----
     200
     160
     ---
      400                  90
      320                  15
      ---                ----
       800               1350
       800
       ---
```

Ex. 2. Let the tranſverſe be 80, the conjugate 60, the ordinate 10, and the abſciſſa 2.1637, required the length of the arch GB. *Anſ.* 10.3

Ex. 3. The tranſverſe is 120, the conjugate 72, the ordinate 48, and the abſciſſa 40, required the length of the arch. *Anſ.* 62.6496.

PROBLEM XVIII.

To find the area of an hyperbola, the tranſverſe, conjugate, and abſciſſa being given.

RULE.

To the product of the tranſverſe and abſciſſa add $\frac{5}{7}$ of the ſquare of the abſciſſa, and multiply the ſquare root of the ſum by 21.

To this product add 4 times the square root of the product of the transverse and abscissa, and divide this sum by 75.

Divide 4 times the product of the conjugate and abscissa by the transverse.

Multiply this last quotient by the former, and the product will give the area of the hyperbola.

EXAMPLE I.

In the hyperbola GBH, the transverse is 60, the conjugate 36, and the abscissa 20, required the area.

60
20
———
1200

20
20
———
400
5
———
7)2000
———
285.714285
1200
———
1485.714285(38.544
9
———
68)585
544
———
765)4171
3825
———
7704)34642
30816
———
77084)382685
308336
———
74349 &c.

21
———
38544
77088
———
809.424

60
20
———
1200(34.641
9
———
64)300
256
———
686)4400
4116
———
6924)28400
27696
———
69281)70400
69281
———
1119

4
———
138.564

809.424

809.424 first product.	36
138.564 second product.	20
75)947.988(12.639	720
75	4
197	60)2880
150	48
479	12.639
450	101112
298	50556
225	606.672 area required.
730 &c.	

Ex. 2. The transverse diameter is 50, the conjugate 30, and the abscissa 25, required the area. *Ans.* 805.09.

Ex. 3. The transverse 30, the conjugate 18, and the abscissa 10, required the area. *Ans.* 151,668.

PROBLEM XIX.

To find the solidity of a spheroid.

RULE.

Multiply the square of the revolving axis by the fixed axis, and multiply the product by .5236 for the solidity.

EXAMPLE. I.

Required the solidity of the prolate spheroid ABCD, the transverse or fixed axis AC is 45, the revolving axis DB 35.

```
      35
      35
    ----
     175
    105
    ----
    1225 sq. of the revolv. axis.
      45 fixed axis.
    ----
    6125
   4900
   -----
   55125
   .5236
  ------
  330750
 165375
110250
275625
----------
28863.4500 Ans.
```

Ex. 2. Required the solidity of the oblate spheroid, whose fixed axis is 60, and revolving axis 100. *Ans.* 314160.

Ex. 3. Required the solidity of a prolate spheroid, whose fixed axis is 9, and revolving axis 7. *Ans.* 230.9076.

Ex. 4. What is the content of an oblate spheroid, whose axis are 50 and 30? *Ans.* 39270.

PROBLEM XX.

To find the solidity of the segment of a spheroid.

RULE.

Case I. When the base is parallel to the revolving axis, multiply the difference between triple the fixed axis and double the height of the segment, by the square of the height, and the product again by .5236. Then,

As

As the ſquare of the fixed axis
Is to the ſquare of the revolving axis,
So is the laſt product
To the ſolidity of the ſegment required.

Case II. When the baſe is perpendicular to the revolving axis, multiply the difference between triple the revolving axis and double the height of the ſegment, by the ſquare of the height, and the product again by .5236. Then,

As the revolving axis
Is to the fixed axis,
So is the laſt product
To the content.

EXAMPLE I.

Required the ſolidity of the ſegment of a prolate ſpheroid, the axis being 20 and 12, the height of the ſegment 2, and its baſe parallel to the revolving axis.

```
  20
   3
 ----
  60 triple the fixed axis.
   4 double the height.
 ----
  56
   4
 ----
  224
 .5236
 ------
  20944
 10472
10472
--------
117.2864
```

Then,

Then as 400 : 144 : : 117.2864

```
        117.2864
             144
       ---------
         4691456
        4691456
       1172864
       ---------
400)16889.2416
       ---------
       42.223104 Ans.
```

Ex. 2. The axis of an oblate ſpheroid being 50 and 30, required the content of the ſegment, its height being 6, and its baſe parallel to the revolving axis. *Anſ.* 4084.07.

Ex. 3. Required the ſolid content of the ſegment of a prolate ſpheroid, the height being 5, and the fixed axis 50, and the revolving axis 30. *Anſ.* 659.736.

Ex. 4. Required the content of the ſegment of an oblate ſpheroid, whoſe height is 5, the axis 50 and 30, its baſe being perpendicular to the revolving axis.

```
      150
       10
      ---
      140
       25
      ---
      700
     280
     ----
     3500
    .5236
    -----
    26180
   15708
   ------
 1832.6000
```

Then,

Then, as 50 : 30 : : 1632.600
3
5)5497.8000
1099.56

Ex. 5. Required the content of the ſegment of the prolate ſpheroid, the fixed axis 50, the revolving 30, and the height 6, its baſe being perpendicular to the revolving axis.

Anſ. 2450.418.

PROBLEM XXI.

To find the content of the middle fruſtum of a ſpheroid.

RULE.

When the ends are circular, or parallel to the revolving axis.

To double the ſquare of the middle diameter, add the ſquare of the diameter of one end, multiply this ſum by the length of the fruſtum, and this product again by .2618 for the ſolidity.

When the ends are elliptical, or perpendicular to the revolving axis.

To double the product of the tranſverſe and conjugate diameters of the middle ſection, add the product of the tranſverſe and conjugate of one end, multiply the ſum by the length of the fruſtum, and the product again by .2618 for the content.

EXAMPLE I.

Required the ſolidity of the middle fruſtum of a ſpheroid, the greater diameter being 50, thoſe of the ends 40, and length 18.

```
         50
         50
       ----
       2500
          2
       ----
       5000
       1600
       ----
       6600
         18
      -----
      52800
      6600
     ------
     118800
      .2618
     ------
     950400
    118800
   712800
  237600
 ----------
 31101.8400 Ans.
```

Ex. 2. Required the solidity of the middle frustum of a spheroid, the greatest diameter being 60, those of the ends 36, and the length 80. *Ans.* 177940.224.

Ex. 3. Required the solidity of the middle frustum of an oblate spheroid, the diameters of each end 20, and middle 25, and the length 9. *Ans.* 3887.73.

Ex. 4. Required the content of the middle frustum of an oblate spheroid, the axis of the middle ellipse are 50 and 30, and those of the ends 30 and 18, height 40.

```
    50        30
    30        18
  ----       ---
  1500       540
     2
  ----
  3000
   540
  ----
  3540
    40
 ------
 141600
  .2618
 -------
 1132800
 1416
8496
2832
-----------
37070.8800 Ans.
```

Ex. 5. Required the solidity of the middle frustum of an oblate spheroid, the axis of the middle ellipse are 25 and 15, and those of the ends 15 and 9, height 20. *Ans.* 4633.86.

Ex. 6. Required the solidity of the middle frustum of a spheroid, the axis of the middle section being 100 and 60, those of the ends 80 and 48, the length 36. *Ans.* 149288.832.

PROBLEM XXII.

To find the solidity of an elliptical spindle.

RULE.

1. From 3 times the square of the middle diameter subtract 4 times the square of the diameter between the middle and end; and from 4 times this last diameter subtract 3 times the middle diameter, then divide the former difference by the latter, and $\frac{1}{4}$ the

the result will be the central distance, or distance between the centre of the spindle and centre of the generating ellipse.

2. Then find the axis of the ellipse by problem 5 and 6, and the area of the segment which generated the spindle by problem 8.

3. Divide 3 times that area by the length of the spindle, from the quotient subtract the greatest diameter, and multiply the remainder by 4 times the central distance.

4. Subtract this product from the square of the greatest diameter, and multiply the remainder by the length of the spindle, and that product again by .5236 for the solidity.

EXAMPLE I.

Required the solidity of the elliptic spindle ACBD, the length AB being 40, the greatest diameter CD 12, and the diameter CF at $\frac{1}{4}$ the length, 9.49546.

1. To find the central distance and axis of the ellipse.

4 times EF=37.98184		3 times CD^2=432.0000	
3 times CD 36.00000		4 times EF^2=360.6546	
Difference 1.98184		1.98184)71.3454(36=4 OG	
		594552	
		1188920	9=OG
		1189104	6=CG
			15=OC
			36=CH
			24=GH
			6=CG
			144=CG × GH

$\sqrt{144}$=12 mean between CG and GH

Then, as 12 : 20 : : 30 : 50=IK the transverse.

2. For

2. For the generating elliptic segment.

```
CH=30)6 =CG
      —
     .2 tabular versed sine.
 .111823 tabular corresponding area.
      50=IK the transverse.
 ————
 5.591150
      30
 ————
167.734500 area of the generating segment ABC.
```

3. To find the solidity of the spindle.

```
              167.7345
                     3
              ————
           40)503.2035
              ————
              12.5800875
Sub. CD       12
              ————
               .5800875
                     36 four times central distance.
              ————
               34805250
              17402625
              ————
Take          20.8831500
From          144
              ————
Rem.          123.11685
                     40
              ————
           4924.67400
                 .5236
           ————
            2954804400
           14774022
           9849348
          24623370
          ————
        2578.559306400 Ans.
```

Ex. 2. Required the solidity of an elliptic spindle, whose length is 40, the middle diameter 12, the diameter at $\frac{1}{4}$ the length 9.49546. *Ans.* 2578.56.

Note. The following rule will serve for any other solid, generated by the revolution of any conic section.

RULE II.

To the square of the greatest diameter add 4 times the square of the diameter at $\frac{1}{4}$ the length; multiply the sum by the length, and the product again by .13 for the solidity, very nearly.

The first example wrought as follows:

```
      9.49546
      9.49546
      -------
      5697276
     3798184
    4747730
   8545614
  3798184
 8545914
 ------------
 90.1637606116
             4
 ------------
 360.6550424464
 144
 ------------
 504.6550424464
             40
 ------------
 20186.2016978560
              .13
 ------------
  6055860509356680
 2018620169785 60
 ------------
Ans. 2624.206220721280 nearly.
```

Ex.

Ex. 2. Required the solidity of the elliptic spindle, whose length is 10, the greatest diameter 3, and the diameter at $\frac{1}{4}$ the length 2.37386. *Ans.* 322.32.

PROBLEM XXIII.

To find the solidity of the middle frustum or segment of an elliptic spindle.

RULE.

Add together the squares of the greatest and least diameters, and 4 times the square of the diameter in the middle between the two; multiply the sum by the length, and the product again by .13 for the solidity.

EXAMPLE I.

Required the solidity of the middle frustum or segment of a spindle, the length being 20, greatest diameter 16, at the ends 12, and the intermediate diameter is $14\frac{1}{2}$.

$$\begin{array}{r}16\\16\\\hline 96\\16\\\hline 256\end{array}\qquad\begin{array}{r}12\\12\\\hline 144\end{array}\qquad\begin{array}{r}14.5\\14.5\\\hline 725\\580\\145\\\hline 210.25\\2\\\hline 420.50\\2\\\hline 841\end{array}$$

$$\begin{array}{r} 256 \\ 144 \\ 841 \\ \hline 1241 \\ .13 \\ \hline 3723 \\ 1241 \\ \hline 161.33 \\ 20 \\ \hline \end{array}$$

Anſ. 3226.60

Ex. 2. Required the content of the ſegment of any ſpindle, the length being 10, the greateſt diameter 8, the leaſt $4\frac{1}{2}$, and the middle 6. *Anſ.* 235.3.

Ex. 3. Required the content of the middle fruſtum of an hyperbolic ſpindle, whoſe length is 40, the greateſt diameter 16, thoſe at the ends 12, and that at $\frac{1}{4}$ the length, $14\frac{1}{2}$. *Anſ.* 6453.2.

PROBLEM XXIV.

To find the ſolidity of the parabolic conoid.

RULE.

Multiply the ſquare of the baſe diameter by .3927, and the product again by the height, for the ſolidity. Or,

Multiply the area of the baſe by $\frac{1}{2}$ the altitude, and the product will give the ſolidity.

EXAMPLE I.

Required the ſolidity of the parabolic conoid, whoſe height is 30, and the diameter of its baſe 20.

20	Or, .7854
20	400
400	314.1600
.3927	15
157.0800	15708000
30	3141600
4712.4000 *Anſ.*	4712.4000 *Anſ.* as before.

Ex. 2. Required the ſolidity of the parabolic conoid, whoſe altitude is 21, and the diameter of its baſe 12.

Anſ. 1187.5248.

Ex. 3. Required the ſolidity of a paraboloid, whoſe height is 30, and baſe diameter 40. *Anſ.* 18849.6.

N. *B.* The paraboloid conoid is $=\frac{1}{2}$ its circumſcribing cylinder.

PROBLEM XXV.

To find the ſolidity of the fruſtum of a paraboloid.

RULE.

Multiply the ſum of the ſquares of the diameters of the two ends by .7854, and this product by $\frac{1}{2}$ the altitude for the ſolidity.

EXAMPLE. I.

Required the content of the fruſtum of a paraboloid, the greateſt diameter being 30, the leaſt 24, and the altitude 18.

$$
\begin{array}{r}
30^2 = 900 \\
24^2 = 576 \\
\hline
1476 \\
.7854 \\
\hline
5904 \\
7380 \\
11808 \\
10332 \\
\hline
1159.2504 \\
9 \\
\hline
10433.2536
\end{array}
$$

Ex. 2. Required the ſolidity of the fruſtum of a paraboloid, the diameter of the greater end being 60, of the leſs 48, and length 18. *Anſ.* 41733.0144.

Ex. 3. Required the ſolidity of the fruſtum of a parabolic conoid, whoſe diameters are 58 and 30, and the height 36. *Anſ.* 60281.0208.

PROBLEM XXVI.

To find the ſolidity of a parabolic ſpindle.

RULE.

Multiply the ſquare of the middle diameter by .7854, and the product by the length, and $\frac{8}{15}$, the laſt product, will be the ſolidity.

Note. The parabolic ſpindle is equal to $\frac{8}{15}$, the circumſcribing cylinder.

EXAMPLE I.

Required the ſolidity of a parabolic ſpindle, whoſe length is 18, and middle diameter 6.

6

6
6

36
.7854

47124
23562

28.2744
18

2261952
282744

508.9392
8

15)4071.5136(271.43424 *Anf.*
30

107
105

21
15

65
60

51
45

63
60

36
30

60
60

Or rather:

.7854
36

47124
23562

28.2744
18

2261952
282744

3)508.9392
.53

25446960
1696464

271.43424 *Anf.*

Ex. 2. Required the solidity of a parabolic spindle, whose length is 40, and middle diameter 10. *Ans.* 1675.52.

Ex. 3. Required the solidity of a parabolic spindle, whose length is 100, and middle diameter 10. *Ans.* 16755.2.

PROBLEM XXVII.

To find the solidity of the middle frustum of a parabolic spindle.

RULE.

Add into one sum 8 times the square of the greatest diameter, 3 times the square of the least diameter, and 4 times the product of the two diameters; multiply the sum by the length of the frustum, and the product again by .05236 for the solidity.

EXAMPLE I.

Required the solidity of the middle frustum of a parabolic spindle, the length being 20, the greatest diameter 16, and the least 12.

$$
\begin{array}{rr}
16^2 = 256 \times 8 = & 2048 \\
12^2 = 144 \times 3 = & 432 \\
16 \times 12 = 192 \times 4 = & 768 \\
\hline
 & 3248 \\
 & 20 \\
\hline
 & 64960 \\
 & .05236 \\
\hline
 & 389760 \\
 & 194880 \\
 & 129920 \\
 & 324800 \\
\hline
 & 3401.30560
\end{array}
\quad Ans.
$$

Ex.

Ex. 2. Required the ſolidity of the fruſtum of a parabolic ſpindle, whoſe length is 10, the diameters being 8 and 6.

Anſ. 425.1632.

Ex. 3. Required the ſolidity of the middle fruſtum of a parabolic ſpindle, whoſe length is 30 feet, and diameter 16 and 20. *Anſ.* 8243.5584.

SURVEYING.

SURVEYING of Land is conſidered to have been the primitive part of Geometry, and conſiſts of three principal parts, viz. The taking of the dimenſions, and making the neceſſary obſervations on the ground ;—the laying down the ſame in a map or drawing on paper or vellum ;—and the finding the content or area thereof.

The inſtruments commonly uſed in ſurveying of land, are, the gunter's chain, a caſe of inſtruments, a ſet of plotting ſcales, the theodolite, and plain table.

The gunter's chain, whether Scots or Engliſh, is divided into 100 links. The Engliſh chain is 66 feet, and the Scots 74; conſequently a link of the Engliſh chain is 7.92 inches, and that of the Scots 8.88 inches: likewiſe the Engliſh chain is divided into 4 poles or perches, each $16\frac{1}{2}$ feet, and the Scots chain into 4 falls, each $18\frac{1}{2}$ feet. 10 ſquare chains are 1 acre, either Scots or Engliſh; and 4 Scots acres are nearly equal to 5 Engliſh miles.

A Table of Scots Land Measure.

Sq. links.	Feet.				
$1\frac{40716}{49284}$	1	Ells.			
$17\frac{13}{36}$	$9\frac{73}{144}$	1	Falls.		
625	$342\frac{1}{4}$	36	1	Roods.	
25000	13690	1440	40	1	Acre.
100000	54760	5760	160	4	1

The Scots elwand is the foundation of all land-surveying in Scotland, the length of which is $37\frac{1}{5}$ inches, when compared with the English yard. It was first established by King David I.; the standard of which is kept in the Council Chamber of Edinburgh.

A Table of English Land Measure.

Sq. links.	Feet.				
$\frac{322}{1089}$	1	Yards.			
$20\frac{80}{121}$	9	1	Poles.		
625	$272\frac{1}{4}$	$30\frac{1}{4}$	1	Roods.	
25000	10890	1210	40	1	Acre.
100000	43560	4840	160	4	1

Writers

Writers on this ſubject are generally very prolix in deſcribing the method in which ſurveyors take dimenſions, uſe their inſtruments, &c. But it muſt be confeſſed, that the practice of a few hours in the field is preferable to all the deſcription that can be given. We ſhall therefore be very brief as to this particular, and ſhall only point out a method or two by which an irregular field may be meaſured, its plan delineated on paper, and its contents found.

Let the figure ABCDEF *Fig.* 1. repreſent a field, whoſe plan and area is required.

Firſt, walk over the field, and make the neceſſary remarks on the ground, and draw an eye-draught, or a repreſentation of the field, as exact as can be done by the ſight of the eye.

Divide this draught into triangles, rectangles, or trapezias, as the figure of the field directs. Erect poles at the different corners.

Chooſe any of the corners A for your firſt ſtation; provide yourſelf with a perſon to lead the chain, and let him have 10 arrows or iron pins in one hand, and the end of the chain in the other. You take your ſtation at A, while he advances the length of the chain towards B. Direct him, by waving your hand, to the right or left, till you find him in ſo ſtraight a line as to intercept the view of the pole B: Then ſtretch the chain at full length, and let him leave one of the arrows at the far end, as a mark for you to go to. In the mean time let him advance another chain-length towards B, directed to keep in a ſtraight line as above. At the end of the ſecond chain-length let him ſtick another arrow, and you take up the firſt and proceed to the next, where you are to ſtand till the chain is again ſtretched in the direction AB, and he put down another as a mark; which done, you take up the ſecond, and proceed to the third; and ſo on, till you come to B. The number of arrows taken up by you is the number of chain-lengths; and the diſtance

tance between the last arrow and the pole B is taken in links. Thus, when you arrive at B, you will have 6 arrows; and there are 90 links over which, together with the chain-lengths, you are carefully to mark on the corresponding line in your eye-draught. In like manner, proceed to measure the lines BC, CD, DE, EF, FA, and lastly the diagonals DB, DA, and EA.

Or otherwise,

The field may be measured thus:—Step over the straight line DB; and where you imagine the perpendicular Cc will cut it, set up the theodolite, directing the fixed sights in the direction DB, and the index to C: if it cuts the limb of the instrument at an angle of 90°, you have guessed right; but if it does not, go towards B or D till you hit the point, and there fix a pole. In the same manner, find the points b, e, in the diagonal DA, and x in the straight line AF. Then measure BD, Cc, DA, Ee, FA, Ex.

Mark down carefully on the eye-draught the segments into which the perpendiculars cut the lines BD, DA, AF, also the length of the perpendiculars on the corresponding lines.

By either of these methods, the plan of the field may be protracted, and its area truly cast up as follows:—Let ABCDEF be an irregular figure, whose measures and area are required.

AB	690	Cc=	586
BC	750	Ee=	$312\frac{1}{2}$
CD	700	Bb	$597\frac{1}{2}$
DE	450	Ex	$482\frac{1}{2}$
EF	540	DC	383
FA	745	DB	605
DB	850	DE	258
DA	950	Fx	239

When the three sides of each triangle are given, the following method is the best for finding the area. The arithmetical computations being intolerably laborious.

First,

First, LOGARITHMICALLY.

For the area DBC.

750	1150	1150	1150	1150=3.06070
700	750	700	850	400=2.60206
850				450=2.65321
	400	450	300	300=2.47712
2)2300				
				2)10.79309
1150				

Sq. links 249200 = 5.39654

For the area DBA.

850	1245	1245	1245	1245=3.09516
950	850	950	690	395=2.59660
690				295=2.46982
	395	295	555	555=2.74429
2)2490				
				2)10.90587
1245				

Sq. links 283800 = 5.45293

For DAE.

950	1050	1050	1050	1050=3.02119
450	950	450	700	100=2.00000
700				600=2.77815
	100	600	350	350=2.54407
2)2100				
				2)10 34341
1050				

Sq. links 148500 = 5.17170

For the area EAF.

700	992	992	992	992=2.99651
540	700	540	745	292=2.46538
745				452=2.65514
	292	452	247	247=2.39270
2)1985				
				2)10.50973
992				

Sq. links 179800 = 5·25486

249200	
283800	
148500	
179800	
8.61300	8.61300
4	4
2.45200	2.45200
40	40
18.08000	18.08000
36	$30\frac{1}{4}$
48000	240000
24000	2000
2.88000	2 42000

In Scots,					If the English chain,			
Anf.	*A.*	*R.*	*F.*	*Ells.*	*A.*	*R.*	*P.*	*Yds.*
	8	2	18	2	8	2	18	2

It muft be obferved, that, in the above example, the dimenfions are fet down in links, (as being the beft method) and not in chains and decimals of a chain, confequently the area is found in fquare links, and may be reduced to acres by cutting off five figures towards the right hand for decimals ; thofe remaining

ing as an integer are acres, and the reſt brought to value as above.

When the baſes and perpendiculars are given, the following method is to be uſed :—

For the area of DCB.	of DBA	of DAE.
DB=850	DA=950	DA=950
Cc 586	Bb =597½	Ee 312½
5100	6650	1907
6800	8550	950
4250	4750	2850
	475	475
2)498100	2)567625	2)296875
249050	283812	148437

For the area of EAF.	Now for the whole field.
FA=745	249058
Ex =482½	283812
1490	148437
5960	179731
2980	8.61030
372	4
2)359462	2.44120
179731	40
	17.64800
	36
	388800
	194400
	23.32800

	A.	R.	F	E
Anſ.	8	2	17	23.

PROBLEM I.

To find the area of a rectangular field.

RULE.

Multiply the length by the breadth, and the product is the area.

EXAMPLE I.

Required the area of a rectangular field, whose length is 1920 links, and perpendicular breadth 1200 links of the Scots chain.

```
     1920
     1200
---------
23.04000
        4
---------
  .16000
       40
---------
 6.40000
       36
---------
  240000
 120000
---------
14.40000
```

	A.	R.	F.	Ells.
Ans.	23	0	6	14

Ex. 2. How many Scots acres are in a field 3500 links long and 1400 broad? *Ans.* 49 acres.

Ex. 3. How many English acres are in a rectangular field 1400 links long and 1200 broad? *Ans.* 16 ac. 3 ro. 8 p.

Ex.

Ex. 4. Required the content of a rectangular field, the length being 2000 links, breadth 1000 links of the Scots chain. *Anſ.* 20 acres.

Ex. 5. How many acres are in a ſquare garden, whoſe ſide is 6 chain lengths Engliſh? *Anſ.* 3 ac. 2 ro. 16 p.

PROBLEM II.

To reduce Scots acres into Engliſh, and vice verſa.

RULE for Engliſh acres.

As the ſquare of 66
Is to the ſquare of 74,
So is any number of Scots acres
To the number of Engliſh acres required.

For Scots acres:

As the ſquare of 74
Is to the ſquare of 66,
So is any number of Engliſh acres
To the number of Scots acres required.

EXAMPLE I.

How many Engliſh acres are in 14 acres, 3 roods Scots?

```
  66      74
  66      74
 ----    ----
 396     296
396     518
-----   -----
4356  :  5476 :: 14.75
                 5476
                ------
                 8850
               10325
               5900
              7375
              ---------
        4356)80771.00(18 acres.
             4356
             -----
             37211
             34848
             -----
              2363
                 4
             -----
        4356)9452(2
             8712
             -----
              740
               40
             -----
        4356)29600
             26136
             -----
              3464
              40¼
            -------
            103920
              866
            -------
       4356)104786(24
            8712
            -----
            17666
            17424
            -----
              242
```

Ans. *A.* 18 *R.* 2 *P.* 6 *Yds* 24

Ex.

Ex. 2. How many Scots acres are in 17 acres 3 roods English? *Ans.* 14 ac. 0 ro. 19 falls 4 ells.

Ex. 3. How many English acres are in 400 Scots?

Ans. 502 ac. 3 ro. 19 p. 29 yds.

PROBLEM III.

To find the area of a triangular field.

If the base and perpendicular are given, workby Problem 5. of surfaces; if the three sides are given, by Prob. 7.; and if the two sides and the angle contained be given, by Prob. 6.

EXAMPLE I.

How many acres are in a triangular field whose three sides are 5600, 7000, 4200 links of the English chain?

5600				
7000				
4200				
	8400	8400	8400	8400=3.92428
2)16800	5600	7000	4200	2800=3.44716
				1400=3.14613
8400	2800	1400	4200	4200=3.62325
				2)14.14082
		Sq. links 11760000	=	7.07041

117.60000
4
2.40000
40
1600000

	A.	R.	P.
Ans.	117	2	16

Ex.

Ex. 2. Required the area of a triangular garden, whose side is 600, and the perpendicular falling upon it, from the opposite angle, 756 links of the Scots chain.

```
      756
      600
  -------
2)453600
  -------
  2.26800
        4
  -------
  1.07200
       40
  -------
  2.88000
       36
  -------
  528000
 264000
  -------
 31.68000
```

	A.	R.	F.	Ells.
Ans.	2	1	2	31

Ex. 3. How many acres are in a triangular field, whose two sides are 1900, and 1700 links of the English chain, and the angle contained between them 48° 13′?

```
      1900
      1700
  --------
  1330000
  1900
  --------
2)3230000
  --------
  1615000
```

As

As radius		90°	10.00000
Is to fine		48° 13′	9.87255
So is the product of the containing fides		3230000	6.70920
To twice the area	2408500		6.38175
Whereof the half is	12.04250		
	4		
	.17000		
	40		
	6.80000		
	$30\frac{1}{4}$		
	2400000		
	20000		
	24.20000		

	A.	R.	P.	Yds.
Anſ.	12	0	6	24

Ex. 4. Required the area of a triangular field, whofe three fides are 600, 1000, 800 links of the Scots chain.

Anſ. 2 ac. 1 ro. 24 falls.

Ex. 5. How many acres are in a triangular field, whofe bafe is 1900 links, and perpendicular 1500 links of the Englifh chain? *Anſ.* 14 ac. 1 ro.

Ex. 6. Required the area of a triangular field, whereof one of the angles is 54°, and containing fides 1400 and 1500 links of the Scots chain. *Anſ.* 8 ac. 1 ro. 39 f. 7 ells.

PROBLEM IV.

To find the area of a field in the form of a trapozoid. See Problem 8. of furfaces.

EXAMPLE I.

Required the area of a trapozoid, whofe parallel fides are

3000 and 1500 links of the Scots chain, and perpendicular distance 1200 links.

$$\begin{array}{r} 3000 \\ 1500 \\ \hline 2)4500 \\ \hline 2250 \\ 1200 \\ \hline 27.00000 \end{array}$$

Anſ. 27 acres.

Ex. 2. Required the area of a field in the form of a trapozoid, its parallel ſides being 1260 and 1500 links, and perpendicular breadth 1000 links of the Engliſh chain.

Anſ. 13 ac. 3 ro. 8 poles.

Ex. 3. How many acres are in a field in the form of a trapozoid, its parallel ſides being 1000 and 1200 links, and perpendicular breadth 650 links of the Scots chain?

Anſ. 7 ac. 0 ro. 24 falls.

PROBLEM V.

To meaſure off-ſets. Fig. 2.

In actual ſurveying, it often happens that a field is bounded by a river, a crooked hedge, &c. in which caſe it will be neceſſary to obſerve the following directions:—Let A b c d e f repreſent a river or hedge. From A, in the direction of the river, meaſure the ſtraight line AB. In doing of which, obſerve the bendings of the hedge; from thence meaſure the off-ſets perpendicular upon the ſtraight line AB, and note them down on the eye-draught, or record them in a field-book.

When the off-ſets are ſmall, meaſure them with an off-ſet ſtaff

ſtaff of 10 links; but when they are large, the chain is more expeditious.

Let	Ag 300	gb 130
	Ah 400	ch 160
	Ai 450	di 162
	Ak 500	ek 100
	Am 620	fm 78
	Ab 750	

Here the figure is divided into triangles and trapezoids. The moſt accurate method to find the area, is, to compute the area of each ſeparately by the rule for their proper form, and the ſum of theſe will be the area of the whole. Thus,

Ag 300	gh 100	hi 50	ik 50	km 120	mB 130
bg 130	ch+bg 290	ch+di 322	di+ch 262	178	78
2)3900	2)29000	2)16100	2)13100	2)21360	1040
					910
Agb 19500	14500	8550	6550	10680	2)10140
					5070

19500
14500
8050
6550
10680
5070

Anſ. 64350 ſq. links. *A.* 0 *R.* 2 *P.* 22 *Yds.* 29

Sometimes ſuch a figure as that above is computed by finding a mean breadth, and reckoning the product of the mean breadth into the whole length of the ſtation-line AB for the area. Thus, add all the off-ſet lines into one ſum, and divide it

 by

by their number, reckoning 1 for each time the irregular boundary meets the station-line, as at A and B; the quot gives the mean breadth, which, being multiplied into the length, produces the area.

However expeditious this method may be considered, it is always false, except in the case when the off-sets are equi-distant from each other, as may be seen from the following computation of the above figure.

130	750		*A.*	*R.*	*P.*	*Yds.*
160	90	False content	0	2	28	0
162		True ditto	0	2	22	29
100	.67500					
78	4	Difference	0	0	5	$1\frac{1}{4}$
7)630	2.70000					
	40					
90	28.00000					

PROBLEM VI.

To find the area of an irregular field. Fig. 3.

RULE.

Compute the areas of the figures into which the field is divided, whether triangles or trapeziums, &c. by the rules proper for the several figures; add the several results together, and the sum will give the content.

Let

Let AB=820
BC=434
CD=860
DE=400
EA=530
Ee=355
Bb=360
Ex=300
EC=650

When the irregularities of the boundaries of a field are numerous, it may not be improper to recommend a field-book, in which the ſeveral meaſures are to be recorded, to prevent confuſion. But when the field is not very irregular, all the meaſures may, with equal advantage, be marked upon an eye-draught of the field, each againſt the correſponding parts of the figure. And either of theſe methods may be practiſed, whether the ſurvey be large or ſmall.

There is no particular form for the field-book; every one rules and contrives as he judges moſt proper for himſelf; but, to avoid perplexity, the ſimpleſt form is the beſt. The following is a ſpecimen of a method generally practiſed. It is divided into three columns; in the middle is marked the ſtations, bearing, and diſtances meaſured. On the right hand, the offſets are marked againſt their correſponding diſtances in the middle column, together with ſuch other remarks as occur in meaſuring, ſuch as houſes, hedges, ponds, roads, &c. In the left hand column are marked the inlets againſt their correſponding diſtances in the middle column, and remarks, as above.

N. B. The *inlets* are perpendiculars dropt from ſuch irregularities as fall within the ſtation-line. The area of which is to be ſubtracted from the general content of the field.

The meaſures of the preceding figure may be arranged in a field-book as follows:—

THE

THE FIELD-BOOK.

Inlets and Remarks.	Station, Bearing, Distances.	Off-sets and Remarks.
	STATION I.	
	100	40
	135	60
	470	0
	680	0
	700	30
	820	0
	STATION II.	
	0	0
	220	58
	280	0 a hedge.
	432	
	STATION III.	
0	0	
62	175	62
0	330	0
	400	40
	550	0
	700	50
	860	
	STATION IV.	
	100	0
	220	62 a house.
	400	0
	STATION V.	
	0	0
	50	45
	110	55
	130	65
	170	50
	250	0
	340	63
	380	46
	440	52
	530	0

The bearings, diſtances, off-ſets, &c. ought to be recorded in the field-book immediately when taken, otherwiſe material miſtakes may be committed. The field-book may be made up thus: Suppoſe A the firſt ſtation, and AB the firſt line meaſured. In the middle column mark .1 for the firſt ſtation; next find by the theodolite the quantity of the angle BAE, which inſert in the middle column. Then write a cypher below to denote the ſtation, and another in the right hand column to ſignify that at the ſtation A there is no off-ſet; and at the diſtance of 100 links from A, in the direction AB, is an off-ſet of 40 links. Regiſter the diſtance 100 in the middle column; and againſt this diſtance, in the middle column, write the off-ſet 40 in the right hand column. Again, at the diſtance of 135 links from A, in the direction AB, is an off-ſet of 60 links; mark the diſtance 135 in the middle column; and right oppoſite to 135 in the middle column, write the off-ſet 60 in the right hand column. At the diſtance of 470 from A, in the direction AB, the crooked boundary touches the ſtation-line AB; in which caſe the diſtance 470 is marked in the middle column, and a cypher in the right hand column, there being no off-ſet. At the diſtance 680 from A, the irregular boundary again deviates from the ſtation-line AB. The diſtance 680 is marked in the middle column, and the cypher at the right-hand ſide, as above. At the diſtance of 700 links from A, is an off-ſet of 30 links; mark theſe as above. *Laſtly*, Mark the whole length of the line AB 850, then draw a ſtroke. In like manner mark B .2, alſo the angle ABC; and proceed to meaſure BC as above; and ſo on, till all the boundaries are meaſured.

If, in planning the field, the direction EA does not paſs through the point A with the meaſured diſtance EA, ſome error has been committed, and the work muſt be reviſed over again. It may ſave much trouble to know whether the miſtake has ariſen from the angles or from the diſtances: If the angles are right, the diſtances alone are to be meaſured.

To

To know if the angles have been accurately taken, add all the inward angles into one ſum; and when the work is right, their ſum is equal to twice as many right angles as the figure has ſides, wanting 4 right angles, (Euclid 31. 1. Cor. 1.) Or, inſtead of the inward angles, their ſupplements may be added into one ſum; and if it is equal to 360°, the angles have been taken right, (Euclid 31. 1. Cor. 2.) becauſe all the exterior angles of any rectilineal figure are together equal to 4 right angles.

Few directions for planning may ſerve for any one who has ſtudied and underſtands the uſe of his inſtruments. It may, indeed, be neceſſary to mention, that all plans of ſurveys ought to be laid down ſo, as the north ſide may ly towards the top of the paper, the eaſt towards the right hand ſide, the weſt to the left, and the ſouth to the bottom. Likewiſe it is cuſtomary to draw a meridian line, with a *flower-de-luce* directed to the top of the map or plan, to point out the north.

When the plan of riſing-ground is to be made out, the hypothenuſal lines muſt be reduced to a level, otherwiſe the plan will be diſtorted; and when a mountain is to be repreſented on a plan, the baſe only is taken; and in computing its content, as well as in planning it, this ſhould be conſidered, that the baſe of the mountain will contain as many growing trees as its ſurface *.—We ſhall ſubjoin a table for making the neceſſary deductions to reduce hypothenuſal lines to a level; and theſe allowances may be made immediately when meaſured, before the meaſures are recorded in the field-book, or when the plan is to be protracted.

A

* This may appear a paradox to ſome, who perhaps never obſerved, that trees grow perpendicular to the horizon, or parallel to each other.

A TABLE *for reducing Hypothenusal Lines to a Level, from* 1° *to* 45°

Degrees of Inclination.	Deductions.	Level.	Degrees of Inclination.	Deductions.	Level.	Degrees of Inclination.	Deductions.	Level.
1°	0	100	16°	3.9	96.1	31°	14.3	85.7
2°	.1	99.9	17°	4.4	95.6	32°	15.2	84.8
3°	.1	99.9	18°	4.9	95.1	33°	16.1	83.9
4°	.2	99.8	19°	5.5	94.5	34°	17.1	82.9
5°	.4	99.6	20°	6.0	94.0	35°	18.1	81.9
6°	.5	99.5	21°	6.6	93.4	36°	19.1	80.9
7°	7.	99.3	22°	7.3	92.7	37°	20.1	79.9
8°	1.0	99.0	23°	7.6	92.4	38°	21.2	78.8
9°	1.2	98.8	24°	8.0	92.0	39°	22.3	77.7
10°	1.5	98.5	25°	8.6	91.4	40°	23.4	76.6
11°	1.8	98.2	26°	9.4	90.6	41°	24.5	75.5
12°	2.2	97.8	27°	10.1	89.9	42°	25.7	74.3
13°	2.6	97.4	28°	10.9	89.1	43°	26.9	73.1
14°	3.0	97.0	29°	11.7	88.3	44°	28.1	71.9
15°	3.4	96.6	30°	13.4	86.6	45°	29.3	70.7

In the foregoing table, we have the allowance to be made for every chain-length, from 1° to 45°. To reduce an inclined field to a level, let it be required to find the deduction on 10 chain-lengths, upon a declivity of 24°. Againſt 24° in the column of degrees, we have 8 in the column of deductions; that is, 92 links on the level, which, multiplied by 10, gives 920 links; and ſo on of the reſt.

The ſtraighteſt line that can be meaſured by the chain is, for various reaſons, longer than the true quantity. All decimals of links ſhould, therefore, be neglected; and, indeed, a moderate and judicious allowance ſhould be made on integers themſelves, except when the ground is ſmooth and plain.

The area of the above figure is computed as follows: *Fig.* 3.

For the triangle ABE.

```
     820
     355
    ----
    4100
   4100
  2460
  ------
2)291100
  ------
  145550 ſq. links.
```

For the triangle BEC.

```
     650
     360
  ------
   39000
  1950
  ------
2)234000
  ------
  117000
```

For the triangle ECD.

```
     860
     300
  ------
2)258000
  ------
  129000
```

To

To find the off-sets on the first station-line AB.

```
   100          60            140
    40          40             30
  ----         ---           ----
2)4000         100         2)4200
  ----         135           ----
  2000         ---           2100
           2)13500
             -----
              6750
```

To find the area of the off-sets on the second line BC.

```
    280
     58
   ----
   2240
   1400
   ----
2)16240
  -----
   8120
```

To find the area of the off-sets on CD.

```
   220             310
    40              50
  ----           -----
2)8800         2)15500
  ----           -----
  4400            7750
```

To find the area of the off-sets on DE.

```
    300
     62
  -----
2)18600
  -----
   9300
```

To find the area of the off-sets on EA.

50	45	55	65	80	63	46	46
45	55	65	50	50	90	63	52
2)2250	100	120	115	2)4000	2)5670	109	98
1125	60	20	40	2000	2835	40	60
	2)6000	2)2400	2)4600			2)4360	2)5880
	3000	1200	2300			2180	2940

52
90
2)4680
2340

To find the inlets on CD.

330
62
660
1980
2)20460
10230

The triangles,	145550
	117000
	129000
The off-sets,	2000
	6750
	2100
	8120
	4400
	7750
	9300
	1125
	3000
	1200
	2300
	2000
	2835
	2180
	2940
	2340
	451890
Inlets	10230
Ans.	441660 in sq. links.

4.41660
4
1.66640
40
26.65600
36
393600
196800
23.61600

	A.	R.	F.	E.
Anſ.	4	1	26	23

PROBLEM VII.

Of the Plain Table.

The plain table conſiſts of a plain rectangular board of any convenient ſize, fitted in a frame of wood, ſo as it can be taken out or put in at pleaſure for the convenience of putting a ſheet of paper upon it.

One ſide of the frame is divided into degrees for the purpoſe of taking angles; the other is uſually divided into equal parts for drawing lines on the table, either parallel or perpendicular to the ſides.

The plain table is provided with an index, either with open ſights, or a ſmall teleſcope: And that edge of the index, which is in the ſame plane with the ſights, is called the *fiducial edge*.

A magnetic needle and compaſs is fixed in one ſide of the plain table, to point out the direction. It is fixed to a ſtand of a convenient height, and moves upon an univerſal joint, by which means it will incline in any direction, and, being ſcrewed faſt in the ſocket, it will retain any ſituation given it.

The plain table is one of the moſt expeditious inſtruments ſurveyors uſe; for no ſooner are the different angles taken, and the

the diſtances marked on the plain table, than a plan of the field is obtained: and this may be done by taking a ſtation within the field, or by chooſing one of the corners for a ſtation, or otherwiſe by going round the field.

1. By taking a ſtation within the field—

Let ABCDE, *Fig.* 4. be a field, and O an eminence within the field. Plant the table at O, and ſcrew it with the needle north. Mark O upon your paper, and apply the index to O, directing it to the corner A, till through the ſights you ſee A; then draw an obſcure line along the fiducial edge of the index to repreſent the direction OA. Then turn the index, till through the ſights you ſee B, and draw an obſcure line from O along the fiducial edge of the index to repreſent the direction OB. In like manner, apply the index ſucceſſively to O, turning it round with the ſights to the remaining angles C,D,E, drawing the obſcure lines OC, OD, OE; then with the chain meaſure the ſtraight lines OA, OB, OC, OD, OE, and mark the reſults upon the correſponding lines on the table. Join their extremities AB, BC, CD, DE, EA, and the thing is done.

When the plain table has degrees marked on it, the quantity of the angles may be marked immediately when taken; otherwiſe they may afterwards be meaſured from a line of chords or protractors—and the area found, as taught above.

2. When from one of the angles all the reſt may be ſeen, let the point A be an angle from whence the reſt may be ſeen; there fix the ſtation. Turn the table till the middle point to the flower-de-luce; ſcrew your inſtrument faſt; then turn the index till through the ſights you ſee the corner B, and draw an obſcure line along the fiducial edge of the index, to repreſent the direction AB: again turn the index, till through the ſights ſucceſſively you ſee the corners C, D, E, and to each of theſe draw obſcure lines: Then with the chain meaſure the lines AB,

AB, AC, AD, and note them down on the paper from a ſcale of equal parts, each againſt the correſponding line on the ground. Join their extremities, and the thing is done. By this method alſo the angles may be found, and the area computed, as above.

3d, By going round the field,

When woods, waters, or other interruptions happen to be in the way, a field may be maeſured by taking all the angles and ſides in their order: Thus, begin at A, after having fixed your inſtrument, the needle pointing to the flower-de-luce, turn the index till through the ſights you ſee the corners E, B, and along theſe directions draw obſcure lines, meeting at A. Meaſure theſe lines AE, AB. and note them down from a ſcale of equal parts. Then remove your inſtrument from A to B; there fix it again, and turn the table about till through the ſights, along the ſtraight line AB, you ſee the point A. In this poſition, alſo, the needle will, of itſelf, point to the flower-de-luce. Here turn about the index till through the ſights you ſee the corner C; meaſure it, and mark it down from a ſcale of equal parts upon the obſcure line drawn in the direction BC. Again carry the inſtrument to C, there fix it again, and lay the index from C along the ſtraight line DC, and ſcrew the table faſt: obſerve if the needle point to the flower-de-luce; turn the index about to D, and draw the obſcure line CD; meaſure it and mark it down from the ſame ſcale of equal parts. In the ſame manner find the remaining ſides and angles till you end at the firſt ſtation, and it is done.

PROBLEM

PROBLEM VIII.

To ſurvey a field by the theodolite.

I. From a point within the field—

Place the theodolite at O, (*Fig. laſt Prob.*) and turn it about till the fixed ſights point to any object, as A; ſcrew the inſtrument faſt, and turn about the moveable index till through the moveable ſights you ſee B, and note the degrees cut on the limb of the inſtrument in a field-book, or rather on an eye-draught, then turn the index in the directions C, D, E, and record the angles as above. Then meaſure the lines OA, OB, OC, OD, OE, and it is done.

2d, From one of the angles.

Chooſe any angle for a ſtation, from whence all the other angles may be ſeen. Suppoſe angle A, as in *fig. Prob.* 7. Find the number of degrees in the angles BAC, CAD, DAE, and mark them down upon the eye-draught, each againſt the correſponding parts of the field; then meaſure the ſtraight lines AB, AC; AD, AE; mark theſe upon the eye-draught; ſo the plan may be made out, and the area found, as ſhewn above.

PROBLEM IX.

To ſurvey a field by two ſtations.

By this method, grounds may be planned and ſurveyed without entering upon them. This is performed by chooſing two ſtations, either within or without the field, from whence all the angles, ponds, houſes, croſs-hedges, roads, rivers, &c. &c. may be ſeen. Either the theodolite or plain table may be uſed.

Let

Let ABCDEF be a field whose plan is required, and let the points 1. 2. be eminences, from which a sufficient view of the field is obtained. First, Place the instrument at 1, and take the angles AIB, BIC, CID, DIE, EIF, FIA, and draw lines in the several directions of these angles, and any other remarkable object. Then measure the distance from 1 to 2, and place your instrument at 2; thence drawing lines in the directions of all the angles, and of such other objects as have been noted from the former station. Then these observations being planned, the intersection of the lines will point out the several corners and objects required.

When two stations are not sufficient, three or more may be chosen, as the surveyor shall see cause, measuring the distance between each station. And the intersections of the lines point out the objects, with their proper places on the plan. By this method very extensive surveys may be taken.

OF DIVIDING, OR LAYING OUT GROUND.

PROBLEM I.

To lay out rectangular ground.

RULE.

Divide the given area by the given side, and the quotient will be the side required.

EXAMPLE. I.

What length of a rectangular field, whose breadth is 400 links, will make 3 acres 2 roods?

400)3.50000

Ans. 875

Ex. 2. What length of a ridge, 8 ells broad, will make 15 falls? *Ans.* $67\frac{1}{2}$ ells.

Ex. 3. What length of a ridge, 40 feet feet broad, will make 3 roods of land? *Ans.* $816\frac{3}{4}$ feet.

Ex. 4. A field contains 32 acres 3 roods Scots, the breadth being 510 links, required the length. *Ans.* 6422 links nearly.

Ex. 5. The length of a rectangular field, being 6575 links, required its breadth to contain 328 acres 3 roods.

Ans. 5000 links.

Ex. 6. A square field of 15 acres 2 roods 20 poles: It is required to find the length of its side. *Ans.* 1250 links.

Ex. 7. Required the side of a square field in yards, whose content is 30 English acres. *Ans.* 831 nearly.

Ex. 8. What length of a rectangular field will make 25 English acres, the breadth being $193\frac{1}{7}$ yards? *Ans.* 625 yards.

PROBLEM II.

To lay out a triangular field.

RULE I.

When the base is given, to find the perpendicular: Divide the area by the base, and twice the quotient will give the perpendicular.

RULE 2. When the perpendicular is given, to find the base: Divide the area by twice the perpendicular, and the quotient is the base.

RULE 3. When any part of a given triangle is to be cut off by a line parallel to one of the sides, it must be remembered that similar surfaces are to one another as the squares of their corresponding sides; and *vice versa.*

EXAMPLE

EXAMPLE I.

Required the perpendicular breadth of a triangular field, the base being 520 links, and content 9 acres.

$$
\begin{array}{llll}
520)9.00000)1730\tfrac{10}{13} & \\
\quad 520 & \quad\quad 2 \\
\hline
\quad 3800 & 3461\tfrac{7}{13} \text{ links.} \\
\quad 3640 & \\
\hline
\quad\; 1600 & \\
\quad\; 1560 & \\
\hline
\quad\quad 400 &
\end{array}
$$

Ex. 2. A triangular field of 630 acres is to be divided equally between two farmers; the base measures 6000 links, and the march is to be drawn parallel to the base; required how much of the perpendicular will fall to each.

$$
\begin{array}{l}
6.000)63000.000 \\
\quad\quad 10500 \\
\quad\quad\quad\;\; 2 \\
\hline
\quad\quad 21000 \text{ the whole perpendicular.}
\end{array}
$$

$320 : 160 :: 21000^2$

Or rather $2 : 1 :: 441000000$

```
              1
        ---------
       2)441000000
        ---------
         220500000(14849
         1
        ---
     24)120
         96
        ---
    288)2450
        2304
        ----
   2964)14600
        11856
        -----
  29689)274400
        267201
        ------
          7199
```

The whole perpendicular	21000 links.
One of the farmer's ſhare is	14849 links.
Conſequently the other's ſhare is	6151 links.

Note. The greater part of the perpendicular will fall to him whoſe diviſion lies towards the vertex.

Ex. 3. Suppoſe a field in the form of a right-angled triangle, whoſe baſe is 2500, and perpendicular 3000 links of the Engliſh chain, and that a hedge is planted parallel to the perpendicular, cutting off 20 acres, required the expence of planting the hedge, at 1s. 6d. *per* yard. *Anſ.* 36l. 3s. $0\frac{1}{4}$d.

Ex. 4. How long is the baſe of a field of 35 acres 3 roods 20 poles, the perpendicular being 3550 links?

Anſ. $2021\frac{7}{10}$ nearly.

PROBLEM

PROBLEM III.

To lay out a given area from an irregular field.

EXAMPLE I.

Let it be required to cut off $1\frac{1}{2}$ acres towards the north side of the irregular field ABCD. *Fig.* 4.

THE FIELD-BOOK.

Inlets.	Stations and Distances.	Off-sets.
	. 1. A.	
	0	0
	80	40
	160	42
	180	30
	240	40
	270	60
	360	38
	470	12
	. 2. B.	
	535	
	. 3. D.	
0	0	
20	60	
20	120	
0	180	
16	200	
0	230	
	300	38
	375	40
	470	30
	. 4. C.	
	535	

80	40	42	30	40	60	38
40	42	30	40	60	38	12
2)3200	82	72	70	100	98	50
1600	80	20	60	30	90	110
	2)6560	2 1440	2)4200	2)3000	2)8820	2)5500
	3280	720	2100	1500	4410	2750

1600	Now an acre is	100000 sq. links.
3280		
720	Whereof the ½ is	50000
2100		
1500	Equal to 1½ acres,	150000
4410		
2750	Subtract the off-sets on AB,	16360
16360 sq. links.	There remains	133640

Now by Prob. I.

```
470)133640(284 nearly=Ax or Bx
    940
    ----
    3964
    3760
    ----
     2040
     1880
     ----
      160
```

Ex. 2. It is required to lay off 2 roods towards the south side of the same field, and to know how far up the lines AC, BD, the march-line must be struck. *Ans.* $92\frac{3}{4}\frac{8}{7}$ links.

GUNNERY.

GUNNERY.

Gunnery is the art of charging, directing, and exploding fire-arms, ſuch as cannons, mortars, &c. to the beſt advantage. To this art belongs the knowledge of the force of gunpowder, the dimenſions of cannon, the proportion of powder and ball they carry.

From experiment and obſervation alone the hiſtory of nature can be collected, or her *phenomena* deſcribed. By the principles of geometry and mechanics we are enabled to carry on the analyſis from the phenomena to the powers or cauſes that produce them.

The ſame power which renders bodies heavy when at reſt, accelerates their motion when they deſcend in the direction of their gravity; and, if projected in any other direction, bends their motion into a curve line, which, from its properties and flexure, is known to be a parabola. For every body, projected into the air, moves under the influence of two diſtinct forces, viz. its projectile force, and that of gravity. By the firſt, it is carried forward with an equal motion, and deſcribes equal ſpaces in equal times. By the latter, it is drawn downwards in lines perpendicular to the horizon, with a motion inceſſantly accelerated. If either of theſe forces were deſtroyed, the body would move for ever in the direction of the remaining force alone, (if its motion was not hindered by the interpoſition of other bodies;) but, as both continue to act, the courſe of a projectile muſt be determined by a power compounded of theſe two forces.

Definitions.

Definitions.

1. The impetus of a piece is the perpendicular height to which it would ſhoot a ball with its ordinary charge of powder; or the height from which it muſt fall perpendicularly to acquire the velocity with which it was projected.——Thus, BA is the impetus. *Fig.* 1.

2. The diameter, or axis to any point of the curve, is a line drawn from that point perpendicular to the horizon. Thus, HQ is the diameter to the point H.

3. The point H is called the vertex.

4. The ordinates to any diameter are lines drawn parallel to the tangent, where the diameter cuts the curve. Thus GK is an ordinate to the axis HQ.

5. The abſciſs is that part of the diameter intercepted between the ordinate and the curve. Thus, HQ is an abſciſs of the diameter HF.

6. The altitude of the curve is the perpendicular height of the vertex above the horizontal plane. Thus, HQ is the altitude of the curve AHK.

7. The amplitude is the diſtance between the object aimed at and the piece, and is ſometimes called the random, or range. Thus, AK is the amplitude of the curve ABK.

8. The elevation of the piece is the angle its direction makes with the horizontal plane.

9. The inclination of a plane is the angle it makes with the horizon, and is either elevated or depreſſed.

10. The directrix is a line parallel to the horizon, and whoſe diſtance from the horizon is the impetus.

N. B. The vertex is equidiſtant from the directrix and focus.

The focus may be found by various methods. Theſe following are moſt commonly uſed.

PROBLEM.

PROBLEM. *Fig. 1.*

To describe the path of a projectile.

Draw AL the horizontal plane, and, from a scale of equal parts, lay off the amplitude AK, and through the point A erect a perpendicular AB equal to the impetus taken from the same scale; through B draw the directrix parallel to AK; then bisect AK in Q, and draw QN at right angles to AK; upon A, as centre with the distance AB, describe the semicircle BFfR, and the point F is the focus. Or,

If the direction AD is given, upon AB, as diameter, describe a semicircle BDA; and through the point of intersection D draw BD, and produce it to F; so shall BD and DF be equal, and F will be the focus. Or,

Through the point D draw PD parallel to the horizon; then shall PD=DH, and NH=HF, and H will be the vertex.

Cor. 4. times PD is equal to the amplitude.

Then proceed as shewn in Prob. 9. *conic sections.*

PROBLEM I.

The impetus of a piece and the angle of elevation being given, to find the amplitude.

EXAMPLE I.

How far will a cannon, whose impetus is 1200 feet, carry, at an elevation of 30°?

Geometrically.

Let AB represent the impetus of the piece, or the velocity a heavy body would acquire in falling from B to A. Through the point A draw the horizontal line AL, and make the angle

LAM equal to the angle of elevation. From the centre A, with the radius AB, describe the semicircle BFOfR; its circumference shall be the locus of the foci of all the parabolas that can be described by a projectile thrown from A, with the velocity it could acquire in falling from B to A; for, by a known property of the parabola, the distance of the focus from A is always equal to one-fourth of the parameter of the diameter that passes through A, that is, to AB; all the foci must, therefore, be found in the semicircle BFOFR. It will therefore be easy to determine the parabolas, when the direction of the projectile is given; for if, upon the impetus AB, you describe a semicircle BDdA, you need only join BD, and lay off BD equal to DF, and F will be the focus; and if through F you draw the line QF perpendicular to the horizontal line AL, it shall be the axis; and H, the middle point between F and N, shall be the vertex of the parabola. $4 \times FH$ is the length of the parameter of the axis.

If a line HP be drawn through the point H perpendicular to AB, the straight line BF and PH will bisect each other; also AM, the line of direction, will pass through the point of intersection in D, and bisect the line BF at right angles; and therefore the semicircle BDdA will pass through the same point D.

The amplitude of any parabola is equal to four times the sine of twice the complement of the angle of elevation: PD is the sine of the angle PCD, and the angle PCD is twice the angle PAD, because the one is at the centre and the other at the circumference; but the angle PAD is the complement of the angle of elevation DAK; therefore PD is the sine of twice the complement of the angle of elevation; and 2PD is equal to PH; but 2PH is equal to AK; therefore AK is equal to 4PD.

Hence it will follow, that when the angle of elevation becomes 45°, the points F and Q shall fall in the point O, and AK becomes twice the impetus. The sine PD is the co-sine of double

double 45°, which is the fine of 90°, or the radius; and, as the fine of 90° is the greateft, we may infer, that if a body is projected with an elevation of 45°, it will be carried farther on the horizontal plain, than, if projected with the fame velocity, in any other direction.

Alfo, If of two directions the elevation of the one exceeds 45° as much as the elevation of the other wants of 45°, their amplitudes will be equal, for the angles are complements of each other, and the fines of double of thefe angles muft be equal, becaufe they are fupplements to two right angles to one another; but the amplitudes of the parabola is always quadruple of thefe fines, and therefore they muft alfo be equal.

To find the amplitude by trigonometry.

As radius 90	-	-	-	-	10.00000
Is to twice the impetus 2400		-	-		3.38021
So is fine twice the elev. 60°		-	-		9.93753
To amplitude 2078 =		-	-	-	3.31774

EXAMPLE II.

Let the impetus be 3600, and the angle of elevation 75°. re- the amplitude.

As radius 90	-	-	-	-	10.00000
Is to twice the impetus 7200,		-	-		3.85733
So is fine twice elev. 150°	-	-	-		9.69897
To amplitude 3600 =		-	-	-	3.55630

From the preceding example, it is evident, that the impetus of a piece is equal to the amplitude, when fired off at the angles of 15° or 75°.

By Scale and Compasses.

In Ex. I. Extend the compasses from the radius to the sine of 60°, the same extent will reach from 2400 on the line of numbers, to 2078, the amplitude required.

PROBLEM II.

The amplitude and impetus being given, to find the elevation.

EXAMPLE I.

At what elevation will a mark be hit, distant 5100 yards, the impetus being 3000?

As twice the impetus 6000 - - -	3.77815
Is to radius 90° - - - - -	10.00000
So is amplitude 5100 - - -	3.70757
To sine twice, elevation 58° 13′ = -	9.92942

$$\left.\begin{array}{ll}\text{Lower elevation} & 29\ \ 6\\ \text{Higher elevation} & 60\ \ 54\end{array}\right\}$$

Ex. 2. At what elevation will a mark be hit, distant 1800 yards, the impetus being 900 yards? *Ans.* 45°

Ex. 3. At what angle will an object be hit, distant 4200 yards, the impetus being being 4000? *Ans.* $\left\{\begin{array}{l}15°\ \ 50'\ \text{lowest.}\\ 74°\ \ 10'\ \text{highest.}\end{array}\right.$

By Scale and Compasses.

The extent from twice the impetus on the line of numbers, to the amplitude, will reach from the radius on the lineof sines, to the sine of double the elevation.

PROBLEM

PROBLEM III.

Given the amplitude and the angle of elevation, to find the impetus.

EXAMPLE I.

What impetus will carry a ball 3520 yards, at an elevation of 30° or of 60°?

As fine twice elev. 30°	-	-	-	9.93753
Is to radius 90°	-	-	-	10.00000
So is $\frac{1}{2}$ amplitude 1760	-	-	-	3.24551
To impetus 2032 =	-	-	-	3.30798

By Scale and Compasses.

The extent from twice the angle of elevation on the line of fines, will reach from $\frac{1}{2}$ amplitude; on the line of numbers, to the impetus.

Ex. 2. The amplitude is 3000, and the direction 45°, required the impetus. *Anf.* 1500.

Ex. 3. The amplitude is 5200, and elevation 75, required the impetus. *Anf.* 5200.

PROBLEM IV.

The amplitude and direction being given, to find the height of the projection.

EXAMPLE. I.

The amplitude being 1200 yards, and elevation 30°, required the height of the projection.

As

As radius 90	- - - -	10.00000
Is to tangent elev. 30°,	- -	9.76144
So is $\frac{1}{4}$ amplitude 300	- - -	2.47712
To the height of the projection 174.3 =	-	2.23856

By Scale and Compasses.

The extent from the tangent of 45°, or radius, on the line of tangents to the angle of elevation, will reach backward on the line of numbers from $\frac{1}{4}$ the amplitude to the altitude required.

Ex. 2. Given the impetus 4000, and amplitude 4200, required the greatest altitude of the ball. *Ans.* 298.

PROBLEM V.

The altitude and elevation being given, to find the amplitude.

EXAMPLE I.

Let the altitude be $173\frac{1}{5}$ yards, and elevation 30°, required the amplitude.

As tangent angle elevation	30° =	- -	9.76144
Is to radius	90	- -	10.00000
So is the greatest alt.	173.2	- - -	2.23855
To $\frac{1}{4}$ amplitude	300	- - -	2.47711
	4		
Amplitude	1200		

By Scale and Compasses.

The extent from the tangent of 30° to 45°, or radius, will reach forward, on the line of numbers, from 173.2, to 300 one-fourth part amplitude.

Ex.

Ex. 2. The altitude 368, and elevation 40° 15′, required the amplitude. *Anʃ.* 1738.

PROBLEM VI.

The elevation and amplitude being given, and any other direction, to find the amplitude for that direction.

EXAMPLE.

The direction MAK, 50° 15′; its amplitude AK is 7000; any other direction, 32° 30′ being given, to find the amplitude for that direction, the piece being the ſame.

As the ſine of twice the 1ſt elev. 50° 15′	-	9.99267
Is to the 1ſt amplitude 7000,	- -	3.84510
So is the ſine of twice the 2d elev. 32° 30′	-	9.95728
		13.80238
To the amplitude required, 6452,	- -	3.80971

By Scale and Compaſſes.

The extent from 79° 30′ to 65° on the line of ſines, will reach backward, on the line of numbers, from 7000 to 6452, the amplitude required.

Ex. 2. The angle of elevation is 28° 12′; its amplitud is 5100, and any other direction 37° 28′, required the amplitude for that direction. *Anʃ.* 5912.

PROBLEM VII.

The greateſt altitude of a ball, with the elevation, and any other altitude, not greater than the impetus, being given, to find the elevation with which the ball was projected.

EXAMPLE.

EXAMPLE.

A cannon being fired at an angle of 24° 5′, the greatest altitude of the ball 180 yards; another was fired off, and the greatest altitude of the ball was 400 yards; at what angle of elevation was the cannon fired off the second time?

As the first altitude 180 =	-	-	2.25527
Is to the versed sine of twice the first elev. 48° 20′			4.52249
So is the second altitude 400	-	-	2.60206
			7.12455
To the versed sine of twice the ang, of the second elevation,	74° 56′		4.86928
The elevation required,	37° 28′		

By Scale and Compasses.

The extent from 180, on the line of numbers, to 400, will reach from 48° 10′ to 74° 56′ on the line of versed sines.

Ex. 2. A ball was projected at an angle of 40° 30′, its greatest altitude being 500 yards; afterwards another was projected, whose altitude was 400, required the elevation of the piece.

Ans. 35° 31′.

Ex. 3. The greatest elevation of a ball being 450 yards, the elevation 36° required the elevation of another projection, the greatest altitude being 240 yards. *Ans.* 25° 25′

PROBLEM VIII.

The elevation and amplitude being given, to find the time of the flight.

EXAMPLE.

How long will a ball, fired off at an angle of 58°, remain in the air, the amplitude being 528c feet?

As

As radius - - - -	90° =	10.00000
Is to amplitude - - -	5280 =	3.72263
So is tangent elevation - -	58° =	10.20421
To the ſquare of 4 times the ſeconds	8450	3.92684
The ſquare root of which is	92 nearly.	
Whereof the one-fourth is	23 ſeconds of time.	

This Problem is neceſſary in adjuſting the fuſee of bombs, which are generally fired off at an angle of 45°.

It is common among gunners to find the angle between the object and the zenith, and take the complement of half that angle for their elevation. And becauſe a leſs charge of powder will ſerve with this elevation than with any other, they find, by trial, what charge will reach the object.

PROBLEM IX.

The amplitude of the projectile, with a given charge of powder being given, to find what charge of powder will be neceſſary to hit an object at any other diſtance, (not greater than the outmoſt range) the elevation being the ſame.

EXAMPLE.

If 16 lb. of powder will ſhoot a cannon ball to the diſtance of 6000 yards, required the neceſſary charge to ſhoot the ſame ball 5000, with the ſame elevation.

As the firſt amplitude 6000 - -	3.77815
Is to a charge of 16 lb. - - -	1.20412
So is the given amplitude 5000 - -	3.69897
To the charge required, $13\frac{1}{3}$ lb. = -	1.12492

P p Or

Or ſay, numerically,

As 6000 : 16 : : 5000 : $13.\frac{1}{3}$ lb.

PROBLEMS *on Aſcents and Deſcents.*

A projectile thrown on an aſcent, with the velocity it would acquire in falling from B to A, in the direction AE, will ſtrike the line AN in K, ſo that AK will be equal to 4CD. Suppoſing the angle KAG a right angle, the angles GAB=GBA, and that a ſemicircle on G, as centre with the radius GB, cuts the line of direction in D, and that DC is parallel to AN, meeting AB in the point C.

Becauſe the angles KAD, ADC, are equal, being the alternate angles, and AK touches the circle, and AD cuts it, the angles KAD, DBA, are equal; therefore the angle DBA= CDA, conſequently the triangles ACD, ADB, are ſimilar, having the angle at A common; therefore AC : AD : : AD : AB.

Again: Becauſe the triangles ACD, PAK, are ſimilar, AP : PK : : PK : 4AB; therefore $AD=\frac{PK}{4}$, conſequently $CD=\frac{AK}{4}$.

Cor 1. Through D draw a line parallel to AB, cutting the circle in Dd, and join AD, then will the projectile, thrown in the direction Ad, ſtrike the line NA in the point k, for CD= $cd=AB=\frac{AK}{4}$.

Cor. 2. Parallel to AB draw HL, a tangent to the circle in H, join AH, then ſhall AH be the direction which ſhall carry the projectile fartheſt on the line AN; becauſe, when D coincides

cides with H, CD is the greateſt poſſible, and conſequently AK (4CD) is the greateſt diſtance the projectile can be carried to, with the velocity acquired in falling from B to A.

Cor. 3. It is plain that the angle HAN=HBA=HAB; therefore the direction AH biſects the angle BAN.

Cor. 4. The lines AD, Ad, make equal angles with AR, conſequently the angles DAN, dAN, are equal, and the diſtance AK is invariably the ſame.

Cor. 5. When AK is given, and the direction required, take $AR=\frac{AK}{4}$, and through RD parallel to AB, meeting the circle in D, d, draw AD, Ad, and theſe will be the directions.

PROBLEM I.

The horizontal diſtance, and the perpendicular height of the object above the level of projection, alſo the impetus being given, to find the elevations.

EXAMPLES.

Let the horizontal diſtance be 7000, impetus 4200, and the horizontal height 744, required the directions.

As the horizontal diſtance 7000 = -	3.84510
Is to radius 90 - - - -	10.00000
So is the height of the object 744 -	2.87157
To tangent angle of obliquity 6° 4′ = -	9.02647

Half of which, added to 45°, makes 48° 2′.

Ax : AK : : AC : AG, that is,

7000 : 7040 : : 2100 : : 2112.

As tangent 48° 2′	-	-	-		10.04607
Is to radius 90	-	-	-	-	10.00000
So is $\frac{1}{2}$ impetus 2100		-	-	-	3.32222
To 1888	-	-	-	-	3.27615
As 2112	-	-	-	-	3.32469
Is to 137.5	-	-	-	-	2.13830
So is radius 90		-	-	-	10.00000
To the versed sine of 20° 48′			-	-	8.81361

Half of which added to or subtracted from 48° 2′, gives $\begin{cases} 58° \; 26' \text{ higher.} \\ 37° \; 38 \text{ lower.} \end{cases}$

PROBLEM II.

Given the angles of direction, obliquity of the plane, and amplitude, to find the impetus.

As sine ang. dAz × into sine ang. BAD,	31° 34′ = 9.71891 52° 22′ = 9.89869		
		19.61760	
Is to the square of the sine of BAz	83° 56′ = 9.99756	19.99512	
So is $\frac{AK = 1760}{4}$	- -	3.24551	
		23.24063	
To the impetus 4198	- -	3.62303	

PROBLEM III.

The angles of direction, obliquity of the plane, and impetus being given, to find the random.

EXAMPLE

EXAMPLE.

The obliquity of the plane is 6° 4′, the angles of the direction $\left\{\begin{matrix} DAx \\ dAx \end{matrix}\right\} = \left\{\begin{matrix} 58^\circ\ 26' \\ 37^\circ\ 38' \end{matrix}\right\}$ and impetus 4200, to find the distance of the object.

As square of the sine DAx 83° 56′=9.99756=19.99512

Is to sine DAz × into sine BAz	31° 34′ / 52° 22′	9.71891 / 9.89869	
			19.61760
So is impetus 4200	- -	-	3.62325
			23.24085
To $\frac{1}{4}$ amplitude 1761	-	-	3.24573

PROBLEM IV.

The angles of direction, obliquity of the plane, and amplitude being given, to find the amplitude of any given elevation.

EXAMPLE.

The angle of obliquity KAx is 6° 4′, any angle of direction 37° 38′, and its amplitude is 7040, any other angle of direction 33° being given, to find the amplitude for that other direction.

As the sine dAz × into sine BAd	31° 34′=9.71891 / 52° 22′=9.89869	
		=19.61760
Is to the sine MAx × into sine MAB	25° =9.62595 / 58° 56′=9.93276	
		=19.55871
So is first amplitude 7040	-	3.84757
		23.40628
To the amplitude required 6147		3.78868

PROBLEM

PROBLEM V.

The impetus and obliquity being given, to find the greatest random.

EXAMPLE.

Let the impetus be 4200, obliquity of the plane 6° 4′, required the greatest random.

As tangent 48° 2′	-	-	-	10.04607
Is to twice impetus 8400		-	-	3.92428
So is secant obliquity 6° 4′		-	-	10.00244
				13.92672
To the greatest random 7596		-	-	3.88065

If to 45° you add half the angle of obliquity, the sum is the direction that carries farthest up an ascent.

If from 45° you subtract half the angle of obliquity, the remainder is the direction which carries farthest on a descent.

The greatest distance up an ascent is equal to twice the impetus, wanting the height of the mark above the horizontal plane. And the greatest distance down a descent is equal to twice the impetus, together with the depression of the object below the horizontal line.

In actual service, cases on ascents and descents are seldom attended to.

COMPUTATION OF SHOT.

It is customary to pile iron balls and shells in horizontal rows; the piles are denominated according to the figure of their respective bases. The base is commonly an equilateral triangle, square, or rectangle. Triangular and square piles, when complete

plete, terminate in a ſingle ball, and a rectangular pile in a ſingle row. The two firſt, when complete, form a pyramid, the laſt a wedge.

PROBLEM I.

To find the number of balls in a triangular pile.

RULE.

Put n for the number of balls in a ſide of the baſe row, then $\dfrac{n\times\overline{n+1}\times\overline{n+2}}{6}$, gives the number of balls in the pile.

EXAMPLE. I.

Required the number of balls in a triangular pile, a ſide of the baſe tire contains 30 balls.

$$\begin{array}{r}
30=n \\
31=n+1 \\
\hline
930 \\
32=n+2 \\
\hline
1860 \\
2790 \\
\hline
6)29760 \\
\hline
4960 \text{ balls in the pile.}
\end{array}$$

Ex. 2. How many balls are in a triangular pile, the ſide of the bottom-row being 25? *Anſ.* 2925

Ex. 3. Required the number of balls in a triangular pile, the ſide of the baſe-row being 20. *Anſ.* 1540.

Ex. 4. How many balls are in a triangular pile, the baſe-row being 10? *Anſ.* 220.

Ex.

Ex. 5. How many balls are in a triangular pile, whose base-tire is 4? *Ans.* 20

PROBLEM II.

To find the number of balls in a square pile.

RULE.

Put n for the number of balls in the side of the square base, then $\frac{n \times \overline{n+1} \times \overline{2n+1}}{6}$ is the number of balls in the pile.

EXAMPLE I.

How many balls are in a square pile of 30 balls to the side of the base-row?

$$
\begin{array}{r l}
30 & =n \\
31 & =n+1 \\
\hline
930 & \\
61 & =2n+1 \\
\hline
930 & \\
5580 & \\
\hline
6)56730 & \\
\hline
9455 & \text{balls in the pile.}
\end{array}
$$

Ex. 2. How many balls are in a pile, the side of the square base being 15 balls? *Ans.* 1240.

Ex. 3. How many balls are in a square pile of 13 tires? *Ans.* 819.

Ex. 4. How many balls are in a square pile of 12 tires? *Ans.* 650.

Ex.

Ex. 5. How many balls are in a ſquare pile, whoſe baſe-row conſiſts of 10 balls? *Anſ.* 385.

PROBLEM III.

To find the number of balls in a pile, whoſe baſe is a rectangle or oblong.

RULE.

Put l for the number of balls in the length, and b for the breadth, then $\frac{\overline{3l+1-b} \times b \times \overline{b+1}}{6}$, will give the number of balls in the oblong pile.

EXAMPLE I.

How many balls are in an oblong pile, the length of the baſe courſe is 40 and breadth 20?

$$\begin{array}{r}
40 = l \\
3 \\
\hline
120 \\
1 \\
\hline
121 \\
20 = b \\
\hline
101 \\
20 = b \\
\hline
2020 \\
21 \\
\hline
2020 \\
4040 \\
\hline
6)42420 \\
\hline
\end{array}$$

Anſ. 7070 balls.

Ex. 2. How many balls are in an oblong pile, the length of the base tire being 36 and breadth 24? *Ans.* 8500.

Ex. 3 How many balls are in an oblong pile, the length of the base row is 24 and the breadth 20? *Ans.* 3710.

Ex. 4. Required the number of balls in an oblong pile, whose length is 12 and breadth 8. *Ans.* 348.

PROBLEM IV.

To find the number of balls in an incomplete pile.

RULE.

From the number in the whole pile, considered as complete, subtract the number in the pile which is wanting at the top, (both computed by the rule for their proper form) and the remainder is the number in the broken pile.

EXAMPLE. I.

Required the number of balls in the incomplete triangular pile, one side of the bottom course being 30 and the uppermost course 21.

To find the pile complete.

$$\begin{array}{r} 30 \\ \underline{31} \\ 930 \\ \underline{32} \\ 1860 \\ \underline{2790} \\ 6)29760 \\ \hline 4960 \\ \underline{1540} \end{array}$$

Ans. 3420 balls.

To find the pile at the top.

$$\begin{array}{r} 20 \\ \underline{21} \\ 420 \\ \underline{22} \\ 840 \\ \underline{840} \\ 6)9240 \\ \hline 1540 \end{array}$$

Ex

EXAMPLE II.

To find the number of balls in an incomplete ſquare pile of 10 tires, the ſide at the top being 20.

To find the whole pile.

29 the baſe row.
30
870
59
7830
4350
6)51330
8555
2470

Anſ. 6085 balls.

To find the top one.

19 the baſe row.
20
380
39
3420
1140
6)14820
2470

EXAMPLE III.

To find the number of ſhot in a rectangular pile of 11 courſes, the baſe being 30 by 20.

To find the complete pile.

30
3
90
1
91
20
71
20
1420
21
1420
2840
6)29820
4970
735

Anſ. 4235 balls in the fruſtum.

To find the pile at top.

19
3
57
1
58
9
49
9
441
10
6)4410
735

EXAMPLES in practice.

1. How many shot are in a complete oblong pile, whose length is 32 and breadth 20? *Ans.* 5390.

2. Required the number of shot in a triangular pile, the side of the base row being 42. *Ans.* 13244.

3. Required the number of shot in a square pile, the side of the bottom course being 40. *Ans.* 22140.

4. How many balls in an incomplete triangular pile, the side of the base course being 40 and top 20? *Ans.* 10150.

5. Required the number of balls in an incomplete triangular pile of 17 courses, the side at top being 8. *Ans.* 2516.

6. Required the number of shot in an incomplete square pile of 17 courses, the side of the base row being 24. *Ans.* 4760.

7. How many balls are in an incomplete oblong pile, the base being 40 by 20, and top 29 by 9? *Ans.* 6146.

8. How many shot are in a triangular pile of 21 courses? *Ans.* 1771.

9. How many in a square pile of 21 courses? *Ans.* 3311.

10. How many balls in an oblong of 13 courses, the single row at top being 12 balls? *Ans.* 1820.

A

A TABLE *of Triangular Piles of Shot.*

0	0	1	2	3	4	5	6	7	8	9
0	0	1	4	10	20	35	56	84	120	165
1	220	286	364	455	560	680	816	969	1140	1330
2	1540	1771	2024	2300	2600	2925	3276	3654	4060	4495
3	4960	5456	5984	6545	7140	7770	8436	9139	9880	10660
4	11480	12341	13244	41190	15180	16215	17296	18424	19600	20825

EXPLANATION.

The figures in the left-hand column represent 10's, those in the top line units. Thus in the triangular table—Required the number of balls in a triangular pile, the side of the base-row being 32. Find 3 in the left-hand column, and 2 in the top line, and opposite to 3 and below 2 is 5984, the number required.—And so of the rest.

A Table of Square Piles of Shot from 1 to 89.

0	0	1	2	3	4	5	6	7	8	9
0	0	1	5	14	30	55	91	140	204	285
1	385	506	650	819	1015	1240	1496	1785	2109	2470
2	2870	3311	3795	4324	4900	5525	6201	6930	7714	8555
3	9455	10416	11440	12529	13685	14910	16206	17575	19019	20540
4	22140	23821	25585	17434	19370	31395	33511	35720	38024	40425
5	42925	45526	48230	51039	53953	56980	60116	63365	66729	70210
6	73810	77531	81375	85344	89440	93665	98021	102510	107134	111895
7	116795	121836	127020	132349	137825	143450	149226	155155	161239	167480
8	173880	180441	187165	194054	201110	208335	215731	223300	231044	238965

Specific Gravity.

THE abſolute gravity of a body is the force with which it tends downwards, and is always proportional to the denſity of the body, without any regard to its magnitude; ſo that a pound of cork is as heavy as a pound of gold. But the ſpecific gravity of bodies are their relative weights under the ſame magnitude, and are proportional to their denſity. Thus a cubic foot of lead is heavier than a cubic foot of fir; for lead, being more denſe than fir, contains a greater quantity of matter.

A cubic foot of water weighs 1000 ounces averdupois.—The ſpecific gravities of bodies, their magnitudes, and their weights, may be found, each from the others, by the following problems.

PROBLEM I.

To find the ſpecific gravity of a body heavier than water.

RULE.

Find the weight of the body in air, and alſo in water, and their difference is the weight loſt in water. Then,

As the weight loſt in water
Is to the weight of the body in air,
So is the ſpecific gravity of water
To the ſpecific gravity of the body.

EXAMPLE I.

A piece of brafs weighed 3 cwt. in air, and only 2 cwt. 2 qrs. 14 lb. in water; required its fpecific gravity.

The weight in air	336	As 42 : 336 : : 1000
The weight in water	294	1000
Weight loft in water	42	42)336000(8000 *Anf.*
		336
		000

A piece of fteel weighed $39\frac{1}{4}$ lb. in air, and $34\frac{1}{4}$ lb when weighed in water; what is the fpecific gravity of fteel?

Anf. 7850.

Ex. 3. A bar of lead weighed 15 cwt. in air, but only 13 cwt. 2 qrs. 19 lb. $10\frac{1}{2}$ oz. in water; required the fpecific gravity of lead. *Anf.* 11325.

PROBLEM II.

To find the fpecific gravity of a body lighter than water.

RULE.

Affix to it a piece of lead, iron, or any other body heavier than water, fo that they may fink together; then weigh the heavier body, and the compound mafs, each in water and out of water; fubtract the difference of the one from the difference of the other; then fay,

As the laft remainder
Is to the fpecific gravity of water,
So is the weight of the light body in air
To its fpecific gravity.

EXAMPLE

EXAMPLE I.

A piece of ash weighs 20 lb. in air, to which is affixed a piece of copper, which weighs 15 lb. in air, and $13\frac{1}{3}$ lb. in water; this compound weighs $8\frac{1}{3}$ lb. in water; required the specific gravity of the ash.

Copper.		Compound.	
In air	15	In air	35
In water	$13\frac{1}{3}$	In water	$8\frac{1}{3}$
	$1\frac{2}{3}$		$26\frac{2}{3}$
			$1\frac{2}{3}$
			25

As 25 : 1000 : : 20 :

$$\begin{array}{r} 20 \\ \hline 25)20000 \\ \hline \end{array}$$

Ans. 800 the specific gravity of ash.

Ex. 2. A piece fir weighs 1 cwt. 2 qrs. and a piece steel being affixed which weighed 3 cwt. in air, but in water 2 cwt. 2 qrs. $13\frac{3}{10}$ lb.; supposing the compound to weigh 1 cwt. 1 qr. $16\frac{1}{2}$ lb. in water, required the specific gravity of fir.

Ans. 550.

Ex. 3. Suppose a piece cork weighs 25 lb. in air, and that a piece lead, which weighs 100 lb. in air, and 91.17 lb. in water, is fixed to it; and that the compound mass, being immersed in water, weighs 12 lb. required the specific gravity of cork.

Ans. 240.

PROBLEM III.

A mixture of two ingredients being given, to find the quantity of each.

RULE.

Find the ſpecific gravity of the mixture, and of each of the ingredients, and multiply the difference of each by the other. Then ſay,

As the greateſt product
Is to each of the leſs products,
So is the weight of the compound maſs
To the weight of each ingredient reſpectively.

EXAMPLE I.

Suppoſe a maſs of gold and ſilver weighs 9 lb. and that its ſpecific gravity is 15618, (the ſpecific gravity of gold is 19640, and of ſilver 11091) required the quantity of each ingredient.

19640	19640	15618
11091	15618	11091
8549	4022	4527
15618	11091	19640
68392	4022	181080
8549	36198	27162
51294	40220	40743
42745	4022	4527
8549		
133518282	44608002	88910280

As 133518282 : 44608002 : : 9 : 3 lb. ſilver nearly.
133518282 : 88910280 : : 9 : 6 lb. gold.

Ex.

Ex. 2. A mixture, whoſe ſpecific gravity is 8784, is made of tin and copper, and weighs 112 lb.; the ſpecific gravity of tin is 7320, and of copper 9000; how much of each ingredient?

Anſ. $\begin{cases} \text{100 lb. copper.} \\ \text{12 lb. tin.} \end{cases}$

Ex. 3. A goldſmith mixes 18 ounces gold with alloy, and forms a maſs of 24 ounces, whoſe ſpecific gravity is 18000; the ſpecific gravity of gold is 19637; required the ſpecific gravity of the alloy. *Anſ.*

PROBLEM IV.

To find the weight of a body from its magnitude.

RULE.

As 1 cubic foot
Is to the content of the body,
So is its ſpecific gravity
To its weight.

EXAMPLE I.

Required the weight of a block of marble 6 feet long, 5 broad, and 6 Inches thick.

```
   6           As 1 : 15 : : 2700
   5                           15
  ---                       -------
  30                         13500
   5                         2700
  ---                       -------   28    4   20
 15.0                    16)40500(2531(90(22(1
                            32      252    8  20
                            --      ---   --  --
                             85      11   10   2
                             80            8
                             --           --
                              50           2
                              48
                              --
                               20
                               16
                               --
                                4
```

Ans. 1 ton, 2 cwt. 2 qrs. 11 lb. 4 oz.

Ex. 2. Required the weight of a log of oak 24 feet long, $1\frac{1}{2}$ broad, and 1 foot thick. *Ans.* 18 cwt. 2 qrs. 9 lb. 4 oz.

Ex. 3. How many deals fir will a ſhip of 400 ton burden carry, each being 16 feet long, 9 inches broad, and 6 inches thick? *Ans.* 4344 $\frac{8}{3}$.

Ex. 4. A ſhip of 300 tons burden carries 96 ſlabs marble, each 10 feet long and 6 feet broad; it is required to find the thickneſs. *Ans.* 8.296 inches.

PROBLEM

PROBLEM V.

To find the magnitude of bodies from their weights.

RULE.

Divide the weight of the body in averdupois ounces by the tabular ſpecific gravities, for the number of cubic feet. Or ſay,

As the ſpecific gravity of the body
Is to its weight,
So is 1 cubic foot
To the ſolidity in feet.

EXAMPLE I.

How many cubic feet are in a ton weight of fir?

$$550 : 35840 :: 1 : 65\tfrac{2}{55}$$

Ex. 2. How many cubic feet are in a ton-weight of braſs?
Anſ. 4.48.

Ex. 3. How many cubic feet in a block common ſtone, whoſe weight is 8 ton? *Anſ.* $113\frac{7}{9}$ feet.

A Table *of Specific Gravities of several Solids and Fluids.*

		Brick -	2000
Very fine gold -	19637	Nitre -	1900
Standard gold -	18888	Alabaster -	1875
Guinea gold -	17793	Dry ivory -	1825
Moidore gold -	17140	Brimstone -	1800
Quick-silver -	14019	Solid gunpowder	1745
Lead -	21325	Allum -	1714
Fine silver -	11087	Sand -	1520
Standard ditto -	10535	Pit-coal -	1240
Copper -	8843	Pitch -	1150
plate brass -	8000	Ebony -	1117
Steel -	7850	Human blood -	1054
Iron -	7645	Amber -	1030
Cast iron -	7425	Cows milk -	1030
Block-tin -	7321	Sea-water -	1030
Speltar -	7065	Goats milk -	1003
Lead-ore -	6800	Pump-water -	1000
Glass of antimony	5280	Spring-water -	999
Copper-ore -	3775	Distilled water -	993
Diamond -	3400	Proof of spirits	931
Clear glass -	3150	Dry oak -	925
White marble -	2707	Gunpowder shaken	922
Black ditto -	2700	Pure spirits -	866
Rock crystal -	2658	Ash - -	800
Green glass -	2620	Crabtree -	765
Cornelian stone	2568	Maple - -	755
Common ditto	2520	Elm - -	600
Flint -	2542	Fir - -	550
Hard paving stone	2460	Cork - -	240
Live sulphur -	2000	Air - -	$1\frac{1}{4}$

BALLS

BALLS AND SHELLS.

PROBLEM I.

To find the weight of a shell, the external and internal diameters being given.

RULE.

MULTIPLY the difference of the cubes of the diameters in inches by .14, and the product gives the answer in pounds nearly.

EXAMPLE I.

Required the weight of an iron shell, whose external and internal diameters are $9\frac{4}{5}$ and 7 inches.

9.8	7	598.192
9.8	7	.14
784	49	2392768
882	7	598192
96.04	343	*Ans.* 83.74688 lb.
9.8		
76832		
86436		
941.192		
343		
598.192		

Ex.

Ex. 2. What is the weight of an iron ſhell, the external and internal diameters being 13 and $9\frac{1}{2}$ inches. *Anſ.* 187.54625 lb.

Ex. 3. Required the weight of an iron ſhell, the diameters being 8 and 10 inches. *Anſ.* 68.32 lb.

Ex. 4. Required the weight of an iron ſhell, whoſe diameters are $10\frac{1}{2}$ and 8 inches. *Anſ.* 90.3875 lb.

PROBLEM II.

To find how much powder will fill a ſhell.

RULE.

Divide the cube of the internal diameter in inches by 57.3; the quotient gives the pounds in powder.

EXAMPLE I.

What weight of powder will fill a ſhell whoſe internal diameter is 7 inches?

```
         7
         7
        ---
        49
         7
       ----
57.3)343.0(5.986 lb.
     2865
     ----
      5650
      5157
      ----
       4930
       4584
       ----
        3460
        3438
        ----
          22
```

Ex.

Ex. 2. How much powder will fill a ſhell whoſe internal diameter is $9\frac{1}{2}$ inches? *Anſ.* 14.962.

Ex. 3. How much powder will fill a ſhell whoſe internal diameter is 8 inches? *Anſ.* 8.935 lb.

Ex. 4. What weight of powder will fill a ſhell whoſe diameter is 10 inches? *Anſ.* 17.45 lb.

PROBLEM III.

To find the ſize of a ſhell to contain a given weight of powder.

RULE.

Multiply the pounds of powder by 57.3, and the cube root of the product will be the ſhell's diameter in inches.

EXAMPLE I.

Required the diameter of a ſhell that will hold 5.986 lb.

LOGARITHMICALLLY.

To the logarithm of	5.986 =	0.77714
Add the logarithm of	57.3 =	1.75815
		3)2.53529
The logarithm of of 7		0.84509

Ex. 2. Required the diameter of a ſhell that will hold 14.962 lb. *Anſ.* $9\frac{1}{2}$ inches.

Ex. 3. What is the diameter of a ſhell that will contain 8.935 lb. *Anſ.* 8 inches.

Ex. 4. A ſhell contains 6 lb. powder; required its diameter. *Anſ.* 7 inches.

PROBLEM IV.

To find the diameter of an iron ball from its weight.

RULE.

An iron ball of 4 inches diameter weighs 9 lb. and ſimilar ſolids are in the triplicaie rate of their homologous ſides ; therefore, if d is the diameter and w the weight,

$$\text{As } 9 : 64 :: w : d^3 \text{ and } \sqrt[3]{d^3} = d \text{ the diameter.}$$

EXAMPLE I.

Required the diameter of an iron ball whoſe weight is 42 lb.

$$\begin{array}{r} 9 : 64 :: 42 \\ 64 \\ \hline 168 \\ 252 \\ \hline 9)2688 \\ \hline 298.6 \end{array}$$

298.6 the cube root of which is 6.684 inches.

Ex. 2. What is the diameter of an iron ball whoſe weight is 52 lb. ? *Anſ.* 7.1777 inches.

Ex. 3. Required the diameter of an iron ball whoſe weight is 36 lb. *Anſ.* 6.3496 inches.

Ex. 4. Required the diameter of an iron ball whoſe weight is 12 lb. *Anſ.* 4.403 inches.

PROBLEM

PROBLEM V.

To find the weight of an iron shot, its diameter being given.

RULE.

As $64 : 9 :: d^3 : w$.

N. B. *d* and *w* are the same as in prob. 4.

EXAMPLE I.

Required the weight of a ball whose diameter is $6\frac{1}{2}$ inches.

$64 : 9 :: 6.5^3 :$

$64 : 9 :: 274.625 :$

$\quad 9$

64)2471.625(38.619
192
551
512
396
384
122
64
585
576
9

Ex. 2. Required the weight of an iron ball whose diameter is 5.0397 inches. *Ans*. 18 lb.

Ex. 3. What is the weight of a ball whose diameter is 6.3496 inches? *Ans*. 36 lb.

Required the weight of an iron ſhot whoſe diameter is 2.4228 inches. *Anſ.* 2 lb.

PROBLEM VI.

To find the caliber of a gun to anſwer a ball of a given weight.

RULE.

To the diameter in inches add one-twentieth of the ſame, and the ſum gives the caliber for Engliſh guns.

EXAMPLE. I.

Required the caliber of a 42 pounder.

$$\begin{array}{lr} \text{The diameter of a ball of 42 lb. is} & 6.684 \\ & \underline{334} \\ & 7.018 \end{array}$$

Ex. 2. Required the caliber of a 32 pounder. *Anſ.* 6.410.
Ex. 3. Required the caliber of a 12 pounder. *Anſ.* 4.623.
Ex. 4. Required the caliber of a 24 pounder. *Anſ.* 5.824.

It is cuſtomary in Britain to allow one-twentieth part more for the diameter of the bore than for the diameter of the ſhot; *i. e.* the diameter of the bore is to that of the ſhot in the proportion of 21 to 20.

A

A Table *of the Diameters of the Shots and Calibers of English Guns.*

lb.	0	1	2	3	4	5	6	7	8	9	
0	0	1.923	2.423	2.775	3.053	3.288	3.498	3.679	3.846	4.000	Diameters.
	0	2.019	2.544	2.913	3.204	3.568	3.668	3.861	4.038	4.200	Calibers.
1	4.143	4.277	4.403	4.522	4.635	4.743	4.846	4.945	5.040	5.131	Diameters.
	4.349	4.490	4.623	4.748	4.866	4.981	5.088	5.192	5.292	5.368	Calibers.
2	5.220	5.305	5.388	5.409	5.547	5.623	5.697	5.769	5.839	5.908	Diameters.
	5.480	5.570	5.661	5.742	5.824	5.893	5.982	6.057	6.129	6.203	Calibers.
3	5.975	6.041	6.105	6.168	6.230	6.290	6.350	6.408	0.465	6.521	Diameters.
	6.273	6.343	6.410	6.475	6.541	6.604	6.666	6.707	6.788	6.846	Calibers.
4	6.576	6.631	6.684	6.737	6.789	6.640	6.890	6.940	6.989	7.037	Diameters.
	6.904	6.962	7.018	7.076	7.128	7.182	7.234	7.287	7.338	7.383	Calibers.

Explication of the above Table.—In the left-hand column ſtand the tens, in the top line the units; *e. g.* When you would know the diameter of a 42 lb. ball, and the caliber of its gun, look for 4 in the left-hand column, and for 2 at the top, and below 2 and oppoſite to 4 is the diameter and caliber required: And ſo of the reſt.

PROBLEM VII.

To find how many pounds of gunpowder will fill a rectangular box.

RULE.

Divide the ſolidity of the box in inches, by 30, and the quotient will give the number of pounds.

EXAMPLE I.

How much powder will a box contain, whoſe length is 16 inches, breadth 10, and depth 9 inches?

$$\begin{array}{r} 16 \\ \underline{10} \\ 160 \\ \underline{9} \\ 30)\underline{1440} \\ \textit{Anſ.}\ 46\ \text{lb.} \end{array}$$

Ex. 2. How much powder will a cubical box contain whoſe depth is 12 inches? *Anſ.* $57\frac{3}{5}$ lb.

Ex. 3. What quantity of powder will fill a box 15 inches long, 12 broad, and 8 inches deep? *Anſ.* 46 lb.

PROBLEM VIII.

To find the ſide of a cubical box that ſhall contain a given weight of powder.

RULE.

RULE.

Multiply the weight in pounds by 30, and the cube root of the product will give the ſide of the box in inches.

EXAMPLE I.

Required the ſide of a cubical box that ſhall contain $57\frac{3}{5}$ lb. powder.

$$\begin{array}{r|l} & 57.6 \\ & 30 \\ \hline & 1728.0(12 \text{ lb.} \\ & 1. \\ \hline 300 & 728 \\ 60 & \\ 4 & \\ \hline 364\times 2= &)728 \\ \hline & 0 \end{array}$$

Ex. 2. Required the ſide of a cubical box to contain 900 lb. powder. *Anſ.* 30 inches

Ex. 3. Required the ſide of a cubical box to contain $112\frac{1}{2}$ lb. gunpowder. *Anſ.* 15 inches.

PROBLEM IX.

To find what quantity of powder will fill a given cylinder.

RULE.

Multiply the ſquare of the diameter by the length, and divide the product by 38.197 for the pounds of powder.

EXAMPLE

EXAMPLE I.

How many pounds powder will fill a cylinder whofe diameter is 8 inches and the length 10 inches.

```
             8
             8
            ——
            64
            10
           ———
38.197)640.000(16.75 lb.
       38197
       ——————
       258030
       229182
       ——————
        288480
        267379
        ——————
         211010
         190975
          ——————
          10035
```

Ex. 2. How much powder will fill a cylinder whofe diameter is 10 inches and length 16 inches? *Anf.*

Ex. 3. Required the weight neceffary to fill a cylinder of 8 inches diameter and 20 in length. *Anf.*

Diameters

GAUGING.

THE art of Gauging is that part of the Mathematics called *Stereometry*, or the measuring of Solids, because the capacity of all vessels used for liquids, &c. are computed as if they were real solids . And since the contents of all sorts of vessels, that come under the consideration of the gauger, are computed by the standard gallon or bushel of its kind, whose content is known to be a certain number of cubic inches, all dimensions used in gauging should therefore be taken in inches and decimal parts of an inch.

The common wine gallon sealed at Guild-hall in London, contains 231 cubic inches: It is used for measuring all wines, brandy, spirits, strong waters, mead, perry, cyder, vinegar, oil, honey, &c.—from which standard gallon the following table is computed.

Table of Wine measure.

Cubic inches.	Gall.	Tierce	Hogsh	Puncheon	Pipe.	Tun.
231	1					
9702	42	1				
14553	63	$1\frac{1}{2}$	1			
19404	84	2	$1\frac{1}{3}$	1		
29106	126	3	2	$1\frac{1}{2}$	1	
58212	252	6	4	3	2	1

Gallons.

18 = 1 Runlet.
$31\frac{1}{2}$ = 1 Wine or vinegar barrel.

 The

The gallon, whether beer or ale, contains 282 cubic inches: From thence the following tables are computed.

1. *Ale meaſure.*

Note. A firkin of ſoap and of herrings are the ſame with that of ale.

Cubic inches.

282= 1 gall.

2256= 8=1 firkin.

4512=16=2=1 kilderkin.

9024=32=4=2=1 barrel.

13536=48=6=3=$1\frac{1}{2}$=1 hogſhead.

2. *Beer meaſure.*

Cubic inches.

282= 1 gallon.

2538= 9=1 firkin.

5076=18=2=1 kilderkin.

10152=36=4=2=1 barrel.

15228=54=6=3=$1\frac{1}{2}$=1 hogſhead.

This diſtinction between beer and ale meaſure is attended to in London, Edinburgh, &c.; but in many country places, both in England and Scotland, the following table is uſed, whether it be ſmall or ſtrong.

Cubic inches.

282= 1 gallon.

2397= $8\frac{1}{2}$=1 firkin.

4794=17 =2=1 kilderkin.

9588=34 =4=2=1 barrel.

14382=51 =6=3=$1\frac{1}{2}$ hogſhead.

The

The ſtandard Wincheſter buſhel * contains 2150.42 cubic inches, conſequently the gallon muſt be 268.8 cubic inches: Hence the following table of

Dry meaſure.

Cubic inches.

268.8= 1 gallon.

537.6= 2= 1 peck.

2150.4= 8= 4=1 buſhel.

17203.2=64=32=8=1 quarter.

Note. 4 buſhels=a coomb, 10 quarters=a wey, and 12 weys=a laſt of corn.

In gauging, All ſuperficies or areas are underſtood to be 1 inch deep, otherwiſe it could not be ſaid (as in the gauger's language it is) that the area of any ſquare, circle, &c. is ſo many gallons.

Moſt of the following problems are ſuch as have been already propoſed in the former part of this treatiſe, and are only here applied to practice.

PROBLEM I.

To find diviſors, multipliers. and gauge-points, with their uſes.

282	cubic inches make 1 ale gallon.
231	cubic inches make 1 wine gallon.
268 8	cubic inches make 1 corn gallon.
2150.42	cubic inches make 1 corn or malt buſhel.

---

* A cylindric buſhel, 18 one-half inches diameter, and 8 inches deep, is eſteemed a legal Wincheſter buſhel, according to the ſtandard in his Majeſty's Exchequer, ſettled by act of Parliament in the year 1697.

RULE.

Divide 1 by these numbers, and the quotient will give equivalent multipliers, and their square roots will be the gauge-points *.

TABLE I. *For right-lined surfaces.*

Divisors.		Multipliers.		Gauge-points.	
282	A. G.	.003546	A. G.	16.79	A. G.
231	W. G.	.004329	W. G.	15.19	W. G.
268.8	C. G.	.0037202	C. G.	16.39	C. G
2150.42	M. B.	.0004650	M. B.	46.37	M. B.

PROBLEM II.

To find the area of any rectangular tun, back, or cooler, &c. in ale, wine gallons, and malt bushels.

RULE.

Multiply the length by the breadth, (both being in inches) and divide the product by the divisors, or multiply it by the multipliers in Table I. the result will be ale gallons, wine gallons, corn gallons, or malt bushels.

EXAMPLE I.

Required the area of a square cooler, whose side is $124\frac{1}{2}$ inches, in ale, wine, corn gallons, and malt bushels.

124.5

* Gauge-points are the sides of squares whose area is 1 gallon, 1 bushel, &c.

$$124{,}5 \times 124.5 = 15500.25$$

First by division,	then by multiplication.
282)15500.25(54.96 A.G.	15500.25 × .003546=54.96 A. G
231)15500.25(76.10 WG.	15500.25 × .004329=76.10 W.G
268·8)15500.25(57.66 C.G.	15500.25 × .0037202=57.66 CG
250·42)15500.25(7.209 M.B.	15500.25 × .0004650= 7.209MB

These areas, being multiplied by the depth, produce the content of the whole vessel.

Ex. 2. A vessel in the form of a rectangle, 232 inches in length and 64 in breadth, what is its area in ale, wine, corn gallons, and malt bushels?

Ans. 52.652 ale gallons, 64.277 wine gallons, 55.238 corn gallons, and 6.904 malt bushels.

Ex 3. Suppose the length of a brewer's tun, back, or cooler, be $217\frac{1}{2}$ inches, and its breadth $85\frac{3}{5}$ inches, required its area in beer, wine, corn gallons, and malt bushels.

Ans. 66.014 ale gallons, 80.59 wine gallons, 69.02 corn gallons, and 8.6 malt bushels.

Ex. 4 Required the area of a square back, whose side is 30 inches, in ale, wine gallons, and malt bushels.

Ans. 3.19 ale gallons, 3.89 wine gallons, and .418 malt bushels.

By the sliding rule.

Set the divisor upon B to the side of the square on A, and against the side of the square on B you have the content on A, in ale, wine, corn gallons, or in malt bushels, in terms of the divisor.

If the tun, back, &c. be a rectangular oblong, set the proper divisor on B to the breadth on A, then against the length on B is the content on A, as above.

PROBLEM

PROBLEM III.

To find diviſors, multipliers, and gauge-poiuts for circular areas.

RULE.

Divide 282, 231, 268.8, and 2150.42 *, by .7854, (the area of a circle whoſe diameter is 1) and the quots will be a ſet of diviſors; and divide .7854 by the ſame numbers, the quot will give a ſet of multipliers as exhibited in the following table. Alſo the ſquare roots of theſe diviſors will give their reſpective gauge-points †.

TABLE II. *For circular areas.*

Diviſors.		Multipliers.		Gauge-points.	
359 05	A. G.	.00278	A. .G	18.95	A. G.
294.12	W. G.	.0034	W. G.	17.15	W. G.
342.24	C. G.	.00292	C. G.	58.5	C. G.
2737.	M. B.	.000365	M. B.	22.32	M. B.

PROBLEM IV.

To find the area of a circle in ale, wine, corn gallons, and malt buſhels.

RULE.

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* In practice, the decimal part is neglected, 2150 being reckoned ſufficiently accurate: We ſhall therefore only uſe the integer for the future. The ſame is to be obſerved of the diviſors in the preceding table.

† The gauge-point for circular areas is the diameter of a circle whoſe area, at 1 inch deep, is 1 gallon, 1 buſhel, &c.

RULE.

Divide the ſquare of the diameter by the diviſors, or multiply the ſame ſquare by the multipliers, the reſult will be the area in ale, wine, corn gallons, or malt buſhels.

EXAMPLE I.

Required the area of a circle, whoſe diameter is 80 inches, in ale, wine gallons, and malt buſhels.

$$80 \times 80 = 6400.$$

First by diviſion, then by multiplication:

359)6400(17.827	A. G.	6400 × .00278 = 17.792	A. G.
294)6400(21.768	W. G.	6400 × .0034 = 21.76	W. G.
2737)6400(2.338	M. B.	6400 × .000365 = 2.336	M. B.

Ex. 2. Required the area of an ellipſe, whoſe diameters are $173\frac{1}{3}$ and 90, in ale, wine gallons, and malt buſhels.

$$173\tfrac{1}{3} \times 90 = 15600,$$

This may be reckoned as the ſquare of the diameter in circular ſurfaces.

Firſt by diviſion, then by multiplication.

359)15600(43.4	A. G.	15600 × 00278 = 43.4	A. G.
294)15600(53.06	W. G.	15600 × 0034 = 53.0	W. G.
2737)15600(5.7	M. G.	15600 × 000365 = 5.7	M. B.

Required the area of a circular back, whoſe diameter is 50 inches, in ale, wine gallons, and malt buſhels.

Anſ. 6.95 ale gallons, 8.5 wine gallons, and .91 malt buſhels.

By

By the ſliding rule.

Set the diviſor upon B to the diameter on A, and againſt the diameter on B is the area on A.

SUPERFICIES having already been ſo largely treated of, it will perhaps be thought needleſs to give rules for each particular figure.

In general, the area of any back, tun, or cooler, or of any other veſſel, may be obtained thus:—Find the area of its bottom or top * by the rule for its proper form, and divide this area (in ſquare inches) by the diviſors, or multiply by the multipliers in Table I. *of right-lined ſurfaces*, and the reſult will be the areas in ale, wine, corn gallons, or malt buſhels.

But when the veſſel is of a polygonous form, the following method is one of the moſt practical:—Divide it into triangles, by the help of a chalk'd line, ſuch as carpenters uſe, by ſtriking diagonals: And, having found the diagonals, the perpendiculars may be aſcertained thus:—Fix one end of the chalk'd line in one of the angles; move it *to* and *fro* upon the ſtretch till you find the neareſt diſtance from the angle to the ſide which ſubtends it, there ſtrike a line. In like manner find the other perpendiculars; then, by a ſcale of inches and decimals of an inch, meaſure the perpendiculars, and the diagonals or ſides on which they fall. Compute the area of each triangle ſeparately, and divide or multiply their ſum as above, for gallons and buſhels reſpectively.

EXAMPLES

* The veſſel is here ſuppoſed to be of equal width from top to bottom.

EXAMPLES *for practice.*

EXAMPLE I.

How many ale, wine gallons, and malt bushels are in the area of a rhombus, whose side is 60 inches, and perpendicular breadth 50 inches?

Ans. 10.63 ale gallons, 12.98 wine gallons, and 1.395 malt bushels.

Ex. 2. How many ale, wine gallons, and malt bushels are in the area of a rectangle, whose length is 96 inches, and breadth 50 inches?

Ans. 17.02 ale gallons, 20.779 wine gallons, and 2.23 malt bushels.

Ex. 3. How many ale, wine gallons, and malt bushels are in the area of a rhombus, whose length is 120 inches, and perpendicular breadth 100 inches?

Ans. 42.55 ale gallons, 51.948 wine gallons, and 5.58 malt bushels.

Ex. 4. How many ale, wine gallons, and malt bushels are in the area of a triangle, whose three sides are 80, 100, and 60 inches?

Ans. 8.51 ale gallons, 10.389 wine gallons, and 1.116 malt bushels.

Ex. 5. How many ale, wine gallons, and malt bushels are in the area of a triangle, whose base is 25 inches, and perpendicular 24 inches?

Ans. 1.0638 ale gallons, 1·2987 wine gallons, and .1395 malt bushels.

Ex. 6. How many ale, wine gallons, and malt bushels are in the area of a trapezoid, whose parallel sides are 120 and 80 inches, and their perpendicular distance 50 inches?

Ans. 17.73 ale gallons, 21.645 wine gallons, and 2.325 malt bushels.

Ex. 7. How many ale, wine gallons, and malt buſhels are in the area of a trapezium, whoſe diagonal is 175 inches, and perpendiculars falling upon it, from the oppoſite angles, 80 and 120 inches?

Anſ. 62.056 ale gallons, 75.757 wine gallons, and 8.13 malt buſhels.

Ex. 8. How many ale, wine gallons, and malt buſhels are in the area of a pentagon, whoſe ſide is 20 inches?

Anſ. 2.44 ale gallons, 2.978 wine gallons, and .32 malt buſhels.

Ex. 9. How many ale, wine gallons, and malt buſhels are in a hexagon, whoſe ſide is 20 inches?

Anſ. 3.686 ale gallons, 4.498 wine gallons, and .4833 malt buſhels.

Ex. 10. How many ale, wine gallons, and malt buſhels are in a circle whoſe diameter is 40 inches?

Anſ. 4.456 ale gallons, 5.44 wine gallons, and .584 malt buſhels.

Ex. 11. How many ale, wine gallons, and malt buſhels are in a ſegment of a circle, whoſe diameter is 60 inches, and height of the ſegment 10 inches?

Anſ. 1.449 ale gallons, 1.769 wine gallons, and .19 malt buſhels.

Ex. 12. How many ale, wine gallons, and malt buſhels are in the ſector of a circle, when the arch is 60 inches and radius 50 inches?

Anſ. 5.319 ale gallons, 6.493 wine gallons, and .697 malt buſhels.

Ex. 13. How many ale, wine gallons, and malt buſhels are in an ellipſe, whoſe tranſverſe and conjugate diameters are 40 and 30 inches?

Anſ. 3.341 ale gallons, 4.08 wine gallons, and .438 malt buſhels.

Ex.

Ex. 14. How many ale, wine gallons, and malt bushels are in an elliptic segment, cut off at the distance of 36 from the centre, the axis being 120 and 40 inches?

Ans. 1.903 ale gallons, 2.323 wine gallons, and .249 malt bushels.

These examples are so essentially necessary to the practice of gauging, that an officer in the excise ought not to be satisfied with less than the very principles on which they are performed, before he venture upon the following part of gauging.

PROBLEM IV.

To find the content of any cube, parallelopiped, prism, or of the cylinder, in ale, wine gallons, and malt bushels.

RULE I.

Find the area of the base in ale, wine gallons, or malt bushels, and multiply that area by the height, or depth; the product will give the content in ale gallons, &c.

RULE 2. Find the solid content, (in inches) as taught in Prob. II. IV. *of solids*, and this content, divided by the divisors, or multiplied by the multipliers in Table I. will give the content in ale, wine gallons, &c.

EXAMPLE I.

How many ale, wine gallons, and malt bushels will a vessel, in the form of a parallelopipedon, contain, the length being 60 inches, breadth 50, and depth 36 inches?

By Rule I.

$$60 \times 50 = 3000$$

282)3000(10.63	A. G	10.63 ×36=382.68	A. G.
231)3000(12.98	W. G.	12.98 ×36=467.28	W. G.
2150)3000(1.395	M. B.	1.395 ×36= 50.22	M. B.

By Rule II.

$$60 \times 50 \times 36 = 10800$$

282)108000(382.97	Content in ale gallons.
231)108000(467.54	Content in wine gallons.
2150)108000(50.23	Content in malt bushels.

And so of the rest.

The small difference in the answers is not to be imputed to any defect in either of the preceding rules; for if the quots are extended to a few more decimal places, the answers will agree to the greatest nicety. In practice, however, it is customary to add 1 to the integral part of the answer, when the decimal exceeds .5, and to neglect it when less. The same is to be observed of the following examples.

EXAMPLE II.

Required the content of a cubical vessel in ale, wine gallons, and malt bushels, the side being 20 inches.

Ans. 28 ale gallons, 35 wine gallons, and 4 malt bushels.

Ex. 3. How many ale, wine gallons, will a cylinder contain whose diameter is 25 inches and depth 20?

Ans. 35 ale gallons, and 42 wine gallons.

Ex. 4. How many bushels malt will a vessel contain whose base is a rectangle of 50 inches by $40\frac{1}{2}$, and depth 40 inches?

Ans. 38.

Ex. 5. Required the content of a triangular priſm, whoſe length is 18 inches, and one of the ſides 25 inches, and perpendicular 16 inches, in ale, wine gallons.

Anſ. 13 ale gallons and 15 wine gallons.

Ex. 6. A cylinder, whoſe diameter is 72 inches and depth 48; it is required to find its content in ale and wine gallons.

Anſ. 693 ale gallons and 846 wine gallons.

Ex. 7. How many ale, wine gallons, and malt buſhels will a cylinder contain, whoſe diameter is $56\frac{1}{2}$ inches, and height 96 inches?

Anſ. 854 ale gallons, 1042 wine gallons, and 112 malt buſhels.

Ex. 8. A parallelopipedon is 40 inches long, 20 broad, and 15 inches deep; required its content in ale, wine gallons, and malt buſhels.

Anſ. 43 ale gallons, 52 wine gallons, and 6 malt buſhels.

PROBLEM V.

To find the content of any pyramid, or of the cone, in ale, wine gallons, and malt buſhels.

RULE.

Compute the ſolidity of the cone or pyramid, in cubic inches, by Problem VI. *of ſolids*, then divide this ſolidity by the diviſors, or multiply by the multipliers, and the reſult will give the gallons or buſhels required.

EXAMPLE I.

How many ale, wine gallons, and malt buſhels will a conical

cal vessel contain, whose base diameter is 40 inches, and altitude 60 inches?

Ans. 89.136 ale gallons, 108.843 wine gallons, and 11.68 malt bushels.

Ex. 2. How many ale, wine gallons, will a vessel, in the form of a pentagonal pyramid, contain, the side of whose base is 90 inches, and perpendicular depth 140 inches?

Ans. 2306.1 ale gallons and 2815.2 wine gallons.

Ex. 3. Required the content of a square pyramid, whose side is 40 inches, and height 60 inches, in ale, wine gallons, and malt bushels?

Ans. 113.47 ale gallons, 138.52 wine gallons, and 14.88 malt bushels?

PROBLEM VI.

To find the content of the frustum of a cone, or any pyramid, in ale, wine gallons, and malt bushels.

RULE I.

For the cone.—Add the square of the two diameters to their product, multiply the sum by .7854, and again by $\frac{1}{3}$ the height; then divide or multiply as in Table I. for gallons or bushels, as required.

RULE II.

For the pyramid.—Add into one sum the area of both ends, and the mean proportional between them; multiply the sum by $\frac{1}{3}$ the height, and the product is the content in cubic inches; which divided or multiplied by the divisors or multipliers in Table I. gives the content in ale, wine gallons, &c.

EXAMPLE

EXAMPLE I.

A veſſel, whoſe baſes are rectangles, the greater baſe 100 inches by 70 inches, the leſſer baſe 80 by 56 inches, and depth 42; required the content in ale, wine gallons, and malt Buſhels

Anſ. 847.9 ale gallons, 1035.1 wine gallons, and 111.2 malt buſhels.

Ex. 2. Required the content of the lower fruſtum of a cone, the greater baſe diameter being 38 inches, the leſs $20\frac{1}{5}$, and depth 21, in ale gallons. *Anſ.* 51 ale gallons.

Ex. 3. A fruſtum of a cone, whoſe diameters are 56.5 inches and 19 inches, and the height 62 inches, required the content in ale, wine gallons, and malt buſhels.

Anſ. 266.3 ale gallons, 325.1 wine gallons, and 34.93 malt buſhels.

PROBLEM VII.

The diviſors for ale, wine, and malt for a cylinder being given, to find a diviſor for any of the following ſolids; namely, *the globe, ſpheroid, parabolic conoid, hyperbolic conoid, parabolic ſpindle, and cone.*

RULE *.

Find what part each is of the circumſcribing cylinder; then ſay,

As

* The globe is $\frac{2}{3}$ of the circumſcribing cylinder—the ſpheroid $\frac{2}{3}$—the parabolic conoid $\frac{1}{2}$—the hyperbolic conoid $\frac{5}{11}$—the parabolic ſpindle $\frac{8}{15}$—and the cone $\frac{1}{3}$.

As the numerator
Is to the denominator,
So is the ale, wine, and malt divisors of a cylinder,
To the like divisors for the figures required.

EXAMPLE I.

Required ale, wine, and malt divisors for the cone, those of the cylinders being 359.05 for ale, 294.12 for wine, and 2737.47 for malt.

As 1 : 3 : :	359.05 :	1077.15	ale divisors
1 : 3 : :	294.12 :	882.36	wine divisors.
1 : 3 : :	2737.47 :	8212.41	malt divisors.

And so on for the rest.

EXAMPLE II.

A cone, whose diameter is 40 inches, and altitude 60, required its content in ale, wine gallons, and malt bushels.

$$40 \times 40 \times 60 = 9600.0$$

1077.15)96000(89.1	ale gallons.
882.36)96000(108.8	wine gallons.
8212.41)96000(11.68	malt bushels.

I shall insist no farther on these examples at large, because what is here shewn of the cone is sufficient to inform the meanest capacity how to proceed with the other solids mentioned in the problem; but shall only offer a few practical exercises.

EXAMPLE. I.

How many ale, wine gallons, and malt bushels will a vessel, in

in the form of a ſpheroid, contain, whoſe fixed axis is 100, and revolving 60 inches?

Anſ. 668.4 ale gallons, 816.4 wine gallons, and 87.6 malt buſhels.

Ex. 2. Required the content of the parabolic conoid in ale and wine gallons, the height being 30, and diameter of its baſe 20.

Anſ. 16.7 ale gallons, and 20.4 wine gallons.

Ex. 3. Required the content of the hyperbolic conoid, the baſe being 100 inches, and altitude 60 inches.

Anſ. 696.2 ale gallons, and 850 wine gallons.

Ex. 4. Required the content of a parabolic ſpindle whoſe length is 60 inches, and greateſt diameter is 64, in ale and wine gallons.

Anſ. 103.02 ale gallons, and 125.7 wine gallons.

INCHING *Tuns and Coolers.*

THE practical method of gauging any fixed tun of copper, and of making a table to ſhew what it will hold at every inch deep.

Firſt, You muſt know that moſt (if not all) brewers tuns are ſo fixed as to lean a little, for conveniency of cleanſing their drink, which is uſually called *the drip, or fall of the tun.* Now, the drip, or fall, is the hoof of ſuch a ſolid as the tun is ſuppoſed to repreſent. The beſt and readieſt way is to meaſure into the

the tun, when dry, so much water as will just cover the bottom; for by this means a level is obtained, by the help of which it will be easy to ascertain how far up the surface of the liquor will reach when the tun is full.

Then find the content of that part between the surface of the drip and the surface of the liquor when the tun is full; to which add the drip or fall, and the sum will be the content of the tun.

Next, divide the difference of the head and bottom diameters by the depth, and the quotient is a common addend for 1 inch from top to bottom, by which you may find the diameter in the middle of every 10 inches of the depth.

EXAMPLE. Let the bottom diameter of a conical vessel be 98 inches, top diameter 80, and depth 40 inches; it is required to find how much it will hold upon every inch, and to table the same.

Bottom diameter	98	And
Top diameter	80	40)18.00(.45 the addend.
	18	

Now, 5, 15, 25, and 35 are the inches which fall in the middle of every 10 inches; if these numbers be multiplied by the addend, the products will shew how much ought to be added to the diameters at those depths. Thus,

.45	.45	.45	.45
5	15	25	35
2.25	6.75	11.25	15.75

So

80 top diameter.
 2.25

		Gall.	B.	F.	Gall.
82.25	diameter at 5 inches deep, its area is	18.84=	0	2	1.84

80 top diameter.
 6.75

86.75	diameter at 15 inches deep, its area is	20.96=	0	2	3.96

80 top diameter.
11.25

91.25	diameter at 25 inches deep, its area is	23.19=	0	2	6.29

80 top diameter.
15.75

95.75	diameter at 35 inches deep, its area is	25.53=	0	3	0.03

The ſum of theſe areas are 88.52
which, multiplied by 10, gives 885.2 gallons, the content of the tun.

Now, 18.84 gallons, which is 0 bar 2 fir. 1.84 gall. is the common area to the firſt 10 inches; and, for the next 10 inches, 20.96 gall. which is 0 bar. 2 fir. 3.96. And ſo on for the reſt.——The operation will be as follows.

Whole content.

B.	F.	Gall.	B.	F.	Gall.	B.	F.	Gall.
26	0	1.20	18	0	0.96	8	3	4.18
	2	1.84		2	3.96		2	6.19
25	1	7.86	17	1	5.50	8	0	6.49
	2	1.84		2	3.96		2	6.19
24	3	6.02	16	3	1.54	7	2	0.30
	2	1.84		2	3.96		3	0.03
24	1	4.18	16	0	6.08	6	3	0.27
	2	1.84		2	3.96		3	0.03
23	3	2.34	15	2	2.12	6	0	0.24
	2	1.84		2	3.96		3	0.03
23	1	0.50	14	3	6.66	5	1	0.21
	2	1.84		2	3.96		3	0.03
22	2	7.16	14	1	2.70	4	2	0.18
	2	1.84		2	6.19		3	0.03
22	0	5.32	13	2	5.01	3	3	0.15
	2	1.84		2	6.19		3	0.03
21	2	3.48	12	3	7.32	3	0	0.12
	2	1.84		2	6.19		3	0.03
21	0	1.64	12	1	1.13	2	1	0.09
	2	1.84		2	6.19		3	0.03
20	1	8.30	11	2	3.44	1	2	0.06
	2	3.96		2	6.19		3	0.03
19	3	4.34	10	3	5.75	0	3	0.03
	2	3.96		2	6.19	0	3	0.03
19	1	0.38	10	0	8.06	0	0	0.00
	2	3.96		2	6.19			
18	2	4.92	9	2	1.87			
	2	3.96		2	6.19			
18	0	0.96	8	3	4.18			

ABSTRACT.

In.	Bar.	Fir.	Gall.
0	26	0	1.20
1	25	1	7.86
2	24	3	6.02
3	24	1	4.18
4	23	3	2.34
5	23	1	0.50
6	22	2	7.16
7	22	0	5.32
8	21	2	3.48
9	21	0	1.64
10	20	1	8.30
11	19	3	4.34
12	19	1	0.38
13	18	2	4.92
14	18	0	0.96
15	17	1	5.50
16	16	3	1.54
17	16	0	6.08
18	15	2	2.12
19	14	3	6.66
20	14	1	2.70
21	13	2	5.01
22	12	3	7.32
23	12	1	1.13
24	11	2	3.44
25	10	3	5.75
26	10	0	8.06
27	9	2	1.87
28	8	3	4.18
29	8	0	6.49
30	7	2	0.30
31	6	3	0.27
32	6	0	0.24
33	5	1	1.21
34	4	2	0.18
35	3	3	0.15
36	3	0	0.12
37	2	1	0.09
38	1	2	0.06
39	0	3	0.03
40	0	0	0.00

Victuallers, who brew but little at a brewing, generally cool their worts in tubs. In order to ascertain the quantity of worts, the gauger ought to have the area of each tub marked upon it, otherwise to number the tubs, and enter the number and area of each tub in his stock-book.

PROBLEM VIII. *Fig.* 1.

To gauge a copper with a rising crown, and make allowance for the same.

RULE.

Take a small cord and let it represent the diameter of the head, and, by a plumb-line, find Ee, Gg, the greatest and least depths of the copper. Note Ae, Find the content of CDFE, considered as the frustum of a cone; then find the content of the crown, being reckoned a spherical segment; subtract the latter from the former, and the remainder will shew the quantity of liquor necessary to cover the crown.

Then find the content of the copper from the crown upwards, take the diameter of every 4, 6, or 10 inches*, and insert them, together with their corresponding areas (in barrels, firkins, and gallons) in a table; then multiply each of these areas by their distance, and the sum of the products will be the content after the crown is covered.

A very good and ready method to find how much liquor will cover the crown, is, to measure in as much water as will just cover it.

EXAMPLE.

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* The more curved the sides of the copper are, the more mean diameters and areas you ought to take.

EXAMPLE.

Suppose the figure ABFE to represent a copper, and ExFgE its rising crown, AB=90, CD=82, Ae=5, it is plain that $\overline{AB-2\times Ae}=90-10=EF$, Ee=30, and Gg=27, consequently gx=3.

To cover the crown.

	Gall.		B.	F.	Gall.
In the cone CDFE, the diameter CD=82, EF=80, and gx=3, its content in gallons, &c. is	54.782	=	1	2	3.778
The base diameter 80, and height of the crown 3, its content in ale gallons, &c. will be	26.737	=	0	3	1.235
The liquor that will cover the crown is - - - -	28.945	=	0	3	2.543

The crown being thus covered, it now remains to find the content of the copper from the crown upwards, the depth being 27 inches. In order to this, take the diameter in the middle of every 10 inches from the top, and insert each against the parts of the depth, as in the following table. Find the area of each in ale gallons, by Problem IV. and insert these areas, each against its corresponding diameter, as in column third; also the contents of the several parts of the depth are placed in the fourth column. And these contents, being reduced to barrels, firkins, gallons, are inserted in the last column, as follows.

Parts

Parts of depth.	Diameter.	Area.	In Gall.	Content in B.	F.	Gall.
10	88.	21.568	215.68	6	1	3.18
10	85.5	20.36	203.6	5	3	8.10
7	52.5	18.956	132.692	3	3	5.19
3	To cover the crown.		28.045	0	3	2.54
	Content of the copper		580.027	17	0	2.01

The content being thus found, you may proceed to inch the copper by the ſame directions which were given for inching tuns in laſt problem.

Cask-gauging.

Cask-gauging is the moſt difficult part of the art: This difficulty ariſes from the variety of curves which veſſels may be compoſed of. It is alſo the moſt imperfect, and ever will be; becauſe no caſk can be made in ſuch ſtrick conformity to the ſolid it repreſents, as by the rules of art it is required to be.

Gaugers have reduced all kinds of caſks to four forms, or varieties.

Variety I. The middle fruſtum of a ſpheroid—*Fig.* 2.

Variety II. The middle fruſtum of a parabolic ſpindle—*Fig.* 3.

Variety III. The middle fruſtums of two parabolicc onoids—*Fig.* 4.

Variety IV. The middle fruſtums of two cones—*Fig.* 5.

PROBLEM.

PROBLEM IX.

To find the content of a caſk.

RULE I.

For Variety I. To the ſquare of the head diameter add twice the ſquare of the bung diameter, then multiply the ſum by the length, and divide the product by 1077 for ale gallons. and by 882 for wine gallons.

RULE II.

For Variety II. To 9 times the ſquare of the bung diameter add 6 times the ſquare of the head diameter, then multiply $\frac{1}{5}$ the ſum by the length; divide the product, as above, for ale and wine gallons.

RULE III.

For variety III. To the ſum and half ſum of the ſquares of the head and bung diameters, add $\frac{1}{10}$ of the difference of their ſquares, then multiply the ſum by the length, and divide the product, as above, for ale and wine gallons.

RULE IV.

For variety IV. From the ſum and half ſum of the ſquares of the head and bung diameters, ſubtract half the ſquare of their difference; then multiply the remainder by the length, and divide the product, as above, for ale and wine gallons.

A general Rule for reducing casks to a cylinder.

First, consider which of all the four Varieties the proposed cask resembles, then from the bung diameter subtract the head diameter, and multiply the difference by .7 for the spheroid, by .65 for the spindle, by .6 for the conoids, and by .55 for the cones; add the product to the head diameter, the sum is a mean diameter, or the diameter of a cylinder of equal content and length with the cask proposed.

EXAMPLE.

The length of a cask is 40 inches, bung diameter 32, and head diameter 24 inches; required its content in ale and wine gallons.

Case I. Suppose the cask of the first form; then,

By Rule I.

$32^2 = 1024$
$\times 2$
$= 2048$
$24^2 = 576$
2624
length $\times 40$
104960

1077)104960(98.3 ale gallons.
882)104960(119 wine gallons.

By the General Rule.

$32 - 24 = 8$
$8 \times .7 = 5.6$

$5.6 + 24 = 29.6$ the mean diameter.

$29.6 \times 29.6 \times .00278 \times 40 = 97.6$ ale gallons.
$29.6 \times 29.6 \times .0034 \times 40 = 119.1$ wine gallons.

By the sliding rule.

Set the length of the cask in inches on C to the gauge-point on D, and against the mean diameter on D you have the content on C.

Case II. Suppose the cask of the second form; then,

By Rule II.

$$\begin{array}{rr} & 32^2=1024 \\ & 9 \\ \hline & 9216 \\ 6\times 24 & =3456 \\ \hline & 5)12672 \\ \hline & 2534.4 \\ & 40 \\ \hline & 101376.0 \end{array}$$

1077)101376.0(94.12 ale gallons.
882)101376.0(114.9 wine gallons.

By the general rule.

$8\times.65=5.2$ and $24+5.2=29.2$ the mean diameter.

$29.2\times 29.2\times.00278\times 40=94.98$ ale gallons.
$29.2\times 29.2=.0034\times 40—115.959$ wine gallons.

Case

CASE III. Suppose the cask of the third form; then,

By RULE III.

$32^2=1024$	$1077)97760(90{\cdot}77$ ale gallons.
$24^2=\ \ 576$	$882)97760(110{\cdot}84$ wine gallons.
1600	
800	
44	
2444	
40	
97760	

By the general rule.

$8\times.6=4.8$ and $24+4.8=28.8$ the mean diameter.

$28.8\times28.8\times.00278\times40=\ 92.4$ ale gallons.
$28.8\times28.8\times.0034\ \ \times40=112.8$ wine gallons.

CASE IV. Suppose the cask of the fourth variety.

$32^2=1024$	$1077)94720(87{\cdot}9$ ale gallons.
$24^2=\ \ 576$	$882)94720(107{\cdot}39$ wine gallons.
1600	
800	
2400	
32	
2368	
40	
94720	

By the general rule.

$8 \times .55 = 4.4$ and $24 + 4.4 = 28.4$ the mean diameter.

$28.4 \times 28.4 \times .00278 \times 40 = 89.8$ ale gallons.
$28.4 \times 28.4 \times .0034 \times 40 = 109.69$ wine gallons.

A cask of the 1st variety is the most capacious, and one of the 4th the least capacious. The spindle is most used.

We have now shewn the method by which casks (when full) may be gauged, both by the pen and rule. It now remains to point out a method by which casks, that are not full, may be gauged, and this is called *ullaging*.

PROBLEM X.

To find the ullage of a cask.

A cask may either stand on its end, with its axis perpendicular to the horizon, *fig.* 6. or ly with its axis parallel to the horizon, *fig.* 7.

RULE I.

When standing—Divide the wet inches by the length of the cask; and, if the quotient exceeds .5, add $\frac{1}{10}$ of the excess to the said quotient: but, if it be less than .5, subtract $\frac{1}{10}$ part of the deficiency, so will the sum or remainder be a multiplicand, by which if you multiply the content, the product will be the quantity of liquor.

RULE II.

When lying—Divide the wet inches by the bung diameter;

ter *; find the quotient among the versed sines in the table of circular segments, and multiply the corresponding area by the whole content of the cask, and the product gives the liquor in the cask.

EXAMPLE I.

For a cask standing.

Let the bung diameter be	28	Wet inches	18
Head diameter -	22	Dry inches	14
Length - -	32	Content	60.96 A. G.

32)18.00(.5625 | .5625—.5=.0625 excess.
.00625 | $\frac{1}{10}$ of which is .00625

the multiplicand .56875
60.96

341250
511875
3412500

34.67100000 *Ans.* 34.671 gall. in the cask.

EXAMPLE II.

For a cask lying.

Let the length be	20	Wet inches -	6
Bung diameter	16	Dry inches -	10
Head diameter	12	Content -	12.3

16

* If, instead of the wet inches, you divide by the dry, and proceed as the rule directs, the result will be vacuity.

16)6.0(.375	corresponding area	.269013	
48		12.3	
120		807039	
112		538026	
		269013	
80			
Anſ. 3.3 gallons.	The content	3.3088599	

EXAMPLE III.

Let the length be	32.5	Dry inches	21
Bung diameter	31	Content	75.37 A. G.
Wet inches -	10	Required the ullage lying.	

Anſ. 20.97 ale gallons A.G.

EXAMPLE IV.

Let the bung diameter be	33	How many gallons are wanting to fill up the veſſel?
The dry inches - -	12	
The content 108 gallons.		

Anſ. 35 gallons.

Note. The nearer the form of the caſk approaches to a cylinder, the more exact will the tables give the ullage; but when the bung diameter is much greater than the head diameter, the line of ſegments is truer than the tables.

By the Sliding rule.

When lying. *1ſt,* Set the bung diameter upon the line of numbers to 100 upon the line of ſegments; then againſt the wet inches on the line of numbers is a fourth number; *which reſerve.*

2d, Set 100 upon B to the whole content upon A, and againſt the reſerved number is the anſwer.

When

When ſtandiug. 1*ſt*, Set the length of the caſk on the line of numbers, to 100 on the line of ſegments; then againſt the wet inches on the line of numbers is a fourth proportional, *which reſerve.*

2*d*, Set 100 on B to the whole content on A; then againſt the reſerved number on B is the ullage required.

To gauge a floor of malt.

RULE I.

Meaſure the length and breadth of the floor, and take a number of depths by your gauging-rod, and divide their ſum by their number, the quotient will give a mean depth.

RULE II.

Multiply the length, breadth, and depth continually, and divide the product by 2150; the quotient gives the number of buſhels.

EXAMPLE.

A rectangular malt floor is 490 inches long, 368 inches broad; the depths, taken in ſeveral places, are as follow:

Inches.	Inches.
3.2	5
4	6.2
4	6.7
4.5	Required the content.

For

For the mean depth.

$$\begin{array}{r} 3.2 \\ 4 \\ 4 \\ 4.5 \\ 5 \\ 6.2 \\ 6.7 \\ \hline 6)33.6 \\ \hline 5.6 \end{array}$$ mean depth.

$490 \times 368 \times 5.6 = 1009792$ cubic inches.

$2150)1009792(469.67$ malt bushels.

By the Sliding Rule.

Set the mean depth on MD to the length on N, and against the breadth on the line A is the content on the line B.

Artificers

ARTIFICERS MEASURING.

Sliding Rule.

This instrument is two feet in length, much uſed in taking dimenſions, and in caſting up the contents. One of its ſides is provided with a ſlider, or ſlip, both ſides of which are divided in the ſame logarithmic manner with one of the ſides of the rule. On the other ſide of the ſlider is another line, marked the *girt line*, and is uſeful in caſting up ſolids. Beſides, at the diviſions 17.15, and 18.95, are marked W. G. and A. G. the wine and ale gauge points, for the purpoſe of gauging wine and ale by this rule.

This rule is ſo well known, that it is unneceſſary to give a tedious deſcription of its properties or conſtruction. We ſhall only give a few rules to ſhew its utility.

Prob. I. *To multiply any two numbers.*

Rule. Set 1 on the ſlip to one of the factors on the ſtock, and over againſt the other factor on the ſlip you have the product on the ſtock.

Prob. II. *To divide by the ſliding rule.*

Rule. Set the diviſor on the ſlip to the dividend on the ſtock, and againſt 1 on the ſlip you have the quotient on the ſtock.

PROB. III. *To square a number by the slide rule.*

RULE. Set 1 on the slip to the given number on the stock, and against the given number on the slip you have the square on the stock.

PROB. IV. *To extract the square root by the sliding rule.*

RULE. Set 1 or 100 on the slip to 1 or 10 on the girt line, and over against the given number on the slip you have the root on the girt line.

PROB. V. *To perform the rule of three by the sliding rule.*

RULE. Set the first term on the slip to second term on the stock, and over against the third serm on the slip you have the answer on the stock.

PROB. VI. *To cast up superficies by the sliding rule.*

RULE. Set the breadth in inches on the slip to 12 on the stock, and against the length in feet on the stock you have the content on the slip in square feet and decimal parts.

PROB. VII. *To find the solid content of squared timber.*

RULE. Set the length in feet on the slip to 12 on the girt line, and against the side of the square in inches on the girt line is the content in solid feet on the slip.

PROB. VIII. *To find the solid content of four-sided timber, whose base is an oblong; that is, whose breadth is greater than its thickness.*

RULE. Multiply the breadth by the thickness by Prob. 1. and extract the square root of their product by Prob. 4. With this

this ſquare root, as the ſide of the ſquare, work as directed in Prob. 7. and you have the anſwer.

MEASURING OF BOARD AND TIMBER.

PROBLEM I.

To find the ſuperficial content of a board or plank.

RULE I.

Multiply the length by the breadth, and the product is the area.

RULE 2. If the board is tapering, add the breadth of both ends together, and multiply half the ſum by the length for the area.

EXAMPLE I.

Required the area of a plank 15 inches broad and 18 feet long.

	Decimally.	Duo-decimally.		By Reduction.
	F.	F.	In.	F. In.
Length	18	18	0	1=216
Breadth	1.25	1	3	15
	90	18	8	1080
	36	4	6	216
	18			
		22	6 *Anſ.*	144)3240(22 feet.
	22.50 *Anſ.*			288
				360
				288
				12)72(6 inches.
				72

By

By the Slide rule.

Set 15, the breadth in inches, on the slip, to 12 on the stock, and opposite to 18, the length in feet, on the stock, you have $22\frac{1}{2}$ feet on the slip.

Ex. 2. Required the content of a plank 16 feet 3 inches long, and 18 inches broad. *Ans.* 24 feet 4 inch. 6 parts.

Ex. 3.——of a plank $22\frac{3}{4}$ feet long $19\frac{1}{2}$ inches broad.
Ans. 36 feet 11 inch. 7 parts 6″

Ex. 4.——$30\frac{1}{3}$ feet long and 14 inches broad.
Ans. 35 feet 4 inch. 8 parts.

Ex. 5.——$25\frac{1}{6}$ feet long and 19 inches broad.
Ans. 39 feet 10 inch. 2 parts.

Ex. 6.——$34\frac{1}{2}$ feet long and 21 inches broad.
Ans. 60 feet 4 inch. 6 parts.

Ex. 7.——$27\frac{1}{4}$ feet long and $15\frac{1}{2}$ inches broad.
Ans. 35 feet 2 inch. 4 parts 6″.

Ex. 8.——$32\frac{1}{12}$ feet long and $13\frac{1}{4}$ inches broad.
Ans. 35 feet 5 inch. 1 part 3″.

Ex. 9.——$23\frac{1}{8}$ feet long and 10 inches broad.
Ans. 19 feet 3 inches 3 parts. 5″

Ex. 10.——$12\frac{1}{4}$ feet long and 9 inches broad.
Ans. 9 feet 2 inch. 3 parts.

Ex. 11.——$10\frac{1}{3}$ feet long and 6 inches broad.
Ans. 5 feet 2 inches.

Ex. 12.——$15\frac{1}{12}$ feet long and $8\frac{1}{4}$ inches broad.
Ans. 10 feet 4 inches 5 parts 3″.

Ex. 13.——$19\frac{1}{10}$ feet long and 8 inches broad.
Ans. 12 feet $8\frac{4}{5}$ inches.

Ex. 14.——29 feet long and 22 inches broad.
Ans. 53 feet 2 inches.

Ex. 15.——$39\frac{1}{2}$ feet long and 16 inches broad.
Ans. 52 feet 8 inches.

PROBLEM

PROBLEM II.

To find the ſolid content of ſquare timber.

RULE.

Multiply the three dimenſions continually, and the laſt product will be the ſolidity.

EXAMPLE I.

Required the ſolid content of a tree 18 feet long, and 18 inches the ſide of the ſquare.

By Duodecimals.	Decimally.	By Reduction.
F. In.	F.	F. In.
18 0 length.	18	18=216
1 6 breadth.	1.5	18
18 0	90	1728
9 0	18	216
27 0	27.0	3888
1 6 thickneſs.	1.5	18
27 0	1350	31104
13 6	270	3888
40 6 *Anſ.*	40.50 *Anſ.*	1728)69984(40 feet.
		6912
		144)864(6 inches.
		864

By the Sliding rule.

Set 18 feet, the length, on the ſlip, to 12 on the girt line, and

and against 18, the side of the square, in inches, on the girt line, is $40\frac{1}{2}$ on the slip, which is the answer in feet and fractional parts.

Ex. 2. Required the content, when the length is 19 feet, side of the square 9 inches. *Ans.* 10 feet 8 inch. 6 pts.

Ex. 3.——the length 24 feet, side of the square 14 inches.
Ans. 32 feet 8 inches.

Ex. 4.——$23\frac{1}{2}$ feet, side of the square 30 inches.
Ans. 146 feet 10 inch. 6 pts.

Ex. 5.——$14\frac{1}{3}$ feet, side of the square 15 inches.
Ans. 17 feet 11 inches.

Ex. 6.——$22\frac{3}{4}$ feet, side of the square 12 inches.
Ans. 22 feet 9 inches.

Ex. 7.——$15\frac{1}{6}$ feet, side of the square 10 inches.
Ans. 10 feet 6 inch. 4 pts. 8″.

Ex. 8.——$24\frac{1}{12}$ feet, side of the square 2 feet.
Ans. 96 feet 4 inches.

Ex. 9.——$28\frac{1}{8}$ feet, side of the square 11 inches.
Ans. 23 feet 10 inch. 7 pts. 1″ 6‴.

Ex. 10.——10 feet, side of the square $14\frac{1}{2}$ inches.
Ans. 12 feet 1 inch.

Ex. 11.——6 feet, the side of the square 3 feet.
Ans. 54 feet.

Ex. 12.——$8\frac{1}{2}$ feet, side of the square 14 inches.
Ans. 10 feet 11 inches.

Squared timber, whose breadth is greater than the thickness, may be truly cast up by the above rule with as much expedition as with any other. Unskillful measurers, however, generally add the breadth and thickness together, and they consider half their sum as the side of the square, and work accordingly; but this method is false; and the greater the difference between the breadth and thickness is, the error thence resulting will be the

the more considerable; and in every case of this kind the result is greater than the truth.

EXAMPLE I.

Required the solidity of a log whose length is $19\frac{1}{2}$ feet, breadth 32, and thickness 20 inches.

	The true method.	
The common method.	Decimally.	Duodecimally.
	F.	F. In.
32	3)19.5	19 6
20	2.6	2 8
2)52	390	39 0
	65	13 0
26 side of the square.	65	
26		52 0 0
	3)520	1 8
156	1.6	
52		52 0 0
	520	34 8 0
676	1733	
19.5	1733	86 8 0
3380	86.66	
6084		
676		
144)131820(91 feet.		
1296		
222		
144		

		F.	In.	pts.
12)78(6 inches.	True	86	8	0
72	False	91	6	6
6 parts.	Error	4	10	6

By the Sliding rule.

Set the rule and slip even at the ends; and over against the

area

area of the base on the slip you have the side of the square on the girt line; then the solidity is found, as above.

Ex. 2. How many cubic feet are in a log of wood 40 feet 6 inches long, the base being 32 inches by 20? *Ans.* 180 feet.

Ex. 3. Required the content of a log of wood, whose length is $13\frac{1}{4}$ feet, and base 10 inches by 8. *Ans.* 7 feet 4 in. 4 pts.

Ex. 4. How many cubic feet are in a plank 12 feet long, 1 foot broad, and 6 inches thick? *Ans.* 16 feet 9 in. 3 pts. 10″

Ex. 5. How many cubic feet are in a plank 20 feet long, the base being 30 inches by 20? *Ans.* 83 feet 4 inches.

Ex. 6. How many cubic feet are in a log, whose length is 15 feet 9 inches, the breadth being 12 inches by 5?

Ans. 6 feet 6 inches 9 pts.

Ex. 7. Required the solidity of a tree, $10\frac{1}{2}$ feet long, the base being 15 inches by 9. *Ans.* 9 feet 10 in. 1 prt. 6″

Ex. 8. Required the content of a log 16 feet long, 30 inches broad, and 18 thick. *Ans.* 50 feet.

9. Required the solidity of a plank 19 feet 3 inches long, 9 inches broad, and 6 inches thick. *Ans.* 7 feet 2 in. 7 pts. 6″.

Ex. 10. How many cubic feet are in a plank 17 feet long, its base being 14 inches by 7? *Ans.* 11 feet 6 in. 10 parts.

Ex. 11. Required the content of a plank whose length is $15\frac{1}{2}$ feet, breadth 10 inches, and thickness 8 inches.

Ans. 8 feet 7 inches 4 parts.

Ex. 12. Required the solidity of a plank whose length is 19 feet 10 inches, breadth 15 inches, and thickness 12 inches.

Ans. 24 feet 9 inches 6 parts.

PROBLEM III.

To find the solidity of round or unsquared timber.

RULE,

RULE, *common way*.

Multiply the ſquare of $\frac{1}{4}$ of the circumference by the length, and the product will be the ſolidity.

EXAMPLE I.

Required the ſolidity of a tree whoſe length is 32 feet, and girt 60 inches.

	Decimally.	Duodecimally. F.	In.
$\frac{1}{4}$ of 60 is 15	1.25	32	0
15	1.25	1	3
75	625	32	0
15	250	8	0
	125		
225		40	0
32	1.5625	1	3
	32		
450		40	0
675	31250	10	0
	46875		
144)7200(50 feet.		50	0
720	50.0000		
0			

By the Sliding rule.

Set the length in feet on the ſlip to 12 on the girt line, and over againſt the ſide of the ſquare, (which is $\frac{1}{4}$ of the girt) on the girt line, you have the content on the ſlip.

Now, if we conſider the tree a cylinder, its ſolidity may be found as follows.

The area of a circle, whose circumference is 1, is .0795775; therefore,

```
As 1² : 0795775 : : 3600
                    3600
        ------------
         477465000
        2387325
        ------------
        286.4790000 area of the end.
                  32
        ------------
        5729580000
       8594370000
       ------------
   144)9167.3280000(63.662
      864              12
      ---            -----
       527           7.944
       432              12
       ---           -----
        953          11.328
        864
        ---
         892
         864
         ---
          288
          288
          ---
```

	F.	Ins	Pts.
True content	63	7	11
False content	50	0	0
Error	13	7	11

By comparing the two methods above, we see that the common way is 13 feet 7 inches 11 parts (which is nearly $\frac{1}{4}$) less than the true quantity. It is strange that a method so absurd, and so pernicious in its consequences, should ever be practised. The ease with which it is performed is perhaps the only argument which can be alleged for using it.—The following rule will give the content extremely near the truth: It may be performed with equal ease with the false one, and should on that account be universally used.

RULE.

RULE.

Multiply the ſquare of $\frac{1}{5}$ of the girt by twice the length, and the product is the content very near the truth *.

Circum. 60, of which the $\frac{1}{5}$ is	12
	12
	144
Twice the length, -	64
	576
	864
	144)9216(64 *Anſ.*
	864
	576
	576

Ex. 2. Required the content when the girt is 4 feet 2 inches, and the length 15 feet. *Anſ.* 20 feet 10 inches.

Ex. 3. What is the ſolidity, when the girt is 55 inches, and the length 20 feet 6 inches? *Anſ.* 34 feet 5 inch. 5 pts.

Ex. 4. Required the ſolidity of a tree whoſe girt is 6 feet 8 inches, and length 16 feet 4 inches.

Anſ. 58 feet 0 inch. 10 pts. 8″.

Ex. 5. Required the ſolidity of a tree, the circumference being 30 inches, and the length $6\frac{1}{2}$ feet. *Anſ.* 3 feet 3 inches.

Ex. 6. Required the content of a tree whoſe girt is 35 inches, and length 17 feet 8 inches.

Anſ. 12 feet 0 inches 3 pts. 4″.

Ex. 7. The girt is 90 inches, and the length 19 feet, required the ſolidity. *Anſ.* 85 feet 6 inches.

Ex. 8. How many ſolid feet are in a tree whoſe girt is 95 inches, and length 25 feet? *Anſ.* 125 feet 4 inches 2 pts.

* By this rule theſe 9 examples are computed.

Ex. 9. How many ſolid feet are in a tree 5 feet 5 inches girt, and 20 feet long? *Anſ.* 46 feet 11 inches 4 pts.

Tapering-Timber.

Tapering-timber is that which is thicker and broader at the one end than at the other.

When the tree tapers regularly, the dimenſions may be taken at the middle for the mean dimenſions; or they may be taken at both the ends, and half their ſum will be the mean dimenſions.

If the tree be very irregular, the dimenſions ought to be taken at ſeveral equidiſtant places, and their ſum divided: Or the tree may be divided into a certain number of lengths, the content of each part found ſeparately, and their ſum will give the content of the whole.

The mean girt of round tapering trees is found in the ſame manner. When trees have their bark on, it is cuſtomary to make an allowance, by deducting ſo much from the girt as is judged ſufficient to reduce it to ſuch girt as it would have without the bark. In oak, the allowance is generally $\frac{1}{10}$ or $\frac{1}{11}$ of the girt; but in elm, aſh, beech, &c. the bark not being ſo thick, the deduction ought to be leſs.

EXAMPLE I.

A tapering-tree, whoſe length is 24 feet, the girt at the greater end being 7 feet, and at the leſs 1 foot; it is required to find its content according to the true method, alſo in the common way.

	True.	Common.
7	.8	The $\frac{1}{4}$ girt is 1
1	.8	1
2)8	.64	1
5)4 mean girt.	48	24
.8	512	24
	256	
	30.72	
	12	
	8.64	
	12	
	7.68	
	12	
	8 16	

	F.	In.	Pts.	
True	30	8	7	8″
Common	24	0	0	0
Error	6	8	7	8

EXAMPLE II.

A tree is girt in 6 different places, as follows:—In the first place, 9 feet; in the second, 6 feet 8 inches; in the third, 5 feet; in the fourth, 4 feet 9 inches; in the fifth, 4 feet 2 inches; and in the sixth, 3 feet 5 inches—required its solidity, its length being 12½ feet.

F.	In.	
9	0	1.1 mean girt.
6	8	1.1
5	0	121
4	9	25
4	2	605
3	5	242
6)33	0	30.25
5)5.5 mean girt.		12
1.1		3.00 *Ans.* 30 feet 3 inches.

MASON-

Mason-Work.

To Masonry belongs all sorts of stone-work; paving and causewaying are measured by the square yard.

Digging for foundations or vaults is estimated by the solid yard.

Slabs, chimney-pieces, &c. by the square foot.

Stones for hewn-work, marble blocks, columns, &c. by the solid foot; but marble-facings by the superficial foot, two inches being the standard thickness.

Hewn-work is generally measured by the superficial foot. The dimensions are taken by a measuring-line, which is bent into all the hollows, and over the projections, in order to ascertain the extent of surface which has been shaped by the tool.

Ruble-work is estimated by the rood of 36 square yards; the standard thickness is 2 feet; and walls of any other thickness must be reduced to that standard.

The value of materials is charged by the solid measure, and of workmanship by the superficial measure *.

Circular work, arches, &c. are estimated double measure. Deductions for all vacancies, such as doors, windows, &c. are made with regard to materials, but none with regard to workmanship.

Different

* When the thickness is unequal in different parts, it must be reduced to the standard of 2 feet, by making proper allowances on such parts as are thicker or thinner than others. For example, if the gable of a house is three feet thick, find the superficies, and to this superficies add one half of the same.

Different methods are used in different places for taking the dimensions of a house; and indeed of these there is such a variety, that scarcely any general rule can be given. A measurer, therefore, before he proceeds to measure a house, ought to inform himself as to the nature of the agreement, the customs of the place with regard to the method of taking dimensions, and of making allowances.

It is, however, a pretty general custom to add one half the thickness to the height of the side walls, as an allowance for the extraordinary trouble of levelling, and a foot for every belt. This allowance is given at the stated rate.

A gable-end, where there are no vents, may be considered a triangle, and measured accordingly; but when there are vents, it is a trapezoid, and ought to be computed by the rule for its proper form.

Chimney stalks are measured by multiplying one-half the girt by the height.

EXAMPLE. I.

Required the solid content of a wall whose length is 53 feet 8 inches, its height being $12\frac{1}{2}$ feet, and thickness 3 feet.

F.	In.
53	8
12	6
644	0
26	10
670	10
3	0
2)2012	6
1006	3
3018	9 standard.

9)3018	9		
36)335	3		
9	11	3	9

Ans. 9 roods 11 yards 3 feet 9 inches.

Ex.

Ex. 2. Required the content of a wall whoſe length is 60 feet 9 inches, its height 10 feet 3 inches, and thickneſs $2\frac{1}{2}$ feet. *Anſ.* 1556.718 feet.

Ex. 3. In a chimney-piece, the length of each jamb is 4 feet 4 inches, breadth of both together 1 foot 9 inches, length of the mantle and ſlab each 4 feet 6 inches, breadth of both together 3 feet 2 inches—required the content.

Anſ. 21 feet 10 inches.

Bricklayers-work.

In Scotland, brick-work is meaſured by the ſquare yard—in England, by the ſquare rod, which is 16.5 long, and conſequently contains 272.25 ſquare feet; but 272 is generally eſteemed ſufficiently accurate.

Brick-work is of ſtandard thickneſs, when the wall is the length of one brick and the breadth of another, thick. Brick walls of other thickneſs muſt be reduced to that ſtandard by the following

RULE.

Multiply the ſuperficial content of the wall by the number of half bricks in the thickneſs; divide the product by 3, and the quotient is the content, reduced to ſtandard thickneſs.

EXAMPLE I.

A brick wall is 36 feet 6 inches long, and 17 feet 3 inches high, and $5\frac{1}{2}$ bricks thick—required the content.

36.5

```
            F.
          36.5
          17.25
        -------
          1825
          730
         2555
         365
        -------
        629.625 content.
             11 half bricks.
        -------
      3)6925.875
        -------
272)2308.625(8 rods.
    2176
    --------
  68)132.625(1 quarter.
      68
    --------
      64.          Ans. 8 rods 1 quarter 64 feet.
```

Ex. 2. A brick wall is $84\frac{1}{2}$ feet long, $17\frac{1}{4}$ feet high, and $5\frac{1}{2}$ bricks thick—how many rods of brick?

Ans. 19 rods 2 quarters 35 feet.

Ex. 3. If each side wall of a building be 45 feet long on the outside, each end wall 15 feet broad on the inside, the height of the building 20 feet, and the gable at each end of the wall 6 feet high, the whole being two bricks thick—required the content. *Ans.* 12.1761.

Plaisterers Work.

Plaisterers work is of two kinds, viz. plaistering upon walls, called *rendering;* and plaistering upon laths, called *ceiling*. Deductions are made for doors, windows, &c. All is measured by the square yard.

 EXAMPLE

EXAMPLE I.

The length of a partition is 22 feet, and height 12,—how many yards of plaister-work are in it?

$$\begin{array}{r}
\text{F.} \\
22 \\
12 \\
\hline
9)264 \\
\hline
29 \quad 3
\end{array}$$

Ans. 29 sq. yds. 3 feet.

Ex. 2. If a ceiling be $59\frac{3}{4}$ feet long, and $24\frac{1}{2}$ feet broad—how many yards? *Ans.* 162.652 square yards.

Ex. 3. How many yards, rendering and ceiling, in a room $28\frac{1}{2}$ feet long, $13\frac{1}{2}$ broad, and 8 high?

Ans. 117 yards 3 feet 9 inches.

CARPENTERS WORK.

CARPENTERS or joiners work is that of flooring, roofing, partitioning, &c. and is either measured by the square yard, or by the square of 100 feet.

Doors and window-shutters are reckoned work and half-work. If the door be pannelled on both sides it is esteemed double work. For the architrave, gird it about the outmost part for its length; measure over it as far as can be seen, when the door is open, for the breadth. In the measuring of roofing, for workmanship alone, sky-lights and holes for chimney-shafts are deducted; but when for workmanship and materials together, no deduction is made.

Stairs

Stairs may be measured in the following manner:—Take the breadth of all the steps, and make a line ply over them from top to bottom; multiply the length of this line by the length of a step for the area.

EXAMPLE I.

If a floor is $50\frac{1}{2}$ feet long and 28 feet broad—how many squares?

$$\begin{array}{r} 28 \\ 50\frac{1}{2} \\ \hline 1400 \\ 14 \\ \hline 100)1414 \\ \hline \end{array}$$

Ans. 14 squares 14 feet.

Ex. 2. If a house within walls be 44 feet 6 inches long, and 18 feet 3 inches broad, how many squares of roofing will cover it?

Ans. 12 squares 18 feet.

It is customary to consider the roof as a flat, and half a flat taken within walls.

Tilers and Slaters Work.

The content of a tiled roof is found by multiplying the length by the girt from eave to eave; and that of a slate roof by multiplying the length by the girt plied over the eaves, to meet the wall sor eave boards.

In ſome places, double meaſure is allowed for hips, vallies, gutters, &c. and no deduction for chimneys.

EXAMPLE I.

A ſlate roof is 30 feet long and 26 feet over,—how many ſquares are in it?

F.	In.	
26	0	
1	6	the allowance for the eaves.
27	6	
30	0	
100)825	0	

Anſ. 8. ſquares 25 feet.

Ex. 2. How many ſquares tiling in a roof $35\frac{1}{4}$ feet deep, and whoſe length is 48? *Anſ.* 16 ſquares 92 feet.

Ex. 3. How much ſlating in a pavilion roof, with a platform 50 feet long and 30 broad at the eaves, and 30 feet long by 10 at the platform; 14 feet from the platform to the eaves, taken the ſhorteſt way, and $17\frac{1}{2}$ along the hips?

Anſ. 5 roods $31\frac{2}{3}$ yards.

Glaziers Work.

Glaziers take their dimenſions in feet, inches, and parts. All circular, triangular, &c. windows, are meaſured as if they were ſquares; and the greateſt lengths and breadths are taken on account of the trouble and waſte of glaſs attending the cutting.

EXAMPLE.

EXAMPLE.

A houſe with 3 rows of windows, 5 in a row, the dimenſions of the firſt row is 6 feet 4 inches by 4 feet 2 inches; of the ſecond row, 5 feet 9 by the ſame breadth; of the third, 5 feet 3 inches, ſame breadth; alſo a ſemicircular window above the door, whoſe height is 2 feet;—required the expence of glazing, at 1s. *per* ſquare foot.

F.	In.		F.	In.		F.	In.		F.
6	4		5	9		5	3		2
4	2		4	2		4	2		4
25	4		23	0		21	0		8
1	0	8	0	11	6	0	10	6	
26	4	8	23	11	6	21	10	6	
		5			5			5	
131	11	4	119	9	6	109	4	6	

		F.	In.	
Firſt flat is	-	131	11	4
Second ditto,	-	119	9	6
Third ditto,	-	109	4	6
That above the door		8		
		369	1	4

Which, at 1s. *per* ſquare foot, amounts to 18l. 9s. $1\frac{1}{3}$d.

Painters Work.

Painters work is meaſured in the ſame manner as that of carpenters; and, in taking the dimenſions, the line muſt be bent into all the hollows, and over all the projections, in order

to

to ascertain the surface which has been touched with the brush. The painting of the bars of windows is reckoned whole work, it being troublesome; and railing a whole surface painted on both sides. Doors, window-shutters, &c. are reckoned double work. Deductions are to be made for chimnies, casements, &c.

EXAMPLE I.

The height of a room is 12 feet 9 inches, circumference 60 feet;—how many square yards of painting?

$$\begin{array}{r} 12.75 \\ 60 \\ \hline 9)765.00 \end{array}$$

Ans. 85 sq. yards.

Ex. 2. The height of a room is 11 feet 7 inches, circumference 74 feet 10 inches, the door 7 feet 6 inches by 3 feet 9 inches, 5 window-shutters, each 6 feet 8 inches by 3 feet 4 inches, the breaks in the windows 14 inches deep and 8 feet high, the chimney 6 feet 9 inches by 5 feet; a closet, the height of the room, $3\frac{1}{2}$ feet deep and 4 feet 9 inches in front, with shelving, at 22 feet 6 inches by 10 inches, painted on both sides,—what will the whole amount to, at 6d. *per* square yard?

Ans. 3l. 14s. $0\frac{3}{4}$d.

Miscellaneous Questions.

Question 1.

What will the side of a cube be, when the surface and solidity are expressed by the same number? *Ans.* 6.

2. A

2. A ciftern 25 feet long, 15 broad, and 10 feet deep, is to be enlarged, fo as to contain four times the quantity, its depth will be $7\frac{1}{2}$ feet, and the fides in the proportion of 3 to 5;—required the fides. *Anf.* 34.641013, and 57.735026 feet.

3. Required the diameter of a globe equal to a cube, whofe fide is 6. *Anf.* 7.43.

4. Three men, Wright, Smith, and Turner, bought a grinding ftone of 60 inches diameter; Wright and Smith paid equal fhares, and Turner one-half fhare; each partner is to grind the ftone in his turn, firft Wright, then Smith, and Turner laft. How much of the diameter ought each to grind down for his fhare?

Anf. $\begin{cases} \text{Wright} = 13.525 \text{ inches} \\ \text{Smith} = 19.640 \\ \text{Turner} = 26.835 \end{cases}$

5. What is the diameter of a cylinder when the cylindric area is expreffed by the fame number with the folidity? *Anf.* 4.

6. My wright made me a ladder of fuch a length, that, by planting it on a ftreet 70 feet wide, it will reach a window on one fide 40 feet high; by turning it over, without moving the foot, it will do the fame by a window 30 feet;—required the length of the ladder. *Anf.* 50 feet.

7. My copperfmith agreed to make me a flat-bottomed kettle, that fhould contain $13\frac{11}{20}$ ale gallons: The depth of the kettle is 12 inches, the top and bottom diameters are in the proportion of 5 to 3;—required the diameters.

Anf. $\begin{cases} \text{The head diam. } 25 \text{ inches.} \\ \text{The bottom } 15 \end{cases}$

8. What ought to be the depth of a tub whofe bafe and head diameters are 20 and 10, to contain 9163 folid inches?

Anf. 50 inches.

9. There is a wall containing 18225 folid feet, the height is 5 times the thicknefs, the length is 8 times the height;—required the thicknefs of the wall. *Anf.* 4.5 feet.

10. A tree 100 feet high, ftanding on a horizontal plane, was

was broke by a tempestuous wind, so that the upper part of it struck the ground 30 feet from its root;—required the length of the part broke off. *Ans.* $54\frac{1}{2}$ feet.

11. A square within a circle contains 16 square yards;—required the area of a square circumscribed about the same circle. *Ans.* 32 square yards.

12. The cubical altar at Delphos was 1 cubit;—required the side of the new altar, which was to be twice that size, and of the same form. *Ans.* 1.259921 cubits.

13. The ceiling of a room, at 6d. *per* square yard, amounted to 2l. 10s.; the breadth is $22\frac{1}{2}$ feet;—required the length. *Ans.* 40 feet.

14. A man left two daughters; ordered by his will that the eldest should have annually the rent of a square field, and the younger the rent of a circular one; the rent of each field, at 20s. *per* acre, would exactly surround it in shillings, allowing one inch to each shilling;—required the portion of each of the daughters.

Ans. { The first's annual income is 250905l. 12s.
The second, - 197061l. 5s. 2d.

15. A gentleman had a garden surrounded by a terrace walk; the length of the garden is 500 yards, and the breadth 400 yards; the walk was equal to $\frac{1}{8}$ of the garden;—required the breadth of the walk. *Ans.* 13.6809 yards.

16. A gentleman has a garden 100 feet long and 80 broad, but intends to make a walk half round it that shall take up half the ground;—required the breadth of the walk.

Ans. 25.96876.

17. The length of a room is 40 feet, and breadth 30 feet; the expence of painting the walls, at 6d. *per* square yard, is 7l.—required the height of the room. *Ans.* 18 feet high.

18. The diameter of a malt bushel is 18 inches, and its depth $8\frac{9}{20}$ inches; what ought to be the depth when the diameter is 20 inches? *Ans.* 6.8448 inches.

19. Three

19. Three farmers, A, B, and C, had each an equal ſhare of a triangular field, whoſe baſe, being a river, meaſured half an Engliſh mile, its content being 120 acres: the marches were drawn parallel to the baſe. Now C had his ſhare next the river, B the middle diviſion, and A the reſt. They agreed with an undertaker to dig a ditch from the top of A's diviſion perpendicular to the river ſide. The breadth of the ditch at a medium is 6 feet, and depth $4\frac{1}{2}$ feet. Required the undertaker's charge againſt each of the three farmers, at 4d. *per* ſolid yard.

Anſ. { A pays 38l. 2s $2\frac{1}{2}$d.
B pays 15l. 15s. $7\frac{1}{2}$d.
C pays 12l. 2s. 2d.

20. One horſe chaiſe, in turning round a ring, it was obſerved that the outer wheel made three turns for every two of the inner wheel. The height of the wheels being $4\frac{1}{2}$ feet, and diſtance 5 feet, required the area of the ſpace or ring included betwixt the tracts of the wheels. *Anſ.* 392.7 ſquare feet.

21. The gilding of a ball, at 3d. *per* ſquare inch, coſt as much as the buying of it at 1d. *per* cubical inch: It is required to find its diameter. *Anſ.* 18 inches.

22. A cone, whoſe height is 40 inches, is to be divided into three equal parts, by ſections parallel to the baſe;—required the height of each.

Anſ. { The upper part is 27.73.
The middle part 7.21.
The lower part 5.06.

23. What length of a plank, that is 10 inches broad, will make $4\frac{1}{2}$ ſquare feet? *Anſ.* 5 feet $4\frac{4}{5}$ inches.

24. What length of a log of wood, that is 15 inches broad and 11 inches thick, will be equal to 10 cubical feet?

Anſ. 8 feet $8\frac{8}{11}$ inches.

25. What length of a cord will ſtrike off one-third part of a circular pond, whoſe diameter is 40 yards? *Anſ.* 38.5696 yds.

26. Two men, A and B, bought a round piece of wood, equally thick throughout. A paid one-third ſhare, B the reſt. A propoſes to have a ſlip the whole length of the tree, for his

ſhare. At what diſtance from the centre muſt the line be ſtruck, the diameter being 40 inches?

Anſ. $4\frac{3}{10}$ inches, or A will cut off $15\frac{7}{10}$ of the diameter.

27. A leaden pipe of 4 inches bore weighs 20 lb. averdupois *per* foot in length;—required the thickneſs, the ſpecific gravity of lead being 11325. *Anſ.* .3014 inches.

28. If ſimilar ſolids be to one another in the triplicate ratio of their homologous ſides, and the weight of an iron ball of 4 inches diameter be $9\frac{1}{2}$ lb.—required the weight of a ball of 6 inches diameter. *Anſ.* 32.025 lb.

29. If water iſſue through an orifice with the ſame velocity that a heavy body would acquire in falling freely from the ſurface of the water to the level of the orifice; and ſuppoſing a ſhip to have been bored through by a 32 pounder, 8 feet below water; it is required to determine what weight of water ſhe will draw in 10 minutes. *Anſ.* 13 ton 2 cwt. 3 qr. $24\frac{1}{2}$ lb.

30. Suppoſing three wheels, A, B, and C, ſo combined that three revolutions of A will drive B five times round, and ſeven revolutions of B will drive C nine times round, it is required to find the leaſt number of teeth neceſſary to regulate the above motion. *Anſ.* A 15, B 9, and C 7 teeth.

31. Upon the ſame ſuppoſition, it is required to determine the number of revolutions each muſt take before the ſame teeth be in conjunction. *Anſ.* A takes 21, B 35, and C 45.

32. A millſtone of 5 feet diameter ſtrikes ſeconds; how many miles would ſhe run upon edge at the ſame rate in ſeven days? *Anſ.* 1799 miles 2 f. $9\frac{3}{5}$ p.

33. The ſame conditions being given, in what time would ſhe go round the terraqueous globe, its diameter being 7958 miles? *Anſ.* 97 days 6 hours 20 min 48 ſec.

34. Suppoſing Loch Tay to be ſupplied ſo as to diſcharge a river 100 yards broad and 3 yards deep, flowing at the rate of 3 miles *per* hour, it is required to determine how far below the ſurface

ſurface an orifice of a ſquare yard ought to be, to diſcharge an equal quantity. *Anſ.* 3249 feet.

35. The times in which pendulums vibrate are as the ſquare root of their lengths; and if a pendulum of 39.128 inches vibrate ſeconds, how many ſwings will a pendulum of 156.512 inches take in an hour? *Anſ.* 30.

36. Required the length of a pendulum that vibrates half ſeconds. *Anſ.* 9.782 inches.

37. My plumber has ſold me 600 yards leaden pipes, of 4 inches bore and $\frac{1}{4}$ thick, at 16s. *per* cwt.; at the ſame time he purchaſed a garden in the form of a rightangled triangle, of ſuch dimenſions that the difference between the hypothenuſe and the ſum of the two other ſides was 50 yards. I, being a gardener, have undertaken to make a circular fiſh-pond in the garden to touch the three ſides, at 6d. *per* ſolid yard, and find, upon the balance, 73l. 13s. 9d. due to him; required the depth of the pond. *Anſ.* 3 yards, or 9 feet.

38. What length of a ridge 6 ells broad will make 10 falls? *Anſ.* 60 ells

39. If a pipe of 4 inches diameter is ſufficient to ſupply a town with water, required the diameter of a pipe, ſimilarly ſituated with the former, that ſhall ſerve, after the town is encreaſed by one half. *Anſ.* 4.899 inches.

40. The ſame town, after being ſo encreaſed, to ſave expences, intends to be ſupplied from the old pipe; at what depth in the reſervoir muſt it be placed, it being formerly 4 feet deep? *Anſ.* 9 feet.

41. Given the two ſides of an acute angled triangle 40 and 60 poles; required the included angle, and the third ſide, that the triangle may contain 4 acres of land.

Anſ. 32° 14′, and 33.84 poles.

42. Two porters agreed to drink off a quart of ſtrong beer between them, at a draught each. Now the firſt having drank till the ſurface of the liquor touched the oppoſite edge of the

 bottom

bottom of the quart pot, and gave the remaining part of it to the other, what was the difference of their ſhares, ſuppoſing the pot was the fruſtum of a cone, the depth being 5.7 inches, the diameter at the top 3.7 inches, and that of the bottom 4.23 inches? *Anſ.* 7.07 cubic inches.

43. A heavy body is dropt into an empty pit; ten ſeconds of time elapſed before the ſound from the bottom was heard. It is required to find the depth of the pit. *Anſ.* 1273 feet.

44. If a heavy ſphere, whoſe diameter is 4 inches, be put into a conical glaſs full of water, whoſe diameter is 5 inches and depth 6 inches, how many cubic inches of water will run over? *Anſ.* 26.1674336 inches.

45. A gentleman has a rectangular piece of ground which he intends for a bowling-green. It being a regular declivity of 30°, the ſlant ſide meaſured 500 feet, the other 433 feet; required the expence of levelling it, one part with another, at 3d. *per* cubic yard. *Anſ* 2712l. 10s. $3\frac{1}{2}$d.

46. The ſame bowling-green being levelled, required the expence of dreſſing it at $\frac{1}{8}$d. *per* ſquare foot.

Anſ. 102l. 17s. $2\frac{1}{8}$d.

47. An old mathematician bought a field in the form of a parabola, the length of the baſe or ordinate being 3000 links, and of the axis or abſciſſa 2500. He built a ſummer-houſe 9 chain-lengths diſtant from the ordinate, and 2 chain-lengths from the abſciſſa; and ordered by his will, that the field ſhould be divided between his two ſons, by the leaſt line that could be drawn through the ſummer-houſe, terminating both ways by the parabolic curve; required how much land each of the ſons ſhall have. *Anſ.* $\begin{cases}\text{The one will have 25 ac. 2 r. 16 falls.}\\ \text{The other, 24 ac. 1 r. 24 falls.}\end{cases}$

48. If a ciſtern can be filled with water from one cock in 12 hours, and from another in 8 hours, in what time will it be filled by both cocks running together? *Anſ.* 4 hours 48 min.

49. The paving of a ſquare court, at 6d. a yard, coſt as much

much as the inclosing at 5s. a yard; required its extent.

Ans. 40 yards.

50. An army being drawn up in a square, there were 79 over; but in attempting to enlarge each side of the square by one man, there was a deficiency of 80; required the number of men. *Ans.* 6241 men.

51. An oblong pond was surrounded by a terrace-walk 7 yards broad; the pond measured 1500 square yards, and the walk 3696 square yards; required the length and breadth of the pond. *Ans.* 100 by 150 feet.

42. A gardener and his servant being at work, each digs a square piece of ground, whose side is as many feet long as the labourer is years old. The gardener digged four times the quantity the servant digged. The sum of their ages was 45 years. Required their ages, and the quantity dug by each.

Ans. The lad's age is 15—the gardener's 30 years. The lad dug 225 square feet—the gardener 900.

53. A rectangular plantation of 360 acres, contains 435600 trees; required the distance of the trees. *Ans.* 6 feet.

54 How many trees can be planted in the same area, at the distance of 12 feet? *Ans.* 108900.

55. The sanctuary at Butis in Egypt is formed of one stone, in the form of a cube of 60 feet, open at the top, and hollowed so as to leave the stone 6 feet thick; required the weight, at the rate of 2520 averdupois ounces *per* cubic foot.

Ans. 6439$\frac{1}{2}$ tons.

56. Two neighbouring gentlemen, A and B, had a circular piece ground, containing 785 acres 1 rood 24 poles, whereof A has two-thirds and B the rest. They agreed with a mason to build a mutual wall on the march at a guinea *per* yard, and that each should pay the expence in proportion to his share of the ground. It is required to find the mason's charge against each of the gentlemen.

Ans. A pays 1484.4984l. B pays 742.2592l.

57. A

57. A circular pond occupies half an English acre; required the perimeter of a square circumscribed about the pond.

Anſ. 1009 links.

58. Three farmers, A, B, and C, had each an incloſure. B's incloſure contains 100 Engliſh acres. A's incloſure, and that of B's, are together $1\frac{1}{3}$ times as large as that of C's; and B's and C's are together $3\frac{2}{3}$ times as large as that of A's;—required the extent of A's and C's

Anſ. A had 60 acres and C 120.

59. Suppoſing A' incloſure, as in laſt queſtion, to be in the form of a rhombus, and one of its acute angles 30°, required the expence of incloſing it with a wall 6 feet high, $1\frac{1}{2}$ feet thick, at 5 guineas *per* rood, ſtandard meaſure.

Anſ. 423 391 guineas.

60. The pit wheel of a meal-mill contains 60 cogs, and makes 16 revolutions in a minute. It drives a trundle of 8 teeth. How many revolutions will the ſtone make *per* minute?

Anſ. 120 revolutions.

61. The advantage gained by water-falls of different heights is as the ſquare root of the heights. Now, ſuppoſing a fall of 4 feet ſufficient to overcome 10 cwt. 3 qr. 7 lb. of friction, what friction will the ſame quantity of water overcome from a fall of 16 feet? *Anſ.* 1 ton 1 cwt. 2 qrs. 14 lb.

62. A wright engaged to build a common corn-mill under the following reſtrictions. The ſtone muſt revolve 75 times *per* minute; the trundles to have 8 rungs, the driver 80 cogs, and the fall 16 feet high. Required the diameter of the water-wheel to produce the greateſt effect poſſible.

Anſ. 13 feet 6 inches 11 pts. 8″.

Note. The greateſt effect is obtained when the float-boards move with one-third the velocity of the impinging fluid.

63. A tapering round tree 10 feet long, whoſe diameter at the greater end is 3 feet, at the leſs 2 feet, being hurled down a regular declivity deſcribes a ſegment of a circle. How far diſtant

diſtant from the greater end is the centre of the ſegment?

Anſ. 30 feet.

64. A barrel is filled with pure ſpirits, and weighs, when full, 66 lb. How many gallons does it contain, allowing 6 lb. for the weight of the barrel? *Anſ.* 8.29 gallons.

65. A column of the atmoſphere, whoſe baſe is a ſquare inch, weighs 15 lb.; and ſuppoſing the atmoſphere to preſs equally in all directions,—required the preſſure upon a middle-ſized man, whoſe ſurface may be reckoned 16 ſquare feet.

Anſ. 34560 lb.

66. Suppoſe the atmoſphere, in a mean ſtate, balance mercury in the barometer $29\frac{1}{2}$ inches high, required the height to which water may be raiſed by means of a pump, the ſtate of the atmoſphere being the ſame. *Anſ.* $34\frac{1}{2}\frac{4}{5}$ feet.

67. Suppoſe the earth's mean diſtance from the ſun is 82 millions of miles, and goes round him in 365 days 5 hours 49 minutes, at what rate does it travel *per* hour?

Anſ. 58776 miles.

68. Light paſſes from the ſun to the earth in 8 minutes 15 ſeconds of time, and the velocity of the earth in its orbit is 58776 miles *per* hour; required the proportion they bear to each other.

Anſ. The velocity of light is to that of the earth as $99393\frac{3}{3}\frac{1}{3}$ is to 979.6.

69. In what time would the earth fall to the ſun at the rate of 58776 miles *per* hour?

Anſ. 58 days 3 hours 7 minutes 38 ſeconds.

70. The paving of a ſquare inſcribed in a ſemicircle, whoſe ſide coincides with the diameter, and whoſe oppoſite angles are in the circumference, at 9d. *per* ſquare foot, coſt 33l. 15s. required the diameter of the circle. *Anſ.* 22.3606 yards.

71. A triangle, whoſe three ſides are 800, 640, and 360 feet, is inſcribed in a circle; it is required to find the diameter of the circle. *Anſ.* 820.211 feet.

72. The

72. The three ſides of a triangular pyramid are 312, 360 and 96, and altitude 100 feet; it is required to find the ſolidity of a cone circumſcribed about the pyramid, and whoſe altitude is equal to that of the pyramid. *Anſ.* 3981978 cubic feet.

73. Required the dimenſions of a cone, whoſe area of the baſe, curve ſuperficies, and ſolid content, are in geometrical progreſſion, and the area of the baſe equal to the rectangle of the baſe's diameter and axis.

Anſ. The diameter of the cone's baſe is 24.7036, and axis 19.4022.

74. The area of an equilateral triangle being 720, required the ſide. *Anſ.* 40.7776.

75. Suppoſe I have a circular incloſure of an acre of ground, how long ought a cord be, that, faſtened in the circumference of the incloſure as centre, will ſtrike an arch that will divide the ſaid incloſure into two equal parts?

Anſ. 45.47898 yards.

76. A reſervoir is ſupplied from a pipe of 6 inches bore. How many pipes of 3 inches bore will be ſufficient to diſcharge the ſame quantity? *Anſ.* 4 pipes.

ALGEBRA.

APPENDIX.

ALGEBRA.

ALGEBRA is a general method of computation, by which many useful problems in geometry and arithmetic are solved, which, without its aid, would be impossible. The principles on which the rules are founded are similar to those in common arithmetic.

Certain symbols and characters are admitted into this science, to give it that extent and excellence which it possesses above all other methods of computation. Through all the steps of an algebraic operation, these symbols may be so conducted as to be preserved distinctly in view, with their relations and affections to each other, and at last to produce a canon, or general rule, by which not only the question proposed is solved, but every other question of the like conditions. Whereas, in the course of an arithmetical operation, the original numbers disappear.

NOTATION.

1.—Algebraic signs only affect those symbols to which they are prefixed.

2.—Quantity is that which is made up of parts, or is capable of being encreased by addition or diminished by subtraction. Hence a quantity may be introduced into an algebraic computation

tation two different ways, either as a decrement, or as an increment, that is, as a negative quantity, or as a positive one.

SIGNS.

3. + *(plus)* signifies Addition, or that the quantity to which it is prefixed is positive *.

— *(minus)* signifies Subtraction, or that the quantity to which it is prefixed is negative.

× signifies Multiplication †.

÷ signifies Division.

= Equal to, or the sign of equality.

$\sqrt{}$ *(the radical sign)* denotes the square root of the quantity annexed.

4.—A quantity may be represented by any symbol or character. It is, however, a pretty general custom to use the first letters of the alphabet, a, b, c, &c. for known quantities, and the last letters, x, y, z, for unknown ones. In the following compendium we will follow the general method.

To examplify these signs, let us suppose $a=3$, $b=8$, $c=12$, $d=10$, $e=4$, $m=6$, $p=1$, and $s=5$.

5.—Then the sum of a and b is represented thus, $a+b=11$.

The difference of d and p, - - $d-p=9$.

The

* When no sign is prefixed to a quantity, + is understood.

† When no sign is placed between two quantities, × is understood.

The product of a multiplied by c *, - $d \times c$ or $dc = 36$.

The quotient of d divided by s, - $d \div s$ or $\frac{d}{s} = 2$;

The square root of ac, - - - $\sqrt{ac} = 6$.

6. Plus and minus are always oppoſed to each other. Thus, if + ſignifies gain, — ſignifies loſs; if + ſignifies ſtock, — ſignifies debt; if + ſignifies a poſitive quantity, — ſignifies a negative one. And ſo on.

7.—When letters are placed together without any ſign between them, they denote the rectangle or product of the quantities they repreſent. Thus, the product of a into b is $ab = 24$.

A number prefixed to any quantity is underſtood to multiply the quantity, and is called its *co-efficient*. Thus, $6a = 18$.

8.—Diviſion is often repreſented by placing the dividend in the form of a numerator, and the diviſor in that of a denominator. Thus, b divided by a is $\frac{b}{a} = \frac{8}{3} = 2\frac{2}{3}$.

9.—The continual multiplication of quantities, by others of the like kind and dimenſion, is called powers of that quantity, and are commonly expreſſed by ſmall numbers placed at the corners of the letters, called *indices*, or *exponents*. Thus, the ſquare of a is a^2, the cube a^3, the fourth power a^4, &c. The exponent of the original quantity, or root, is unity, and is ſeldom or never expreſſed.

10.—Quantities of the like dimenſions, which, by their ſucceſſive multiplication, produce any given quantity, are called *roots* of the given quantity. Thus, a is the ſquare root a^2.

---

* When no ſign is marked between two or more quantities, it denotes their product.

Exercises for practice.

1. $2a+b=2\times 3+8=14.$
2. $bc+d=8\times 12+10=106.$
3. $\frac{bc}{a}=\frac{8\times 12}{3}=32.$
4. $cd-am+c^2=\overline{12\times 10}-\overline{3\times 6}+144=246.$
5. $\frac{ad}{s}+c^2-b^2=\frac{30}{5}+144-64=86.$
6. $\frac{s}{p}=\frac{5}{1}=$ - - 5
7. $\frac{a^2+b}{a^2-b}=\frac{3^2+8}{3^2-8}=\frac{17}{1}=$ 17

ADDITION.

Rule I.

11.—When the quantities are alike, and their signs the same, add the co-efficients, and to the sum prefix the sign, and annex the common letter or letters

Rule II. When the quantities are alike, but their signs different, subtract the lesser co-efficient from the greater; to their difference prefix the sign of the greater, and annex the common letter or letters.

Rule III. When the quantities are unlike, write them one after another, with their proper signs and coefficients.

By Rule I.

Ex. 1st,	2d,	3d,	4th,
$3a$	$a+b$	$4m-n$	$2a^2$
$4a$	$3a+2b$	$5m-3n$	$8a^2$
$7a$	$4a+3b$	$9m-4n$	$10a^2$

Rule

By Rule II.

Ex. 5th,	6th,	7th,	8th.
$4a$	$-5c+4b$	$5ab$	$-5ab-4c+de$
$-5a$	$2c+8b$	$-ab$	$+2ab-3de+c$
$-a$	$-3c+12b$	$4ab$	$-3ab-2de-3c$

In example 8th, the articles are to be arranged, so that like may stand under like.

By Rule III.

Ex. 9th,	10th,	11th,	12th,
$a+b$	$-a-c$	$acd+e$	$-a+b+c$
$2c+b$	$a+d$	$bc+m$	$b+d+a$
$a+2c+2b$	$d-c$	$acd+e+bc+m$	$2b+c+d$

Examples for practice.

1st,	2d,	3d,
$a+b$	$32a+4b$	$4a^2+3b$
$3a+2b$	$-a--3b$	$3a-b^2$

4th,	5th,	6th,	7th,
a^2	$a-b+c$	$c+a-b$	$ab+b^2$
$3a^2$	$2b-a+b$	$d-c-a$	$3b-c$

8th,	9th,	10th,	11th.
$10a+c$	a^2b-c^3	$d-a+b$	$a+3d-c$
$2c-8d$	$-c^3+a^2b$	$d+a-c$	$ac-c-5d$

SUBTRACTION.

SUBTRACTION.

Rule.

12. Change the signs of the subtrahend, or suppose them changed, then proceed as in addition.

Ex. 1st,	2d,	3d,	4th,
$7a$	$6a$	$-b$	$4ab$
$-6a$	$4b$	$7b$	$-3bc$
$13a$	$6a-4b$	$6b$	$4ab+3bc$

5th,	6th,	7th.
$3a+b$	$4ac-5b$	$-4d-3e$
$a-b$	$3ac+2b$	$5e+3d$
$2a+2b$	$ac-7b$	$-7d-8e$

Examples for practice.

1st,	2d,	3d,
$7a+3b$	$4c-5b$	$a-c+b^2$
$-3a^2+3b$	$-2a+5b$	$3a+b^2-d$

4th,	5th,	6th.
$12a-12c$	$a+b$	$4ac-5ab$
$-3a+b$	$a-b$	$3ab-3ae$

The reason of the foregoing rule is obvious; for if from any quantity a decrement be subtracted, it is the same as adding an equal increment. For example, If a man owe 100l. more than his stock, the state of his affairs may be represented — 100l. or he is 100l. worse than nothing. But if another add 100l. to his stock, it is the same thing as taking away his debt, for in either of these cases he will be worth nothing.

MULTI-

MULTIPLICATION.

RULE.

14.—Multiply the coefficients, and to their product annex the letters of both factors together. If the signs of the factors be like, the sign of their product is +; but if the signs of the factors be unlike, the sign of the product is —.

Ex. 1st,	2d,	3d,	4th.
$7ab$	$-a$	$3ab$	$-3a$
$10cd$	$+b$	$-8c$	$-2b$
$70abcd$	$-ab$	$-24abc$	$+6ab$

15.—Powers of the same root are multiplied by adding their exponents.

Ex. 1st,	2d,	3d,	4th,
a^2	$a^4b^2c^5$	$ab\ c^2d$	a^2b
a^3	$a^2b^3c^3$	a^2b	$a^3c^3b^3$
a^5	$a^6b^5c^8$	$a^3b^3c^3d$	$a^5b^4c^3$

16.—Radical quantities, under the like sign, are multiplied like others, and the product is placed under the same sign.

Ex. 1st,	2d,	3d.
$\sqrt{a}$	$a^4\sqrt{bc}$	$ab\sqrt{x}$
$3\sqrt{b}$	$a^4\sqrt{ad}$	$ab\sqrt{y}$
$3\sqrt{ab}$	$a^2\sqrt{abcd}$	$a^2b^2\sqrt{xy}$

17.—If one or both factors be compound, multiply each term of the multiplicand by all the terms of the multiplier successively,

cessively, and the sum of the particular products will be the product required.

Ex. 1*st*,

$$\begin{array}{r} a+b-3c \\ ab \\ \hline a^2b+ab^2-3abc \end{array}$$

2*d*,

$$\begin{array}{r} 3a+a^2b-a^3d-8b \\ 2a \\ \hline 6a^2+2a^3b-2a^4d-ab16 \end{array}$$

3*d*,

$$\begin{array}{l} a+b \\ a+b \\ \hline a^2+ab \\ \quad +ab+b^2 \\ \hline a^2+2ab+b^2 \end{array}$$

4*th*,

$$\begin{array}{l} 3a+b \\ 2a-b \\ \hline 6a^2+2ab \\ \qquad -3ab-b^2 \\ \hline 6a^2-ab-b^2 \end{array}$$

5*th*,

$$\begin{array}{l} a+b-c \\ \quad m-n \\ \hline am+bm-cm-an-bn+cn \end{array}$$

6*th*.

$$\begin{array}{l} a+b \\ a-b \\ \hline a^2+ab \\ \quad -ab+b^2 \\ \hline a^2 \quad * -b^2 \end{array}$$

or, a^2-b^2

18.—If one of the factors be a fraction, multiply its numerator by the other factor, and place the product over the given denominator.

$$\frac{2a}{b}\times 2c=\frac{4ac}{b} \qquad \frac{3a}{b}\times 4b=\frac{12ab}{b}=12a$$

19.—If both factors be fractions, multiply their numerators for the numerator of the product, and their denominators for the

the denominator of the product. If the same letters occur in both numerator and denominator, they may be expunged without altering the value.

Ex. 1st,

$$\frac{3a}{b}\times\frac{a}{b}=\frac{3a^2}{b^2}$$

2d,

$$\frac{a+b}{a-b}\times\frac{a}{c}=\frac{a^2+ab}{ac-bc}$$

3d,

$$\frac{4b}{a}\times\frac{a}{b}=\frac{4ba}{ba}=4$$

4th,

$$\frac{a^2}{c}\times\frac{b^2}{a}=\frac{a^2b^2}{ac}=\frac{ab^2}{c}$$

5th.

$$\frac{a+b}{b-c}\times\frac{a-b}{b+c}=\frac{a^2-b^2}{b^2-c^2}$$

Questions for practice.

Ex.

1. multiply $a+b$ by $3c$
2. $bc-4b$ by $3a$
3. $a+b$ by a^2+d
4. $\frac{b-c}{a}$ by $b-m$
5. $\frac{3a-4c}{a-b}$ by $\frac{a+x}{y-z}$
6. $4ab-cd-m$ by $3ab$
7. $3\sqrt{ab}$ by $5\sqrt{bc}$
8. $2\sqrt{ax}$ by $2\sqrt{bz}$
9. $b\sqrt{ab}$ by $a\sqrt{cd}$

DIVISION.

Rule.

20.—Place the dividend in the form of a numerator, and the divisor in that of a denominator; expunge like quantities from both, and divide the coefficients by the greatest common measure. Like signs give +, unlike give —.

Ex. 1. Divide $3ab$ by b, $\frac{3ab}{b}=3a$

Ex. 2. Divide $-abc$ by $-3bc$, $\frac{-abc}{-3bc}=\frac{-a}{-3}$

Ex. 3. Divide $4ax$ by $12\,ba$, $\frac{4ax}{12ba}=\frac{x}{3b}$

21. Powers of the same root are divided by subtracting their exponents.

Ex. 1. Divide a^3 by a^2 $\frac{a^3}{a^2}=a$

Ex. 2. Divide $a^4b^3c^2$ by a^2b^2 $\frac{a^4b^3c^2}{a^2b^2}=a^2bc^2$

Ex. 3. Divide b^5c^4d by $a^3b^4c^4$ $\frac{b^5c^4d}{a^3b^4c^4}=\frac{bd}{a^3}$

22. If the dividend be a compound quantity, all its parts must be arranged according to the dimensions of some of its letters; the divisor also must be arranged according to the dimensions of the same letters: Then divide the first term of the dividend by the first term of the divisor; if compound, multiply

ply the whole divisor by the quotient; from the dividend subtract the product, and the remainder shall give a new dividend; then proceed as before.

Ex. 1. $a+b)a^2+2ab+b^2(a+b$

$$\begin{array}{l} a^2+ab \\ \hline \quad ab+b^2 \\ \quad ab+b^2 \\ \hline \qquad 0 \end{array}$$

Ex. 2. $x-y)x^3-y^3(x^2+xy+y^2$

$$\begin{array}{l} x^3-x^2y \\ \hline \quad x^2y-y^3 \\ \quad x^2y-xy^2 \\ \hline \qquad xy^2-y \\ \qquad xy^2-y^3 \\ \hline \qquad\quad 0 \end{array}$$

$$\begin{array}{l} x^2+xy+y^2 \\ \qquad x-y \\ \hline x^3+x^2y+xy^2 \\ \quad -x^2y-xy^2-y^3 \\ \hline x^3 \text{———————} y^3 \text{ proof.} \end{array}$$

23. It sometimes happens that the operation may be continued without end, in which case the quotient is called an infinite series.

Ex. 1. $1-a)1\ (1+a+a^2+a^3+a^4$, &c.

$$\begin{array}{l}
\underline{1-a} \\
\quad a \\
\quad \underline{a-a^2} \\
\qquad a^2 \\
\qquad \underline{a^2-a^3} \\
\qquad\quad a^3 \\
\qquad\quad \underline{a^3-a^4} \\
\qquad\qquad a^4 \\
\qquad\qquad \underline{a^4-a^5} \\
\qquad\qquad\quad a^5 \text{ \&c.}
\end{array}$$

Ex. 2. $x^2-y^2)x^2y\ \left(y+\frac{y^3}{x^2}+\frac{y^5}{x^4}+\frac{y^7}{x^6}\right.$ &c.

$$\begin{array}{l}
\underline{x^2y-y^3} \\
\quad y^3 \\
\quad \underline{y^3-\frac{y^5}{x^2}} \\
\qquad \frac{y^5}{x^2} \\
\qquad \underline{\frac{y^5}{x^2}-\frac{y^7}{x^4}} \\
\qquad\quad \frac{y^7}{x^4} \\
\qquad\quad \underline{\frac{y^7}{x^4}-\frac{y^9}{x^6}} \\
\qquad\qquad \frac{y^9}{x^6} \text{ \&c.}
\end{array}$$

24. Here, in the two foregoing examples, the quotients obſerve a certain law, which, if attended to, will enable us, after obtaining a few terms, to extend the quotient to any length without dividing further. Thus the firſt quotient is $1+a+a^2+a^3+a^4$, &c. Now, if we obſerve, that in each term the power of *a* encreaſes by unity, we may continue to add to the former quotient $+a^5+a^6+a^7+a^8+a^9$; and ſo on to infinity.

The quotient, in the ſecond example, may alſo be extended, by obſerving that the powers of the numerators encreaſe in the ſeries of the odd numbers, and thoſe of the denominators in the ſeries of the even numbers.

25. To divide fractions, multiply the numerator of the diviſor by the denominator of the dividend for the denominator of the quotient, and multiply the denominator of the diviſor by the numerator of the dividend for the numerator of the quotient. If one of them be a whole quantity, it may be brought into the form of a fraction by placing 1 for its denominator.

Ex. 1ſt,

$$\frac{a}{b}\Big)\frac{bc}{a}\Big(\frac{b^2c}{a^2}$$

2d,

$$\frac{ab}{ca}\Big)\frac{b^2}{ad}\Big(\frac{cab^2}{a^2bd}=\frac{cb}{ad}$$

3d,

$$\frac{a}{1}\Big)\frac{c}{b}\Big(\frac{c}{ab}$$

4th,

$$\frac{c}{a}\Big)\frac{d}{1}\Big(\frac{ad}{c}$$

5th.

$$\frac{3a}{4b}\Big)\frac{4c}{3b}\Big(\frac{16bc}{9ab}=\frac{16c}{9a}$$

Examples

Examples for practice.

Ex. 1.	Divide $3bc$	by $2ac$
2.	$4bac$	by bc
3.	a^4-b^4	by $a-b$
4.	$a^3-3a^2b+3ab^2-b^3$	by $a-b$
5.	$a^3+3a^3b+3ab^2+b^3$	by $a+b$
6.	a^2+b^2	by $a+b$
7.	a^2-b^2	by $a+b$
8.	$\dfrac{a}{2b}+\dfrac{3b^2}{2a}$	by ab
9.	a^3b^3c	by a^2b^3

INVOLUTION.

26. Involution is performed by the successive multiplication of any quantity into itself. A quantity multiplied into itself produces the square of the same; and the square multiplied again by the original quantity produces the cube of the same; and that cube again multiplied by the root gives the biquadratic power. And so on.

27. Simple quantities are involved by multiplying their exponents by that of the power, and prefixing a like power of the coefficient. Thus, the square of b is b^2; the cube of $8a$ is $512a^3$.

28. Positive roots give positive powers; but negative roots give positive and negative powers by turns.

29. Any two quantities connected by the ſign +, are called a Binomial; but if connected by the ſign —, a Reſidual.

Example I.

Required the ſquare, cube, biquadratic, ſurſolid, and ſixth power of $a+b$.

$a+b$=root.
$a+b$

a^2+ab
$\quad +ab+b^2$

$a^2+2ab+b^2$=ſquare.
$\quad a+b$

$a^3+2a^2b+ab^2$
$\quad +a^2b+2ab^2+b^3$

$a^3+3a^2b+3ab^2+b^3$=cube.
$\quad a+b$

$a^4+3a^3b+3a^2b^2+ab^3$
$\quad +a^3b+3a^2b^2+3ab^3+b^4$

$a^4+4a^3b+6a^2b^2+4ab^3+b^4$=biquadratic.
$\quad a+b$

$a^5+4a^4b+6a\ b^2+4a^2b^3+ab^4$
$\quad +a^4b+4a^3b^2+6a^2b^3+4ab^4+b^5$

$a^5+5a^4b+10a^3b^2+10a^2b^3+5ab^4+b^5$=ſurſolid.
$\quad a+b$

$a^6+5a^5b+10a^4b^2+10a^3b^3+5a^2b^4+ab^5$
$\quad a^5b+5a^4b^2+10a^3b^3+10a^2b^4+5ab^5+b^6$

$a^6+6a^5b+15a^4b^2+20a^3b^3+15a^2b^4+6ab^5+b^6$=6th power.

Example

Example II.

equired the ſquare, cube, and biquadratic powers of $a-b$.

$$
\begin{array}{l}
a-b \\
a-b \\
\hline
a^2-ab \\
\quad -ab+b^2 \\
\hline
a^2-2ab+b^2 \\
\qquad\quad a-b \\
\hline
a^3-2a^2b+ab^2 \\
\quad -a^2b+2ab^2-b^3 \\
\hline
a^3-3a^2b+3ab^2-b^3 \\
\qquad\qquad a-b \\
\hline
a^4-3a^3b+3a^2b^2-ab^3 \\
\quad -a^3b+3a^2b^2-3ab^3+b^4 \\
\hline
a^4-4a^3b+6a^2b^2-4ab^3+b^4
\end{array}
$$

30. It appears by reviewing theſe examples, that all the terms of the powers of a binomial are poſitive; but the terms of the powers of a reſidual are poſitive and negative alternately, the firſt poſitive, the ſecond negative; the third poſitive, the fourth negative; and ſo on, + and — by turns. Alſo, that the ſum of the exponents of a and b, in any of the intermediate terms, is equal to the exponent of the firſt or laſt term; and that the exponent of the firſt or of the laſt term is equal to that of the power. In the firſt term b is wanting, and the power of a in every ſucceeding term decreaſes regularly by 1; and that of b encreaſes in each term by 1, until a diſappear.

The coefficient of the firſt term is 1: The coefficient of the ſecond term is equal to the exponent of the firſt. One or more terms being found, the cofficient of the next ſucceeding term may be diſcovered in this manner: Multiply the exponent of *a* in the laſt term by the coefficient of the ſame; divide the product by the number of terms already made up, and the quot will be the coefficient required.

31. From theſe obſervations we may infer the following rule, commonly called the $a+b$, or the Binomial Theorem, by which we may involve either a binomial or a reſidual root to a power of any dimenſion.

Rule

1ſt, *To find the firſt term of the power.*

Multiply the exponent of *a* in the root by that of the power, for the firſt term of the power required.

2d, *To find the ſecond term of the power.*

Multiply the exponent of *a* in the firſt term by the coefficient of the ſame, divide the product by the number of terms already found: the quotient will be the coefficient of the ſecond term of the power; then diminiſh the exponent of *a*, and encreaſe the exponent of *b*, each by 1, for the ſecond term.

3d, *To find the third term of the power.*

Multiply the exponent of *a* in the ſecond term by the coefficient of the ſame; divide the product by 2, (the number of terms already found); the quotient gives the coefficient of the third term; then decreaſe the exponent of *a* in the ſecond term, and encreaſe that of *b* in the ſame, each by unity, for the third term.

4th, *To find the fourth term.*

Multiply the exponent of a in the third term by the coefficient of the ſame; divide the product by 3, (the number of terms already found); the quotient is the coefficient of the next term; then take off a power of a and bring on a power of b, for the fourth term of the power complete: Continue this proceſs till all the powers of a are exhauſted, and the power of b be equal to that of the power required.

EVOLUTION.

32. Evolution is the operation by which roots are diſcovered, and is always oppoſed to Involution.

Roots are quantities by whoſe ſucceſſive multiplication given powers are produced.

33. The roots of ſimple quantities are extracted, by dividing the exponent of the power by the exponent of the root required. Thus, the cube root of a^3 is a, of $8b^6$ is $2b^2$.

The reaſon of this is deduced from § 27.

34. Rules for extracting roots of compound quantities are deduced from a review of the ſteps by which they are involved. Thus, the ſquare of $a+b$ is $a^2+2ab+b^2$; that is, the ſquare of any two quantities is equal to the ſquares of each of the quantities, together with twice their product. See *Euclid*, Book 2d, propoſition 4th. Therefore when a quantity is propoſed, whoſe ſquare root is a compound quantity, you are firſt to arrange the terms as taught in diviſion, (§ 22.)

Thus, Let the ſquare root of $a^2+2ab+b^2$ be required.

$$\begin{array}{rl} & a^2+2ab+b^2\,(a+b \\ & a^2 \\ \hline 2a+b) & 2ab+b^2 \\ & 2ab+b^2 \\ \hline \end{array}$$

We

We will now proceed to lay down ſome rules for extracting the roots of numbers.

The following TABLE exhibits the firſt nine powers of the 9 digits.

1ſt power, or Root.	1	2	3	4	5	6	7	8	9
Square.	1	4	9	16	25	36	49	64	81
Cube.	1	8	27	64	125	216	343	512	729
Biquadr.	1	16	81	256	625	1296	2401	4096	6561
Surſolid.	1	32	243	1024	3125	7776	16807	32768	59049
Cube ſq.	1	64	729	4096	15625	46656	117649	262144	53144
7th power	1	128	2187	16384	78125	279936	823543	2097152	4782969
8th power	1	256	6561	65536	390625	1679616	5764801	16777216	43046721
9th power	1	512	19683	262144	1953125	10077696	40353607	134217728	387420489

Rules *for extracting the square root.*

I. Divide the given number into periods of two figures, reckoning from the unit's place.

II. Find the greatest root contained in the left hand period, and place it as the first figure of the root: Subtract its square from the said period, and to the remainder bring down the next period for a resolvend.

III. Double the first part of the root for the first part of the divisor, and enquire how often this part is contained in the resolvend, neglecting the right hand place; the quot gives the next figure of the root.

IV. Annex the quotient also to the divisor, and multiply this number by the quotient; subtract the product from the resolvend, and to the remainder, if any, bring down the next period for a new resolvend.

V. Use the last divisor for the first part of a new one, doubling the right hand figure; then proceed as before.

Note. Every period gives a figure in the root.

Ex. I. Required the square root of 2025.

```
    2025(45 root.                 45 root.
    16                            45
    ———                          ———
85) 425 resolvend.               225
    425                         180
    ———                         ————
      0                         2025 proof.
```

Here enquire for the greatest root contained in the first period 20, which is 4, then place it as the first figure of the root, and subtract its square (16) from 20; to the remainder 4 annex the next period 25 for a resolvend; then divide 42 (neglect-ing

ing the 5) by twice 4 or 8, and place the quotient 5 in the root; alſo annex it to 8, and multiply this 85 by 5, the laſt figure of the root. And ſubtract this product from the reſolvend; and ſince there is no remainder, 2025 is an exact ſquare, of which 45 is the root.

Ex. II. Required the ſquare root of 58264.

$$
\begin{array}{rl}
 & 58264(241.37 \\
 & \underline{4} \\
44) & 182 \\
 & \underline{176} \\
481) & 664 \\
 & \underline{481} \\
4823) & 18300 \\
 & \underline{14469} \\
48267) & 383100 \\
 & \underline{337869} \\
 & 45231 \text{ rem.}
\end{array}
$$

37. If, after the given number is exhauſted, there be a remainder, annex periods of cyphers thereto, and continue the operation till the decimal part of the root terminate, repeat or circulate, or till you think proper to limit it.

38. RULES *for extracting the ſquare root of vulgar fractions or mixt numbers.*

Extract the ſquare root of the numerator and of the denominator for their reſpective terms of the root required. Thus the ſquare root of $\frac{25}{36}$ is $\frac{5}{6}$. Or,

39. If the numerator and denominator of the fraction proposed be not complete powers, place the square root of their product over the given denominator, and reduce this new fraction to its lowest terms for the fractional root required. Thus,

$$\frac{25}{36}\times\frac{36}{1}=\frac{900}{36} \text{ and } \frac{\sqrt{900}}{36}=\frac{30}{36}=\frac{5}{6} \text{ as before. Or,}$$

40. Reduce the fraction to a decimal, and extract its square root. Thus,

$$\frac{25}{36}=.69416 \text{ and } \sqrt{.69416}=.83=\frac{5}{6}.$$

41. In mixt numbers it will be best to reduce the fractional part to a decimal, to which prefix the integral part, and extract the square root of the whole. Thus,

$$\sqrt{12\tfrac{1}{4}}=3\tfrac{1}{2}, \text{ or rather, } \sqrt{12.25}=3.5$$

Extraction of the Cube Root.

Required the cube root of $a^3+3a^2b+3ab^2+b^3$.

$$
\begin{array}{rl}
 & a^3+3a^2b+3ab^2+b^3(a+b \\
 & a^3 \\
\hline
3a+3ab+b^2) & 3a^2b+3ab^2+b^3 \\
 & 3a^2b+3ab^2+b^3 \\
\hline
\end{array}
$$

The

42. The following rules are inferred from the foregoing operation.

RULES.

I. Divide the given number into periods of three places, beginning at the unit place; each period gives a figure of the root.

II. Find the greatest cube root of the first period, and place it as the first figure of the root; subtract its cube from the first period, and to the remainder annex the next period for a resolvend.

III. To three times the square of the first part of the root annex two cyphers for the first part of the divisor.

IV. Enquire how often this number is contained in the resolvend, and place the result as the next figure of the root.

V. To the product of the first and second parts of the root annex a cypher for the second part of the divisor, and square the last part of the root for the remaining part of the divisor: the sum of these three parts will be the divisor complete.

VI. Multiply this sum by the last part of the root, and subtract the product from the resolvend: if there be more periods, annex the next to the remainder, and proceed as before.

If, after all the periods are used, there happen to be a remainder, annex periods of cyphers as before directed, § 27.

EXAMPLE. Required the cube root of 15625.

$$
\begin{array}{rr|r}
 & & 15625(25 \\
 & & 8 \\
\hline
3a^2+0+0= & 1200 & 7625 \\
3ab+0= & 300 & \\
b^2= & 25 & \\
\hline
3a^2+3ab+b^2 & 1525 & \\
b & 5 & 7625 \\
\hline
3a^2b+3ab^2+b^3 & & 0
\end{array}
$$

43. The number of cyphers annexed to the first part of the divisor is less by unity than the exponent of the power proposed; for $2=a$, the first part of the root, (in regard another figure is to follow) occupies the place of tens, it is therefore equal 20. Hence it is plain, that $3a^2$ or $20^2\times3=1200$, the first part of the divisor; and since $5=b$, that $3ab$, or $3\times20\times5=300$, the second part of the divisor; also that $b^2=5^2=25$, the last part; likewise that the first part of the root is considerably greater than any other figure.

44. The reason of the rule will appear, if we take the case where the root consists of 2 places $a+b$. Let the given number be represented by $a^3+3a^2b+3ab^2+b^3$; and if we place a, the cube root of a^3, in the root, and subtract a^3 from the given number, the remainder or resolvend will be $3a^2b+3ab^2+b^3$; and since it has been shewn, that a in the root is considerably greater than b, it will follow that $3a^2b$, the first part of the resolvend, will be the greatest part of it. If, therefore, $3a^2b$ be divided by $3a^2$, it will quot b, the other part of the root sought, by the help of which the divisor may be completed; but since all the parts of the divisor are multiplied by the last figure of the root, the divisor will be $3a^2+3ab+b^2$, which is obtained by dividing each term by b.

N. B. If the root consists of more than two places, a represents all the places found; and, by repeating the operation for a new divisor, the other part b may be found as before; and so on.

EXAMPLE

EXAMPLE 2. What is the cube root of 99252847

The divisor. $\quad$ 99252847(463 $\quad$ $40=a$ & $6=b$

$3a^2+3ab+b^2$ $\quad$ 64

$3a^2+0+0=4800$ $\quad$ | 35252

$3ab+0=\ 720$

$b^2\ \ 36$

$b\times 3a^2=3ab+b^2=5556\times 6=\ \ 33336$

$3a^2+0+0=634800$ $\quad$ | 1916847 $\quad$ now $460=a$ & $3=b$

$3ab+0=\ \ 4140$

$b^2=\ \ \ \ 9$

$b\times 3a^2+3ab+b^2=638949\times 3\ =1916847$

0

EPAMPLE 3. Required the cube root of .373248.

.373248(.72 $\quad$ here $a=70$ & $b=2$

343

14700 $\quad$ | 30248

420

4

$15124\times 2=$ $\quad$ 30248

0

In this example the given number is a decimal, and decimals are pointed by beginning at the decimal point, and passing over as many places towards the right hand as there are units in the exponent of the root required.

45. *General Rule for extracting any root.*

Having divided the given number into proper periods, involve a like power of $a+b$ with the number proposed. Put n

equal to the exponent of the root. Place the value of a in the root, and ſubtract the value of a^n from the firſt period; then expunge b out of every term of the reſolvend. This will give a general diviſor that will anſwer a power of any dimenſion..

Note. a^n-1 is always the trial diviſor.

Required the biquadratic root of 1679616.

The diviſor is $4a^3+6a^2b+4ab^2+b^3$

$$
\begin{array}{rl|r}
 & & 1679616(36 \\
 & & 81 \\
\hline
4a^3= & 108000 & 869616 \\
6a^2b= & 32400 & \\
4ab^2= & 4320 & \\
b^3= & 216 & \\
\hline
 & 144936 \times 6 = & 869616 \\
\hline
 & & 0
\end{array}
$$

Required the furſolid root of 60466176.

The diviſor $5a^4+10a^3b+10a^2b^2+5ab^3+b^4$

$$
\begin{array}{rl|r}
 & & 60466176(36 \\
 & & 243 \\
\hline
5a^4= & 4050000 & 36166176 \\
10a^3b= & 1620000 & \\
10a^2b^2= & 324000 & \\
5ab^3= & 32400 & \\
b^4= & 1296 & \\
\hline
 & 6027696 \times 6 = & 36166176
\end{array}
$$

Required

Required the root of 887503681, being the ſixth power.

The diviſor $6a^5+15a^4b+20a^3b^2+15a^2b^3+6ab^4+b^5$

```
                              887503681(31
                               729
                              ---------
  6a^5   = 145800000        | 158503681
  15a^4b =  12150000        |
  20a^3b^2 =  540000        |
  15a^2b^3 =   13500        |
  6ab^4  =       180        |
  b^5    =         1        |
  ------------------
        158503681 × 1 = 158503681
                        ---------
                            0
```

If the foregoing examples be well underſtood, the learner will be able to inveſtigate theorems for extracting higher roots.

We will now ſubjoin a few mixt examples for practice.

Ex. 1. Required the ſquare root of 144 *Anſ.* 12

2. of 4635 *Anſ.* 68.

3. of 5776 *Anſ.* 76.

4. of 2985984 *Anſ.* 172.8

5. Required the cube root of 13824 *Anſ.* 24.

6. of 13312053 *Anſ.* 237.

7. of 1906624 *Anſ.* 124.

8. of 34582249.512 *Anſ.* 3258

9. biquadratic root of 6612111737853987761 *Anſ.* 50709

10 ſurſolid root of 33554432 *Anſ.* 32.

11. the eighth root of 28179280429056 *Anſ.* 48

12. the ſquare root of 2 nearly *Anſ.* 1.4142+

13. the cube root of 2 nearly *Anſ.* 1.148699+

14. An army of 7744 men was drawn up in a ſquare. Required the number of men in each rank. *Anſ.* 88 men.

15. A maltſter had a round malt-kiln of 16 feet diameter, but is to build a ſquare one that will contain 3 times as much. The ſide of the new kiln is required.

Anſ. 24.5557 feet.

16. The ſolidity of a ſphere is 47016 cubic inches. Required the ſide of a cube whoſe content is equal to it.

Anſ. 36 inches.

46. PROPORTION.

When two quantities of the ſame kind are compared, their relation or ratio is obtained by enquiring how often the firſt contains the ſecond. Thus, the ratio of 12 to 4 is 3; of 4 to 3 is $1\frac{1}{3}$; and of 3 to 8 is $\frac{3}{8}$, or .375.

47. When four quantities, a, b, c, d, are proportional, it is uſually expreſſed by ſaying, the firſt is to the ſecond as the third is to the fourth; or, $a : b :: c : d$, and the quantities are ſaid to be in geometrical proportion.

48. The quantity whoſe ratio is enquired into is called the antecedent, and the quantity, with which it is compared, the conſequent.

49. The firſt and third terms, a and c, are called antecedents.

The ſecond and fourth, b and d, are called the conſequents.

The firſt and fourth terms, a and d, are called the extremes.

The ſecond and third terms, b and c, are called the means.

50. If $a : b :: c : d$, the product of the means, is equal to the product of the extremes, *Euclid* vi. 16. thus $ad=bc$.

51. If the product of two quantities, ad, be equal to the product of two others, bc, the quantities are proportional, and $a : b :: c : d$; that is, a factor of the firſt is to a factor of the ſecond

ſecond as the remaining factor of the ſecond is to the remaining factor of the firſt. *Euclid*, vi. 16.

52. If $a : b :: c : d$, then $\frac{bc}{a} = d$ and $\frac{ad}{c} = b$; that is, the product of the means divided by either extreme quotes the other extreme, and the product of the extremes, divided by either of the means, quots the other mean.

53. If $a : b :: c : d$, they will remain proportionals under the following varieties. Euclid, v. *Definitions*.

Thus,	$a : b :: c : d$	Componendo,	$a+b : b :: c+d : d$
Alternando,	$a : c :: b : d$	Dividendo,	$a-b : b :: c-d : d$
Invertendo,	$b : a :: c : d$	Convertendo,	$a : a+b :: c : c+d$

In all theſe varieties, the product of the means is equal to that of the extremes.

54. EQUATIONS.

An equation is a propoſition aſſerting the equality of two quantities; it is uſually expreſſed by the ſign $=$. Thus, $2 \times 6 = 12$, or $bd = e$.

55. AXIOMS.

1. Quantities that are equal to one and the ſame quantity are equal to each other.

2. If equal quantities be added to equal quantities the ſums are equal.

3. If equal quantities be taken from equal quantities, the remainders are equal.

4. Quantities

4. Quantities which are double of the ſame quantity are equal; and the contrary.

5. If equal quantities be multiplied by the ſame quantity, the products are equal.

6. If equal quantities be divided by the ſame quantity, the quotes are equal.

7. Like powers of equal roots are equal.

8. Like roots of equal quantities are equal.

56. The value of an unknown quantity is found by changing the form of the equation till it ſtand alone on one ſide, and the known quantities on the other. But it frequently happens that the unknown quantity is variouſly combined with others, and ſo its value not eaſily diſcovered. We ſhall therefore lay down a few general rules for the ſolution of equations, and which depend on the foregoing axioms.

Rules.

1. A quantity is ſaid to be tranſpoſed, if it be taken from one ſide of the equation to the other with the oppoſite ſign. Thus, $2+4=6$, and $4=6-2$. Ax. III.

2. If the unknown quantity be multiplied by any other quantity, divide both ſides of the equation by that other quantity. Thus, if $ax=b$, then $x=\frac{b}{a}$ Ax. VI.

3. If the unknown quantity be divided by any other quantity, multiply both ſides of the equation by the diviſor. Thus, if $\frac{x}{a}=b$, then $x=ab$. Ax. V.

4. If that member of the equation which involves the unknown

known quantity be a surd root, make that member stand alone on one side of the equation, remove the radical sign, and raise the other side to the corresponding power. Thus, $\sqrt{4x+b}=a$; then, $\sqrt{4x}=a-b$, by the rule $4x=a^2-2ab+b^2$. Ax. VII. and VIII.

5. If the same quantity be found on both sides of the equation, with the same sign prefixed, expunge it from both.

6. If $a : b :: c : d$, an equation is obtained by asserting the product of the means equal to that of the extremes.

Examples. I.

A person being asked his age, answers, If to $\frac{1}{3}$ my age you add triple my age, the sum will be 100. Required his age.

Suppose his age x years.

Then $\frac{1}{3}$ his age will be $\frac{x}{3}$

And three times his age $3x$

The sum of these is $\frac{x}{3} + 3x=100$

By Rule 3d, $x+9x=300$

By addition, $10x=300$

By Rule 2d, $x=30$ years old.

Example II.

From London to Edinburgh, by the Carlisle road, is 399 miles. A messenger is dispatched from Edinburgh, who travels at the rate of 36 miles *per* day: and, after six days, another is dispatched from London to meet the former, who travels

vels 25 miles *per* day. Required their distance from London when they meet, and how many days will the latter take.

Suppose they meet in x days, then,

The first travels $36x+216$
The other $25x$
By quest. $36x+216+25x=399$
$61x=183$
$x=3$ days
And $3\times25=75$ miles from London.

EXAMPLE III.

Suppose a messenger, who travels $33\frac{3}{4}$ miles *per* day, is dispatched, and after 8 days another is sent on horseback to overtake the former, who rides 80 miles a day, how many days does each travel before the first is overtaken?

Suppose in x days.

$$33.75x+270=80x$$
$$270=46.25x$$
$$x=5.8377=5\text{ d. }20\text{ h. }6\text{ m.}$$

EXAMPLE IV.

Suppose the sun to proceed one degree *per* day in the ecliptic, and the moon 13°, and that the sun is in the beginning of Capricorn, and after three days the moon enters Aries. Required the place of their next conjunction.

When the moon enters Aries the sun is advanced 273° from Aries.

$$x+273=13x$$
$$273=12x$$
$$22\tfrac{3}{4}=x\text{ days.}$$

$$\begin{array}{r} 22\frac{3}{4}\text{ days} \\ 3 \\ \hline 25\frac{3}{4} \end{array}$$

That is, the next conjunction will be in 25° 45′ of Capricorn.

EXAMPLE

Example V.

A merchant, ſending an adventure to ſea, doubled his ſtock; by his ſecond voyage he loſt 1200l.; by his third he doubled his remaining ſtock, and by his fourth loſt 1200l.; after which he had nothing left. Required his original ſtock.

Suppoſe his ſtock x pounds.

By his 1ſt voyage, $2x$
—— 2d voyage, $2x-1200$
—— 3d voyage. $4x-2400$
—— 4th voyage, $4x-1200-2400=0$ *per* queſt.

$$4x=3600$$
$$x=900\text{l. his original ſtock.}$$

When there are two unknown quantities, the conditions muſt be ſuch as to afford two equations; from each of theſe equations exterminate one of the unknown quantities; then form a new equation, by placing its values equal to one another. This new equation contains only one unknown quantity, and is reſolved as before.

Example I.

Two men diſcourſing of their money, ſays A to B, give me 4 ſhillings and I ſhall have three times as much as you have. B ſaid, Give me 6 ſhillings of yours, and each of us will have equal ſhares. Required how much each had.

Suppoſe A had x and B y ſhillings.

Anſ. $x=20$, and $y=8$.

Example II.

Two travellers, A and B, met at an inn. A aſked B how far he had travelled. B anſwered, that he had travelled ſo many miles and furlongs. Well, ſays A, I travelled only half that diſtance; and the number of miles I travelled correſponds with your furlongs, and my furlongs with your miles. How far did each travel?

Let x repreſent the miles, y the furlongs.

Then A travels $8x+y$
and B $8y+x$

By queſt. $\dfrac{8x+y}{2}=8y+x$

$8x+y=16y+2x$

$6x=15y$

Therefore $x:y:(15:6)\ 5:2$

B travels 5 miles 2 furlongs.
A 2 miles 5 furlongs.

Example III.

Two merchants, A and B, began trade with equal ſtocks; but A, by frugality and application, gained 60l. while B, through miſmanagement and bad luck, loſt 80l. At the year's end A was 8 times richer than B. Required their original ſtock. *Anſ.* 100l.

Quadratic Equations.

When the ſquare and the root of the unknown quantity are joined together, it is called an adfected quadratic equation.

Rule.

Rule.

Transpose the quantities till the unknown quantity stand on one side of the equation. Divide both sides by the coefficient of the square of the unknown quantity. Add the square of one half the coefficient of the simple power to both sides of the equation. Extract the square root, and transpose the half coefficient, which gives the value of the unknown quantity.

Example I.

Required two numbers whose sum is 16, and product 48.

$$x+y=16$$
$$xy=48$$
$$x=16-y$$
$$x=\frac{48}{y}$$
$$16-y=\frac{48}{y}$$
$$y^2-16y=-48$$

Per Rule, $y-16y+64=16$

$$y^2-16y+64=16$$
$$y-8=4$$
$$y=8+4$$

Required two numbers whose product is 108, and sum of their squares 360.

Suppose x the less and y the greater.

$$xy=108$$
$$x^2+y^2=360$$
$$y^2=\frac{108^2}{x^2}$$
$$y^2=360-x^2$$
$$360-x^2=\frac{11664}{x^2}$$
$$x^4-360x^2=-11664 \quad \text{Substitute } z \text{ for } x^2$$
Then, $z^2-360z=-11664$
$$z^2-360z+32400=20736$$
$$z-180=144$$
$$z=x^2=36$$
$$x=6$$
$$y=18$$

Questions for Practice.

1. A man and his wife did usually drink out a barrel of beer in 12 days; and they found, by often experience, that the wife being absent, the man drank it out in 20 days. In how many days would the wife alone drink it out at her rate of drinking?
Ans. 30 days.

2. Two ships, A and B, loaded with the same sort of wine, sailing by a pass, they were obliged to pay toll according to the quantity each had on board. A had 250 hogsheads, out of which she paid 1 hogshead, and 36 shillings more. B had 400 hogsheads, out of which she paid 2 hogsheads, and received back 20 shillings. Required at what rate the wine was valued *per* hogshead. *Ans.* 4l. 14s.

Suppose the minute and hour hands of a common clock to be

be in conjunction, in how many hours will they be in conjunction again? *Ans.* In $1\frac{1}{11}$ hour.

4. Required two numbers, such that the quot of the greater divided by the lesser may be 2 less than their difference, and their product may exceed their sum by 20. *Ans.* 8 and 4.

5. A boy is offered 10 apples for a penny, and 25 pears for 2d: He agreed to buy 100 apples and pears together for $9\frac{1}{2}$d. Required the number of each. *Ans.* 75 apples and 25 pears.

6. Required two numbers whose sum is 108, and proportion as 5 to 4. *Ans.* 60 and 48.

LITERAL EQUATIONS.

In literal equations unknown quantities are represented by x, y, z, as before; known quantities by a, b, c, d, &c. The rules for transposing and exterminating quantities are the same as above. When the value of the unknown quantity is thus discovered, we obtain a general theorem, which will serve for the solution of all questions under the like conditions.

EXAMPLE I.

Required a theorem for determining two numbers whose sum, s, and difference, d, are given.

Let x be the greater and y the less.

$x+y=s$

$x-y=d$ And by exterminating y, the value of x will be $\frac{s-d}{2}$

$x=s-y$

$x=d+y$

$s-y=d+y$

$$\frac{s+d}{2}=y$$

In

In words.——From half the ſum ſubtract half the difference, the remainder will be the leſs.

To half the ſum add half the difference, the ſum will be the greater.

Ex. 2. The powers or forces of three different agents being given, to find a general theorem for determining the time in which they would, all three together, produce a given effect.

Three day-labourers, A, B, and C, have undertaken a piece of work, which A could perform in a days, B in b days, and C in c days. In what time will they perform it, if all the three work together?

Suppoſe in x days.

Then A's ſhare. B's. C's.

$$\frac{x}{a} + \frac{x}{b} + \frac{x}{c} = 1$$

$$x + \frac{ax}{b} + \frac{ax}{c} = a$$

$$bx + ax + \frac{bax}{c} = ab$$

$$cbx + cax + bax = abc$$

$$x = \frac{abc}{cb+ca+ba}$$

The rule obtained may be tranſlated thus. Divide the product of the three given times by the ſum of the products of each two taken ſeparately.

3. Required a theorem for determining two numbers, whoſe ſum (s) and ſum of their ſquares (q) are given.

Anſ.

$$Ans.\ \tfrac{1}{2}s \pm \sqrt{\tfrac{1}{2}q\ \tfrac{1}{4}s^2}$$

Tranſ. From half the ſum of their ſquares ſubtract $\frac{1}{4}$ the ſquare of their ſum, and to the ſquare root or the remainder add or ſubtract the half ſum for the numbers required.

In like manner any other theorem may be tranſlated.

4. Required a theorem for determining two numbers x and y, whoſe product, p, and proportion a to b, are given.

$$Anſ.\ x \text{ the greater} = \sqrt{\frac{ap}{b}}$$

$$y \text{ the leſs} = \sqrt{\frac{pb}{a}}$$

5. Required a theorem for determining two numbers, x and y, whoſe ſum, e, and proportion a to b, are given.

$$Anſ.\quad x \text{ the greater} = \frac{as}{a+b}$$

$$y \text{ the leſs} = \frac{bs}{a+b}$$

6. Required a theorem for determining two numbers, x and y, whoſe ſum, s, and product, p, are given.

$$Anſ.\ \tfrac{1}{2}s \pm \sqrt{\tfrac{1}{4}s^2 - p}$$

7. Required a theorem for determining two numbers, x and y, whoſe proportion, a to b, and the ſum of their ſquares, cubes, &c. are given.

Let

Let n denote the exponent of the power.

Ans. x the greater $= \sqrt{\frac{sa^n}{b^n+a^n}}$

y the less $\sqrt{\frac{sb^n}{b^n+a^n}}$

8. Required a theorem for determining the time and place in which two bodies, moving towards each other, will meet, their velocity, their distance, and the difference of the time of their first motion, being given.

Put $a =$ the velocity of the one,
$b =$ the velocity of the other,
$d =$ their distance, and
$t =$ the time the one moves before the other.

Ans. $\frac{d-t}{a+b} =$ the time of their meeting.

And the product of the velocity of either body, multiplied by the time of its motion, will give the space passed over, and consequently the place of meeting.

GEOMETRICAL PROBLEMS.

When a geometrical problem or question is proposed you are first to construct a figure representing the true one; prepare the figure (if necessary) by drawing more lines as you see cause, according to the method of solution you have chosen, that so, by the help of these lines and mediums, you may deduce a connection between known and unknown quantities or lines. Then proceed to the operation as before directed, which, with a competent knowledge of Euclid's Elements, will be your guide: but the exact manner of proceeding can scarlely be reduced

duced to general rules, but must be collected by a careful examination of each step, assisted by practice.

PROBLEM I.

Given the hypothenuse of a right-angled triangle, and the sum of the sides, to find each of the sides.

Euclid, 47. I. $a^2=x^2+s^2-2sx+x^2$	$AB=a$
$a^2=2x^2+s^2-2sx$	$AC+CB=s$
$a^2-s^2=2x^2-2sx$	$AC=x$
$\frac{a^2-s^2}{2}=x^2-sx$	$BC=s-x$

$$\frac{a^2-s^2}{2}\ \frac{s^2}{4}=x^2-sx+\frac{s^2}{4}$$

$\frac{1}{2}s+\sqrt{\frac{1}{2}a^2-\frac{1}{2}s^2\frac{1}{4}s^2}$ or rather,

Ans. $\frac{1}{2}s+\sqrt{\frac{1}{2}a^2\ \ \frac{1}{4}s^2}$ either of the legs.

PROBLEM II. *Fig.* 1.

Given the hypothenuse and the difference of the legs, to find the legs.

$AB=a$, $AC=x$, $BC=x+d$

Ans. $x=\frac{1}{2}d+\sqrt{\frac{1}{2}a^2-\frac{1}{4}d^2}$

PROBLEM III. *Fig.* 1.

Given the hypothenuse and the product of the two legs, to find each of the legs.

Let AC$=x$, and BC $\frac{p}{x}$, AB$=a$

$$Ans.\ x = \sqrt{\tfrac{1}{2}a^2 + \sqrt{\tfrac{1}{4}a^4}}$$

PROBLEM IV. *Fig.* 1.

Given the hypothenuse, and the proportion of the two legs, to find each of them.

Per quest. $x : y :: c : d$, and AB$=a$

$$Ans.\ y = \frac{\sqrt{d^2\ a^2}}{c+d}$$

PROBLEM V. *Fig.* 1.

Given one of the legs, and the sum of the hypothenuse and the other leg, to find the hypothenuse and that leg.

AB+BC$=s$, AC$=d$, CB $s-x$

$$Ans.\ x = \frac{d^2}{2s} = \frac{s}{2}$$

PROBLEM

PROBLEM VI. *Fig.* 2.

To divide any given line, AB, into two ſuch parts that the rectangle, contained by the whole line and one of the parts, ſhall be equal to the ſquare of the other part.

$$AB=a$$
$$CB=a-x$$

Anſ. $\frac{a}{2} + \sqrt{\frac{5a^2}{4}}$

PROBLEM VII.

Given the difference between the diagonal of a ſquare and one of its ſides, to find the diagonal and the ſide.

Let d be the difference and x the ſide.

Anſ. $x=d+\sqrt{2d^2}$

PROBLEM VIII. *Fig.* 1.

Given the perimeter, and the area of a right-angled triangle, to find the hypothenuſe.

Let $AB+BC+CA=2a$
Area $=bc$
$AB=x$
$AC+CB=2a-x$

Anſ. $x=\frac{a^2-bc}{a}$

PROBLEM IX. *Fig.* 3.

In the right-angled triangle, ABc, given the base AB, and the sum of the perpendicular CD, and the sides BC and CA, to find the triangle.

$$CA+CB+CD=a$$
$$AB=b$$
$$CD=x$$
$$AC+CB=a-x$$

Ans. $x=b+a-\sqrt{2ab+2b^2}$

PROBLEM X. *Fig.* 3.

Given the perimeter of a right-angled triangle, and the perpendicular upon the hypothenuse, to find the several sides.

$$AB+BC+AC=s$$
$$BD=p$$

Ans. The hyp. $AC=\frac{s^2}{2s+2p}=z$

$$\sqrt{\tfrac{1}{2}z^2+\sqrt{\tfrac{1}{4}z^4-z^2p^2}}$$

PROBLEM XI. *Fig.* 3.

Given the sum of the sides of a right-angled triangle, and the perpendicular upon the hypothenuse, to find the hypothenuse and the other legs.

AB+BC=s
BD=c

$$Anf.\quad z = \sqrt{s^2+c^2} - c$$

The other fides may be found as in Prob. 10. or by Eucl. 8. vi. Coroll.

PROBLEM XII. *Fig.* 4.

In a given triangle to infcribe a fquare.

Let BC=a
AD=b
KD=x
KA=$b-x$

Anf. $\frac{bp}{p+b}=x$, the fide of the fquare.

PROBLEM XIII. *Fig.* 5.

In a given triangle, to infcribe a rectangle of a given magnitude.

BC=a
AD=c
HO=$\frac{a}{x}$
KD=x
KA=$a-x$

$$Anf.\quad \tfrac{1}{2}c + \sqrt{\tfrac{1}{2}c^2\ \tfrac{1}{4}ca}$$

PROBLEM

PROBLEM XIV.

The ſides of a rectangle being given, to find the breadth of a piece to be taken off half round the rectangle, and of an equal width, that ſhall take up juſt half the rectangle.

Let l=length,
b=breadth,
a=the area,
s=the ſum of the ſides,
p=their product,
x=the breadth required.

Anſ. $x = \frac{1}{2}s \quad \sqrt{\frac{1}{4}s^2 \quad \frac{1}{2}p}$ the breadth.

PROBLEM XV. *Fig.* 5.

Given all the ſides of a parallelogram, and one diagonal, to find the other diagonal.

Put DC=a,
BA=b
AC=d
DB=x

Anſ. $x = \sqrt{2a^2 + 2b^2 - d^2}$

PROBLEM XVI. *Fig.* 6.

Given the chord of an arch, and the diameter of the circle, of which the arch is a part, to find the verſed ſince, or the height of the arch.

Put

Put AB $=2a$
CF b
CD x
DF $b-x$

See Euclid, 35. III.

$$Ans.\quad x=\sqrt{\frac{\frac{1}{2}b^2}{2}-a^2}+\frac{1}{2}b$$

N. B. The half of the diameter is to be added or ſubtracted according as the arch is greater or leſs than a ſemicircle.

PROBLEM XVII. *Fig.* 7.

Given the ſolidity, depth, and the proportion of the head and baſe diameters, of the fruſtum of a cone, to find the diameters.

Let x AB,
y CD,
d depth,
s ſolidity;
And let $a : b :: x : y$;

Alſo $\frac{s}{pd}$ ſquare of the mean area.

Subſtitute $m=\frac{s}{pd}$

Then the anſwer will be $\sqrt{\frac{mb^2}{a^2+b^2+ab}}=y$ the leſs,

$\sqrt{\frac{ma^2}{a^2+b^2+ab}}=x$ the greater.

FINIS.

PLATE I

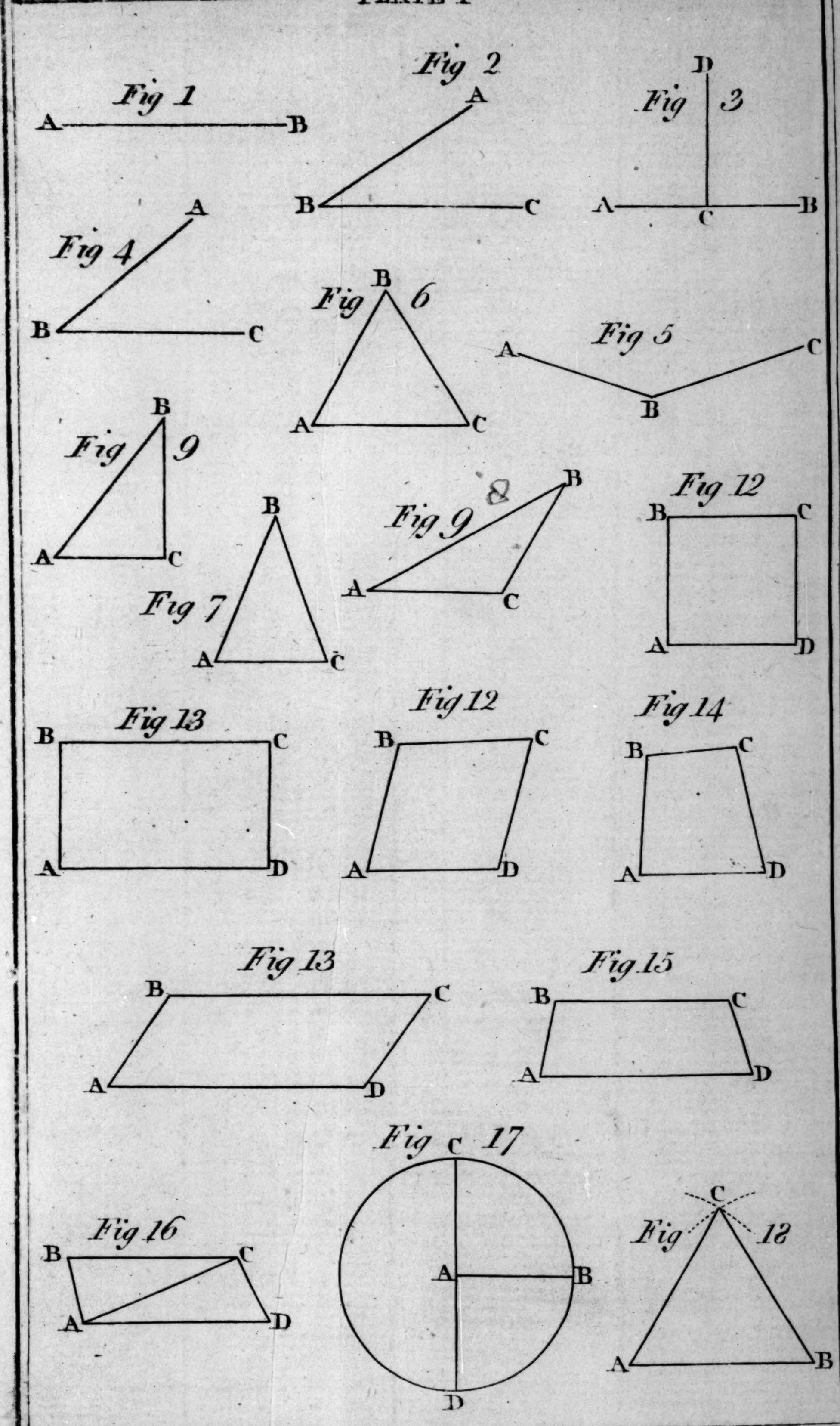

Fig 1
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Fig 2
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Fig 3
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Fig 4
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Fig 5
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Fig 9
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Fig 7
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Fig 9
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Fig 12
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Fig 13
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Fig 12
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Fig 14
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Fig 13
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Fig 15
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Fig 17
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Fig 18
C
A
B

PLATE II

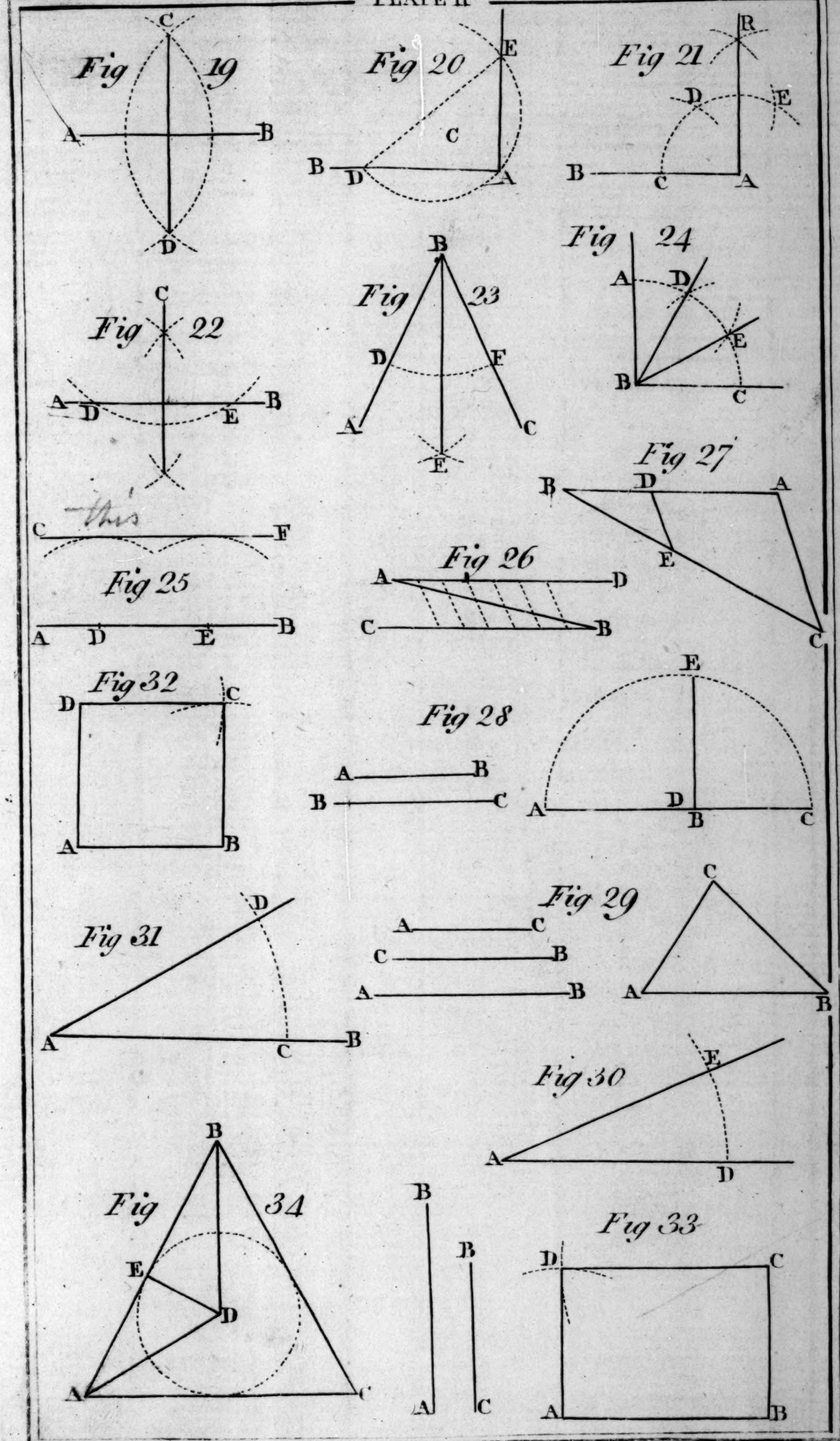
Fig 19
Fig 20
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Fig 33
Fig 34

PLATE III

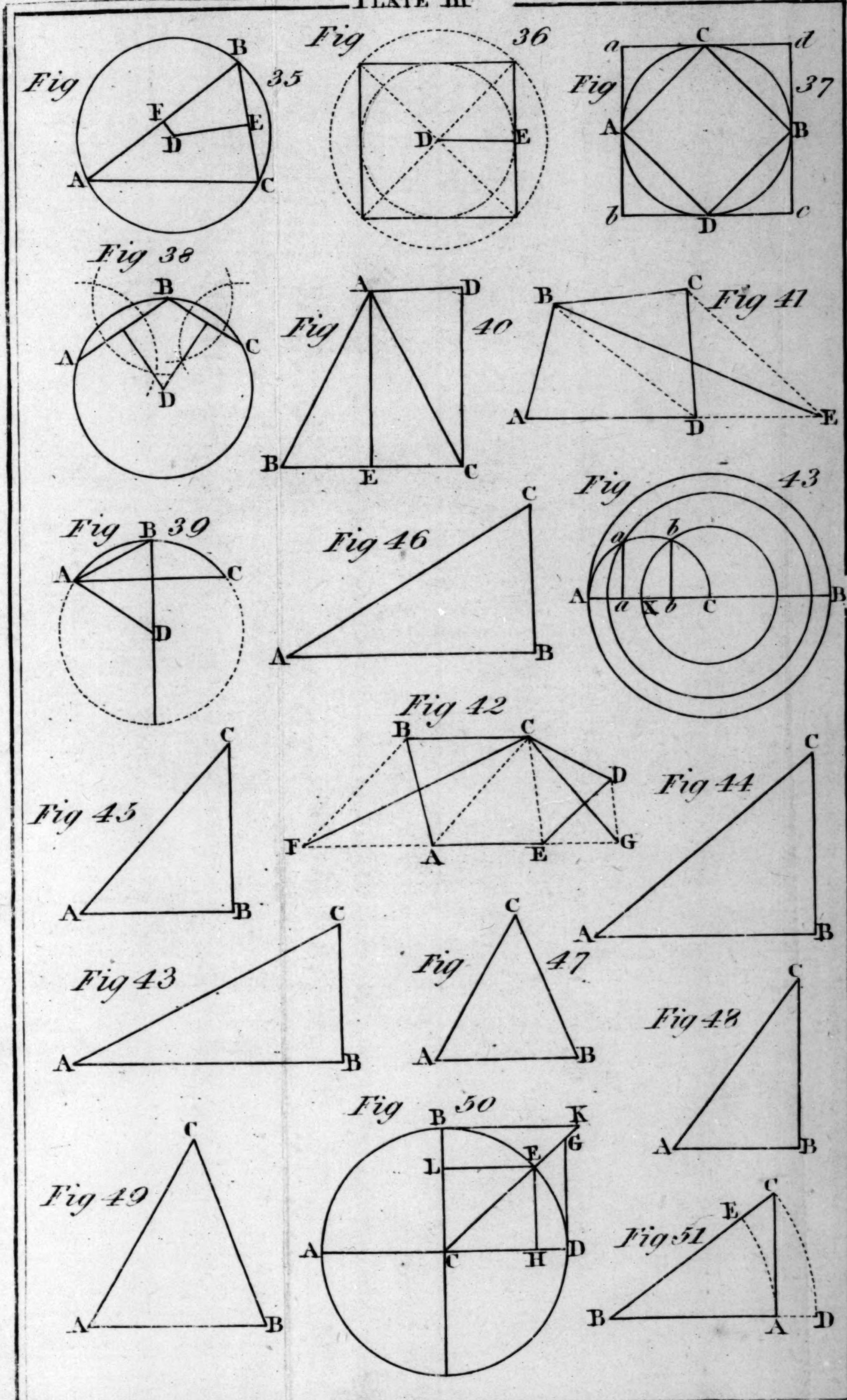
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Fig 49
Fig 51

PLATE IV

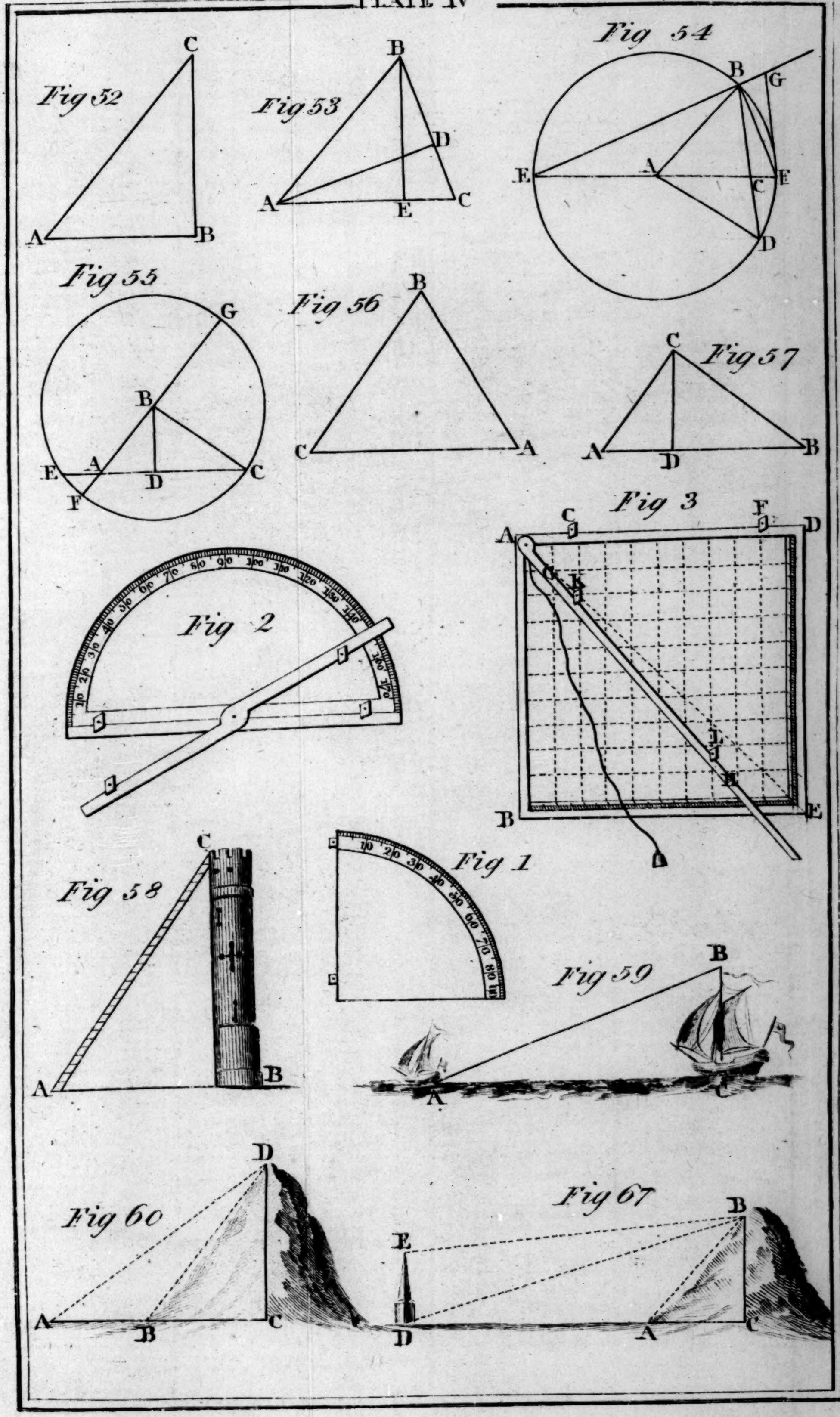
Fig 52
Fig 53
Fig 54
Fig 55
Fig 56
Fig 57
Fig 3
Fig 2
Fig 58
Fig 1
Fig 59
Fig 60
Fig 67

PLATE V.

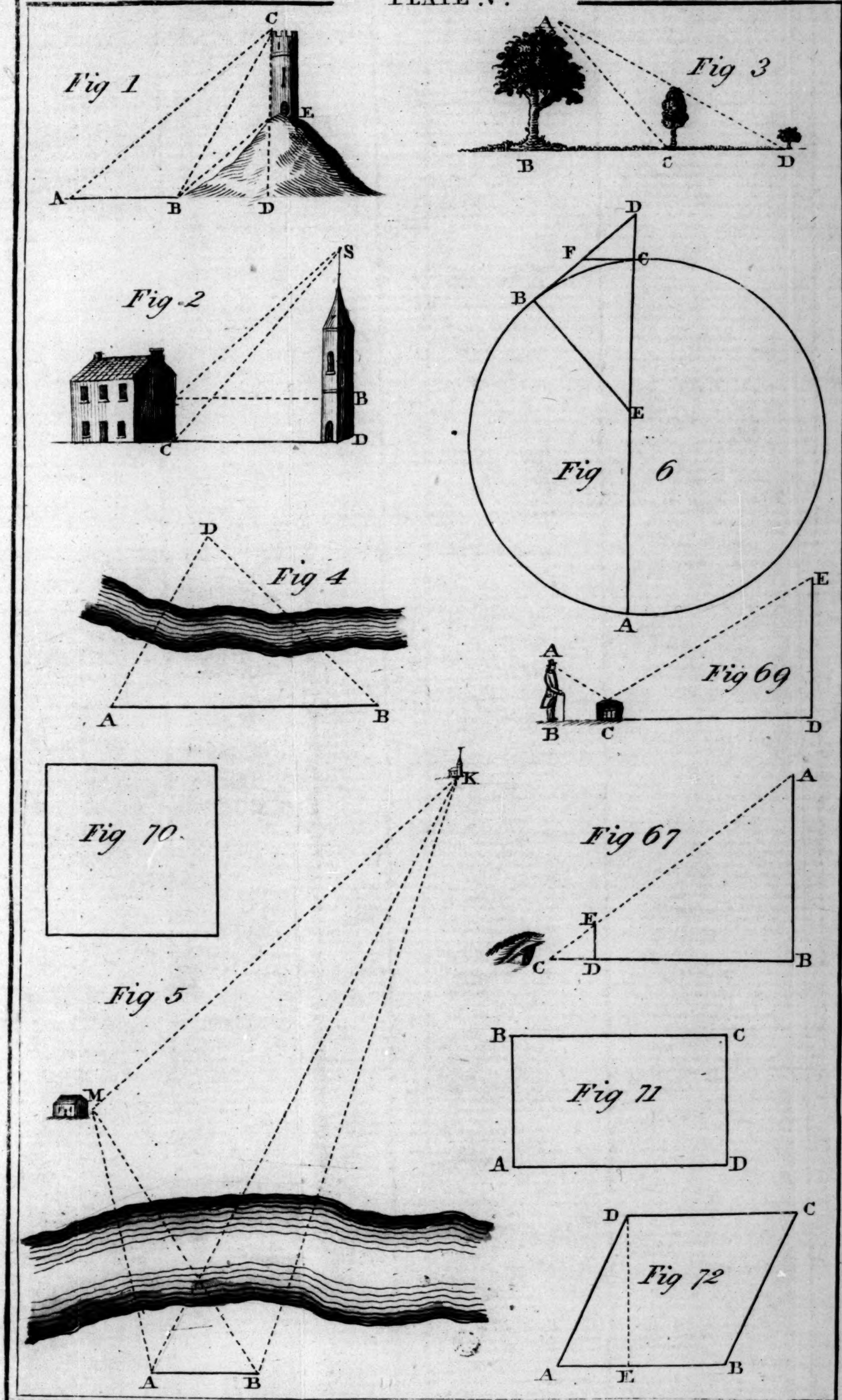
Fig 1
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Fig 3
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Fig 2
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Fig 6
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B
E
A
Fig 4
D
A
B
Fig 69
E
A
B
C
D
Fig 70
K
Fig 67
A
E
C
D
B
Fig 5
M
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B
Fig 71
B
C
A
D
Fig 72
D
C
A
E
B

PLATE VI

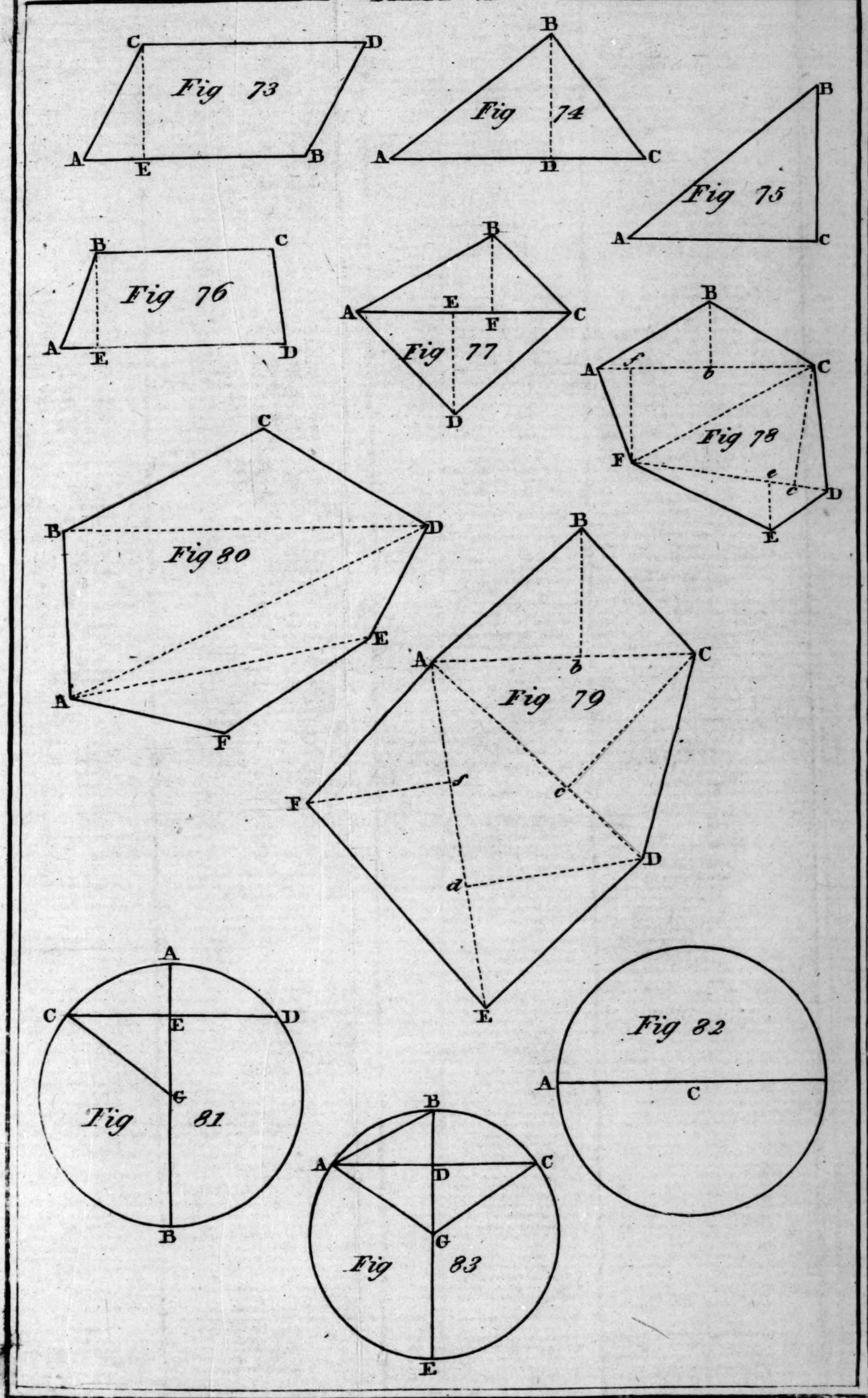
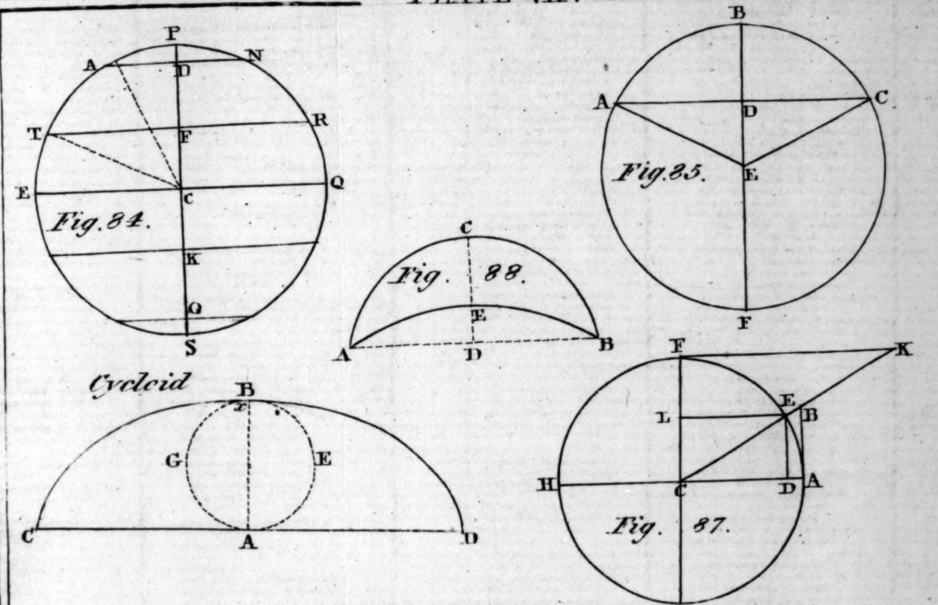


Fig 73
C D A E B
Fig 74
B A D C
Fig 75
B A C
Fig 76
B C A E D
Fig 77
B A E F C D
Fig 78
B A f b C F e c D E
Fig 80
C B D E A F
Fig 79
B A b C f c F d D E
Fig 81
A C E D G B
Fig 82
A C
Fig 83
B A D C G E



Mensuration of Solids.

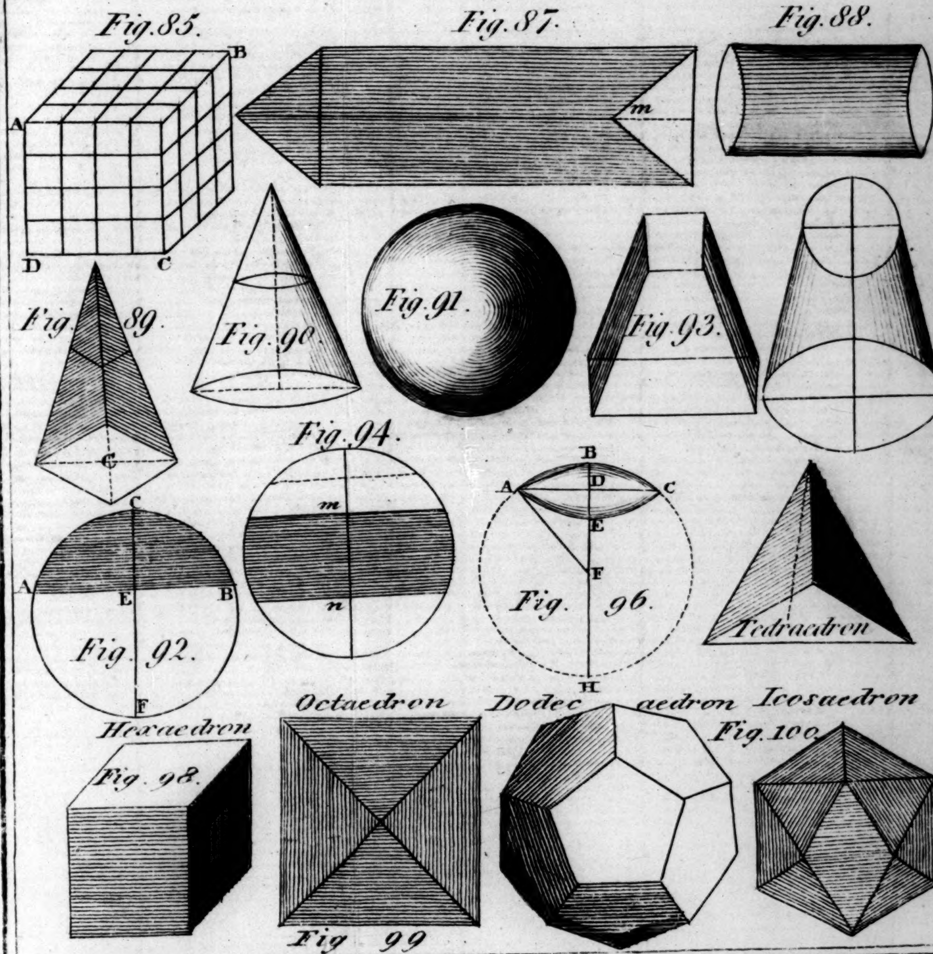


PLATE VIII.

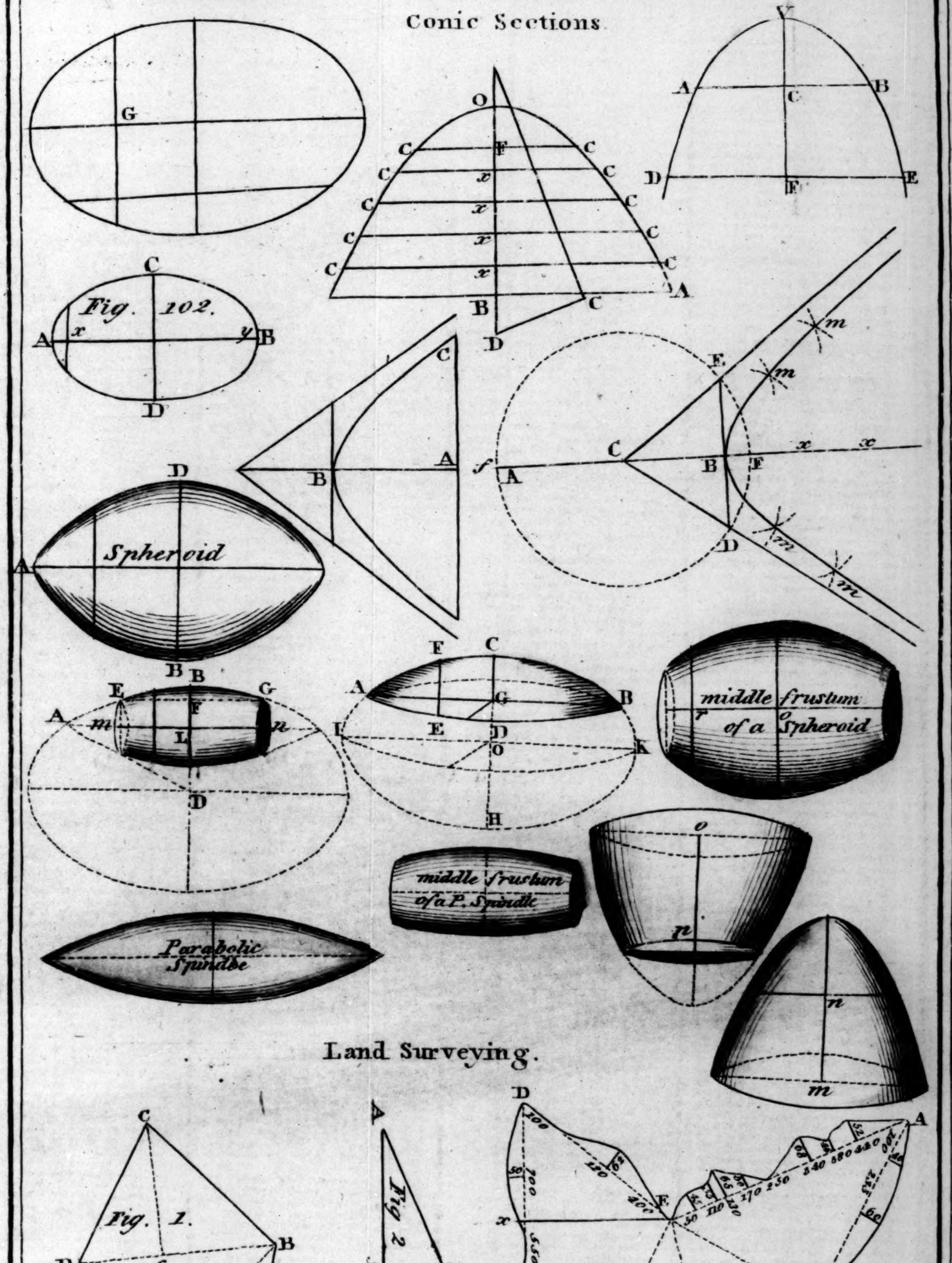
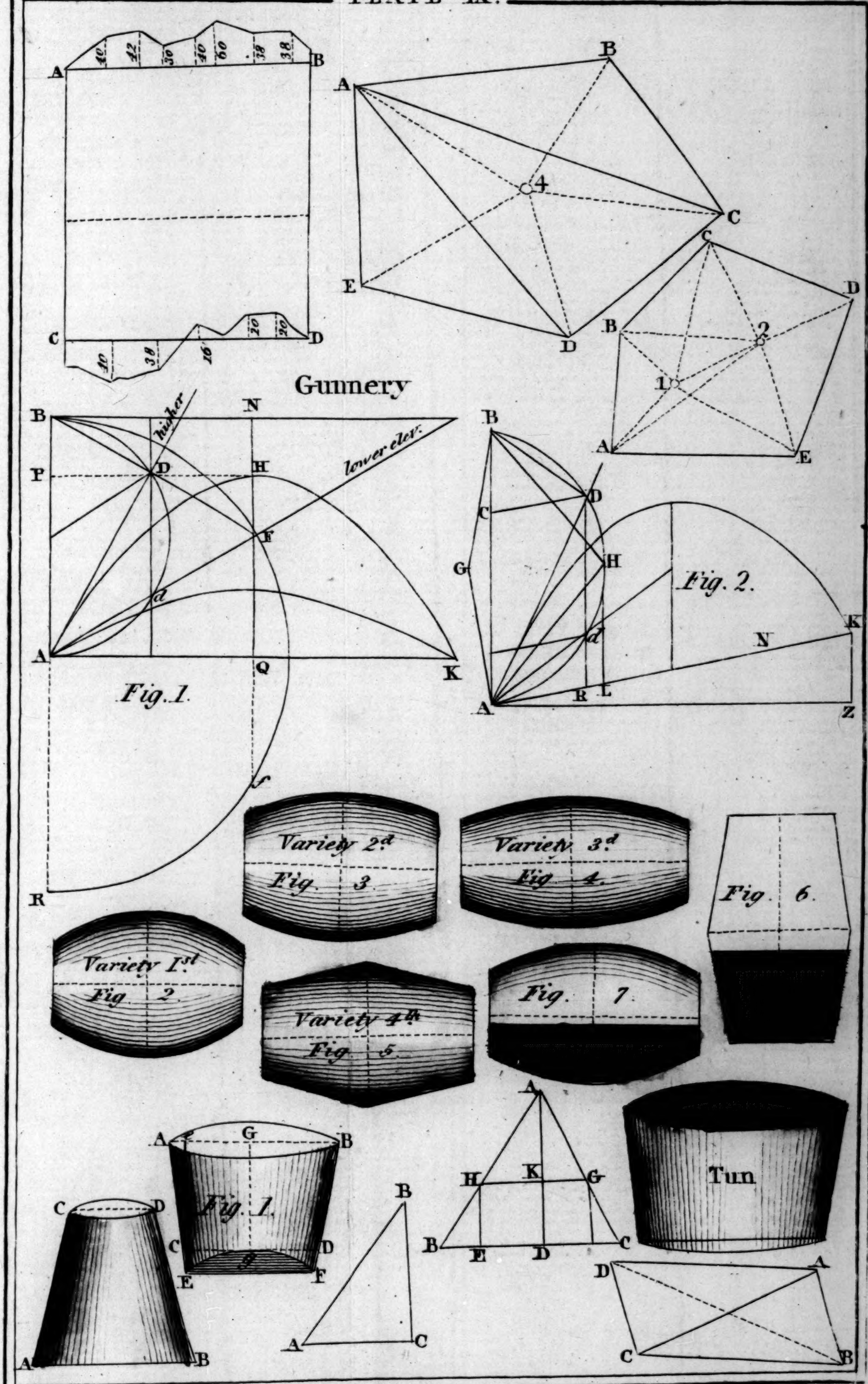
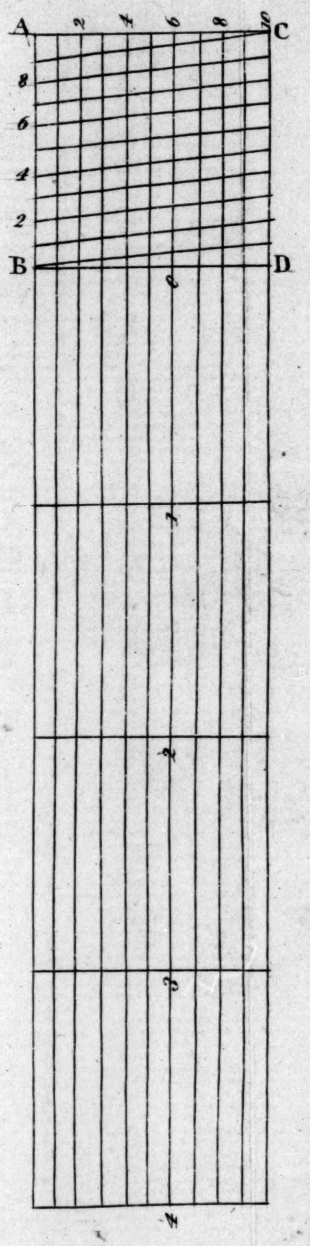

Conic Sections
Fig. 102.
Spheroid
middle frustum
of a Spheroid
middle frustum
of a P. Spindle
Parabolic
Spindle
Land Surveying.
Fig. 1.
Fig 2
Fig. 3.

PLATE IX.


Gunnery
higher
lower elev.
Fig. 1.
Fig. 2.
Variety 2d Fig. 3
Variety 3d Fig. 4.
Fig. 6.
Variety 1st Fig. 2
Variety 4th Fig. 5
Fig. 7
Fig. 1.
Tun


A
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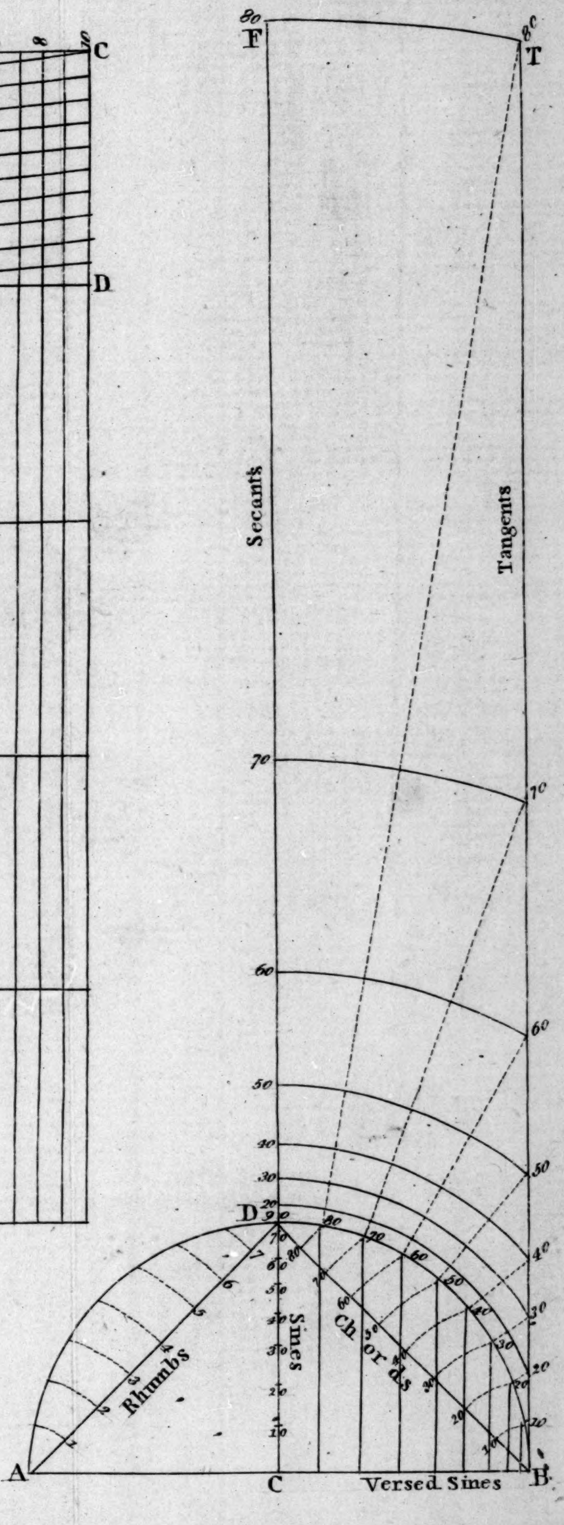
F
T
Secants
Tangents
D
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Rhumbs
Sines
Chords
Versed Sines